

Two Bar Truss Optimization using Fuzzy Posynomial Geometric Programming Technique

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Abstract— This paper presents a method for solving posynomial geometric programming with fuzzy coefficients in a context of structural design model. We have been developed a two bar truss design model in fuzzy environment. By utilizing comparison of fuzzy numbers with different approaching method, the programming with fuzzy coefficients is reduced to the programming with constant coefficient. Then we can solve the two bar truss problem with fuzzy coefficients using a method to posynomial geometric programming. Finally, one comparative example is used to illustrate the advantage of the new method.

Keywords— Fuzzy posynomial geometric programming, Yager's method, A new approach for ranking of trapezoidal fuzzy numbers.

I. INTRODUCTION

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. Decision making that is solely depends on a single criterion appears insufficient when the decision-making process deals with the complex environment. So, one must acknowledge the presence of several criteria that lead to the development of multi-criteria decision making.

Structural Optimization is a kind of decision making, in which decision have to be made to optimize one or more objectives under a prescribed set of circumstances. These structural problems may be single or multi-objective, and are to be optimized (maximized or minimized) under a specified set of constraints. The main objective of structural engineering is to design structures which can withstand external loads safely and at a minimum cost or weight. In practical, the problem of structural design may be formed as a typical non-linear programming problem with non-linear objective functions and constraints functions in fuzzy environment. Some researchers have applied the fuzzy set theory for Structural model. For example, Wang et al. [12] first applied α -cut method to structural designs where the non-linear problems were solved with various design levels α , and then a sequence of solutions are obtained by setting different level-cut value of α . Rao et al.[5,6] showed significant work in multi-objective structural optimization with uncertain parameters. Rao [4] applied the same α -cut method to design a four-bar mechanism for function

generating problem .Structural optimization with fuzzy parameters was developed by Yeh et.al [14]. In 1989, Xu [13] used two-phase method for fuzzy optimization of structures. In 2004, Shih et.al [7] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources .Shih et.al [8] developed an alternative α -level-cuts methods for optimum structural design with fuzzy resources in 2003. A generalized fuzzy number has been used Dey et al.[1] in context of a non-linear structural design optimization. Dey et al. [2] used basic t-norm based fuzzy optimization technique for optimization of structure and Dey et al.[3] developed parameterized t-norm based fuzzy optimization method for optimum structural design.

Geometric programming problems (GPs) have a wide range of applications in production planning, location, distribution, risk management, chemical process designs and other engineering design situations, etc. Since the late 60s, the GP has been known. Duffin, Peterson and Zener [16] in 1967 discussed basic GP theories with engineering applications in their books. Another famous book on the GP and its application appeared in 1976 (Beightler and Philip[15]). In 1987, Cao [11] introduced widespread geometric programming for the first time. There is a good book on the widespread geometric programming of Cao [9] which was the most recent until now. GP is an effective method to solve a particular type of nonlinear programming problem. So it can be applied to optimize nonlinear structural design.

The classification (ranking) of fuzzy numbers plays an important role in making language decisions and some other widespread application systems, such as management, operational research, etc. Many methods have been proposed to treat the classification of fuzzy numbers. Recently, Abbasbandy and Hajjari [17] propose a new approach for the classification of trapezoidal fuzzy numbers. Now, the use of this method to solve two bar truss structural design is proposed in this document.

This paper is organized as follows: In section II, two bar truss mathematical model is discussed. In Section III, we have discussed some mathematical preliminaries. In section IV and section V, we present a review of the Yager’s Method [18] and a New Approach Method to classify the trapezoidal fuzzy numbers. In section VI, A New Approach to FPGP by Ranking of Trapezoidal Fuzzy Numbers is discussed with example respectively. In Section VII, Solution of Fuzzy Two Bar Truss Mathematical Model using Fuzzy Posynomial Geometric Programming Technique is stated and we explain it with an illustrative example in Section VIII. The document ends with the conclusions in Section IX.

II. MATHEMATICAL MODEL FORMULATION OF TWO BAR TRUSS

Two bar truss model is developed and work out under the following notations.

- We define the following variables and parameters;
- 2P = applied load;
- t = thickness of the bar;
- d = mean diameter of the bar (decision variable);
- 2b = the distance between the truss support.
- WT= weight of the structure;
- h = height of the system.
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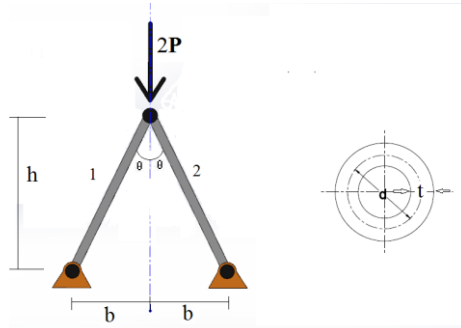


Fig-1 Two bar truss under load

The weight of the structure is $WT = 2\rho AL$, where ρ is the material density of the bar, A is the cross-sectional area and L is the length of the element. The cross-sectional area can be found by the equation

$$A = \frac{\pi(d_0^2 - d_1^2)}{4} = \frac{\pi((d+t)^2 - (d-t)^2)}{4} = \pi dt,$$

The inner and outer diameters of the bar can be found by $d_0