

Economic Load Dispatch Model Optimization using Fuzzy Interactive Optimization Technique

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Abstract— Environmental awareness and the recent environment protecting policies have extremely spurred many electric utilities to regulate their practices to account for the emission impacts. This paper proposes a new fuzzy multi-objective optimization approach to solve a multi-objective nonlinear programming problem in context of a power system optimization model. We have been developed a multi-objective power system optimization model in fuzzy environment. Here, the objectives are (i) to minimize the fuel cost and (ii) to minimize the emission level simultaneously while satisfying the power balance and generation limit constraints. In this model, the design variables are the generator power output. The model is tackled through interactive fuzzy optimization technique and the approach is tested on a system of 3-generators with transmission losses. The empirical results demonstrate the effectiveness of the proposed method.

Keywords—*Economic and emission dispatch, Multi-objective problem, Fuzzy interactive method*

I. INTRODUCTION

In these present days the world is more concerned about the environmental issues. Natural disaster like tsunami, hurricanes, etc. is because of global warming. The main reason for global warming is the emission of greenhouse gasses from the fossil fuel fire plants. This greenhouse gases include carbon (COX), nitrogen (NOX) and sulphur (SOX) components. Emission dispatch is to reduce the emission levels from the thermal plants. The power system scheduling is one of the most important problems in planning and management decision. The main objective of economic power dispatch problem is to determine the optimal combination of power outputs for all generating units, which minimizes the total fuel cost and minimize the total emission while satisfying load demand and operating constants of a power system. Various techniques have been proposed to solve this problem. Nanda et al. [1] were one of the first approaches to solve the Economic environmental dispatch problem considering multi-objective optimization using linear and non-linear goal programming techniques. Yokoyama et al. [2] were used ϵ -constrained method to minimize cost as well as minimize emission level. In 2009, Xuebin [3] had been discussed about the study of multi-objective optimization and multi-attribute decision making for economic and environmental power dispatch

model. Feng et al.[4] optimized load dispatch model based on fuzzy multi-objective optimization.

To find the solution(s) of multi objective problems, it requires the participation of a human Decision Maker (DM), who is suppose to have better insight of the problem and can express preference relations between alternative solutions. In literature, four classes are available according to the role of the DM in the solution processes. In this study, an interactive method in which the DM specifies preference information progressively during the solution processed is used. Interactive methods are widely used in literature. A brief survey is given in [5]. Recently, an interactive method for engineering design problems has been proposed in [6]. In this method, all objective functions and constrains are considered as fuzzy goals. Moreover, solutions are obtained using different combination of weights for objectives and constraints.

This paper is organized as follows: In section II, economic load dispatch model formulation is stated. In section III, multi-objective economic dispatch model is stated, In section IV, preliminaries, In section V, discuss about fuzzy interactive non-linear programming technique to solve multi-objective non-linear programming problem. In section VI, discuss about computational algorithm for multi-objective

economic emission problem using fuzzy Interactive optimization technique. In section VII, Numerical illustration and finally in the section VII, conclusions are drawn.

II. ECONOMIC LOAD DISPATCH MODEL FORMULATION

The purpose of ELD model is to estimate the optimal amount of the generated power by minimizing the both objectives, fuel cost (economy) and emission levels simultaneously while satisfying load demand and operational constraints.

A. Economy objectives:

The fuel cost of a thermal unit is regarded as an essential criterion for economic feasibility. The fuel cost curve is assumed to be approximated by a quadratic function of generator power output as

$$Cost(P) = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) \quad (1)$$

where a_i, b_i and c_i are the coefficients of the i^{th} generator, P_i is the generated power of i^{th} power plants and n is the number of generators.

B. Environmental objectives:

The emission cost can be directly related to the cost curve through the emission rate per Mkal, which is a constant factor for a given type of fuel. Therefore, the amount of emission is given as a quadratic function of generator output as

$$Em(P) = \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \quad (2)$$

where α_i, β_i and γ_i are emission coefficients of the i^{th} generator and n is the number of generators.

C. Constraints in Economic Operation Of Power System

a. Power balance constraints:

The total generated power should be equal to the sum of total load demand and line loss. It can be formulated as

$$\sum_{i=1}^n P_i = (P_D + P_L) \quad (3)$$

where, P_D is the power demand $P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j$

Where P_D and P_L are the total system demand and line loss respectively and B_{ij} is the loss coefficients.

b. Inequality constraints:

Generating output of each generating unit should lie between the maximum and minimum limits as given in

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (i=1, 2, 3, \dots, n) \quad (4)$$

where P_i is the output power of i^{th} generator and P_i^{\min} and P_i^{\max} are the minimum and maximum generated power of i^{th} generator respectively.

III. MULTI-OBJECTIVE ECONOMIC DISPATCH MODEL

In the design of the optimal dispatch model i.e. minimum fuel cost of generation and minimum amount of emission generated. To dispatch system the basic constraints (including power balance equations, minimum and maximum amount of power that can be generated by a plant, etc.) are known and the optimization's target is to identify the optimal power generated by the plants so that the cost of generation and amount of emission emitted are minimum, in a given load conditions.

The multi-objective economic-emission dispatch model can be expressed as:

$$\begin{aligned} &\text{Minimize Cost}(P_i) \\ &\text{Minimize Em}(P_i) \end{aligned} \quad (5)$$

$$\text{Subject to } \sum_{i=1}^n P_i - (P_D + P_L) = 0$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}$$

$$Cost(P_i) = \sum_{i=1}^n a_i P_i^2 + b_i P_i + c_i$$

$$Em(P_i) = \sum_{i=1}^n \alpha_i P_i^2 + \beta_i P_i + \gamma_i$$

where, $P = [P_1, P_2, \dots, P_n]^T$ are power generated, n is the no. of power plants, $Cost(P_i)$ is the cost of power generation, $Em(P_i)$ is the emission function, P_i, P_L and P_D is the power generated, transmission losses and power demand respectively. P_{\min} and P_{\max} is the minimum and maximum power generation from plant respectively.

IV. MATHEMATICAL PRELIMINARIES

A. Fuzzy Set:

Let X is a set (space), with a generic element of X denoted by x , that is $X(x)$. then a fuzzy set(FS) is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is the membership function of FS \tilde{A} . $\mu_{\tilde{A}}(x)$ is the degree of membership of the element x to the set \tilde{A} .

B. Alpha cut of a fuzzy set:

The α -level set of the fuzzy set \tilde{A} of X is a crisp set A_{α} that contains all the elements of X that have membership values greater than or equal to α i.e. $\tilde{A} = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0,1]\}$.

V. MATHEMATICAL ANALYSIS

A. Fuzzy Interactive Non-linear Programming Technique to Solve Multi-Objective Non-linear Programming Problem

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$\text{Min } f(x) = [f_1(x), f_2(x), \dots, f_k(x)]^T \tag{6}$$

Subject to

$$x \in X = \{x \in \mathbb{R}^n : g_j(x) \leq \text{or } = \text{or } \geq b_j \text{ for } j = 1, 2, 3, \dots, m\}$$

and $l_r \leq x_r \leq u_r$ ($r = 1, 2, 3, \dots, n$).

Zimmermann [8] showed that fuzzy programming techniques can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

Step 1: Solve the MONLP(6) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2: From the result of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{matrix} & f_1(x) & f_2(x) & \dots & f_k(x) \\ \begin{matrix} x^1 \\ x^2 \\ \dots \\ x^m \end{matrix} & \begin{pmatrix} f_1^*(x^1) & f_2(x^1) & \dots & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \dots & f_k(x^2) \\ \dots & \dots & \dots & \dots \\ f_1(x^m) & f_2(x^m) & \dots & f_k^*(x^m) \end{pmatrix} \end{matrix}$$

Here $x^1, x^2, x^3, \dots, x^m$ are the ideal solutions of the objective $f_1(x), f_2(x), f_3(x), \dots, f_k(x)$ respectively. The maximum value of each column U_r gives upper bound or upper tolerance or highest acceptable level of achievement for the i^{th} objective function $f_i(x)$, where $U_i = \max \{f_i(x^1), f_i(x^2), \dots, f_i(x^m)\}$ and the minimum value of each column L_i gives lower tolerance or aspired level of achievement for the i^{th} objective function $f_i(x)$ where $L_i = \min \{f_i(x^1), f_i(x^2), \dots, f_i(x^m)\}$ for $i = 1, 2, 3, \dots, k$.

Step 3: Using aspiration level of each objective of the MONLP (6) may be written as follows:

Find x so as to satisfy

$$f_i(x) \lesseqgtr L_i \text{ with tolerance } P_i = (U_i - L_i) \text{ for } i = 1, 2, 3, \dots, k$$

$$x \in X. \quad l_r \leq x_r \leq u_r \quad (r = 1, 2, 3, \dots, n)$$

Define a membership function $\mu_i(f_i(x))$

$$\mu_{f_i}(f_i(x)) = \begin{cases} 1, & \text{if } f_i(x) \leq m_i, \\ \frac{M_i - f_i(x)}{M_i - m_i}, & \text{if } m_i \leq f_i(x) \leq M_i \\ 0, & \text{if } f_i(x) \geq M_i, \end{cases}$$

Where $L_i \leq m_i \leq M_i \leq U_i$. The value of m_i and M_i for objective are chosen by DM on the basis of his/her knowledge of problem.

Having elicited the membership function $\mu_{f_i}(f_i(x))$ for $i = 1, 2, \dots, k$, introduce a general aggregation function

$$\mu_D(x) = F(\mu_{f_1}(f_1(x)), \mu_{f_2}(f_2(x)), \dots, \mu_{f_k}(f_k(x)))$$

So a fuzzy multi-objective decision making problem can be defined as

$$\begin{aligned} & \text{Maximize } (\mu_D(x)) \\ & \text{subject to } x \in X \end{aligned} \tag{7}$$

According to Bellman & Zadeh [7] and Zimmermann [8] using max product operator, the problem (7) is reduced to

$$\begin{aligned} & \text{Max } \prod_i \mu_{f_i}(f_i(x)), i = 1, 2, \dots, k \\ & \text{subject to} \end{aligned} \tag{8}$$

$$x \in X. \quad l_r \leq x_r \leq u_r \quad (r = 1, 2, 3, \dots, n)$$

where μ_{f_i} is membership function of i^{th} objective function.

DM opinion at each interactive phase, is incorporated as in [9], which is based on reference level. Aspiration levels are used as reference levels in [9] and [10]. However, we have used minimum reservation levels for reference levels. It incorporates as constraint for each objective, to make sure that the minimum reservation level satisfies, in above formulation. Therefore, equation (8) becomes:

$$\begin{aligned} & \text{Max } \prod_i \mu_{f_i}(f_i(x)), i = 1, 2, \dots, k \\ & \text{subject to} \\ & \mu_{f_i}(f_i(x)) - \bar{\mu}_{f_i}(f_i(x)) \geq 0, i = 1, 2, \dots, k \\ & x \in X. \quad l_r \leq x_r \leq u_r \quad (r = 1, 2, 3, \dots, n) \end{aligned} \tag{9}$$

Where $\bar{\mu}_{f_i}$ is minimum threshold level, between 0 and 1, for i^{th} objective. At each interactive phase DM can change his/her reservation levels, on the basis of outcome of previous iteration, for some or all objective functions. The process is repeated till DM is satisfied. The Pareto optimal solution, which is expected to satisfy DM, may be obtained by solving the problem (9). Where it is assumed that fuzzy goal of each objective have equal importance to the DM.

Step 4: The above non-linear programming problem (9) can be easily solve an appropriate mathematical programming algorithm.

Definition: A decision vector $X^* = (x_1, x_2, \dots, x_m)$ in feasible region is Pareto optimal if there does not exist another vector X in feasible region such that $f_i(X) \leq f_i(X^*)$ for all $i = 1, 2, \dots, k$ and $f_j(X) < f_j(X^*)$ for at least one index j .

VI. COMPUTATIONAL ALGORITHM FOR MULTI-OBJECTIVE ECONOMIC EMISSION PROBLEM USING FUZZY INTERACTIVE OPTIMIZATION TECHNIQUE

Step 1: Taking the first objective function from set of objectives of the problem (5) and solve as a single objective subject to the given constraints. Find the value of objective functions and decision variables.

Step 2: Repeat the Step 1 for remaining objective functions. After that according to step 2 pay-off matrix formulated as follows:

$$\begin{matrix} & Cost(P_i) & Em(P_i) \\ P_i^1 & \left(\begin{matrix} Cost^*(P_i^1) & Em^*(P_i^1) \end{matrix} \right) \\ P_i^2 & \left(\begin{matrix} Cost^*(P_i^2) & Em^*(P_i^2) \end{matrix} \right) \end{matrix}$$

The bounds are $U_1 = \max\{Cost(P_i^*), Cost(P_i^{2*})\}$, $L_1 = \min\{Cost(P_i^*), Cost(P_i^{2*})\}$ for cost function ($L_1 \leq Cost(P_i) \leq U_1$) and the bounds of objective are $U_2 = \max\{Em(P_i^*), Em(P_i^{2*})\}$, $L_2 = \min\{Em(P_i^*), Em(P_i^{2*})\}$ for emission function $Em(P_i)$ (where $L_2 \leq Em(P_i) \leq U_2$) are identified.

Use following linear membership function $\mu_F(Cost(P_i))$ and $\mu_{Em}(Em(P_i))$ for the objective functions $Cost(P_i)$ and $Em(P_i)$ respectively are defined as follows:

$$\mu_{Cost}(Cost(P_i)) = \begin{cases} 1 & \text{if } Cost(P_i) \leq m_1 \\ \frac{M_1 - Cost(P_i)}{M_1 - m_1} & \text{if } (m_1 \leq Cost(P_i) \leq M_1) \\ 0 & \text{if } Cost(P_i) \geq M_1 \end{cases}$$

$$\mu_{Em}(Em(P_i)) = \begin{cases} 1 & \text{if } Em(P_i) \leq m_2 \\ \frac{M_2 - Em(P_i)}{M_2 - m_2} & \text{if } (m_2 \leq Em(P_i) \leq M_2) \\ 0 & \text{if } Em(P_i) \geq M_2 \end{cases}$$

where $L_i \leq m_i \leq M_i \leq U_i, i = 1, 2, \dots, n$. The value of m_i and M_i for objective are chosen by DM on the basis of his/her knowledge of problem.

Step 3: Having elicited the above membership functions crisp non-linear programming problem is formulated as follows:

Using max-product operator:

$$\max \left[\left(\mu_{Cost}(Cost(P_i)) \times \mu_{Em}(Em(P_i)) \right) \right]$$

$$\mu_{Cost}(Cost(P_i)) - \bar{\mu}_1 \geq 0, \quad i = 1, 2, 3, \dots, n \tag{10}$$

$$\mu_{Em}(Em(P_i)) - \bar{\mu}_2 \geq 0, \quad i = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^n P_i = P_D + P_L; \quad P_i^{\min} \leq P_i \leq P_i^{\max}$$

$$Cost(P_i) = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i)$$

$$Em(P_i) = \sum_{i=1}^n (\alpha_i P_i^2 + \beta_i P_i + \gamma_i)$$

Step 4: The above non-linear programming problem (10) can be easily solve an appropriate mathematical programming algorithm.

VII. NUMERICAL ILLUSTRATION

In this section a system consisting of 3 thermal units is considered. The cost coefficient, emission coefficient and generating limits of the 3-generator system are given as in table-1 and table-2 : $P_D = 700$

B- Coefficients of 3-generator system are as follows

$$B_{ij} = \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix}$$

where B_{0i} and B_{00} are considered as zero.

Solution: According to step 2 pay off matrix is formulated as follows:

$$\begin{matrix} & Cost(P) & Em(P) \\ P^1 & \left(\begin{matrix} 35424.44 & 660.7492 \end{matrix} \right) \\ P^2 & \left(\begin{matrix} 35473.32 & 651.4851 \end{matrix} \right) \end{matrix}$$

Here $U_1 = 35473.32$, $L_1 = 35424.44$, $U_2 = 660.7492$, $L_2 = 651.4851$.

$L_1 \leq m_1 \leq Cost(P) \leq M_1 \leq U_1$, $L_2 \leq m_2 \leq Em(P) \leq M_2 \leq U_2$
 $L_1 \leq m_1 \leq M_1 \leq U_1$, $L_2 \leq m_2 \leq M_2 \leq U_2$ and we take $M_1 = 35460, m_1 = 35425$ and $M_2 = 659, m_2 = 651.5$

Considering the Pareto optimal solutions of the MOSOP (1) with different weights by GFNLP method based on different operator is given in table 3.

Table 1: Input data for cost coefficient, generating limits of the 3-generator system

Unit	a_i	b_i	c_i	P^{\min}	P^{\max}
1	0.03546	38.30553	1243.53110	35	210

2	0.02111	36.32782	1658.56960	130	325
3	0.01799	38.27041	1356.65920	125	315

Table 2: Input data for emission coefficient of the 3-generator system

Unit	α_i	β_i	γ_i
1	0.00683	-0.54551	40.26690
2	0.00461	-0.51160	42.89553
3	0.00461	-0.51160	42.89553

Table 3. Pareto optimal solutions of MOSOP based on max-product operator with different weights

Iteration	I	II	III	IV	V	VI
$\bar{\mu}_1$	0.3	0.7	0.8	0.3	0.3	0.4
$\bar{\mu}_2$	0.3	0.3	0.3	0.7	0.8	0.4
μ_1	0.6950462	0.7000000	0.8000000	0.6591206	0.5205426	0.6950462
μ_2	0.6672674	0.6624760	0.5474055	0.7000000	0.8000000	0.6672674
P_1	169.4666	169.3508	166.7805	170.2812	173.0916	169.4666
P_2	279.7721	279.8467	281.5012	279.2472	277.4345	279.7721
P_3	274.3008	274.3437	275.2964	273.9996	272.9631	274.3008
$Cost(P)$	35435.67	35435.50	35432.00	35436.93	35441.78	35435.67
$Em(P)$	653.9955	654.0314	654.8945	653.7500	653.00	653.9955

VIII. CONCLUSION

In this paper, an interactive method is introduced for multi-objective economic emission dispatch problem in which the product operator is used to aggregate the different fuzzy goals of objectives. Economic emission dispatch problem application of the proposed interactive approach shows that it is able to find Pareto optimal solution as close as the desire of decision maker. Change in reservation levels at different interactive phase give trade-off between different objectives, which is helpful for DM to obtain desire solution. This procedure is able to find the Pareto optimal solution for any number of objective functions in the problem.

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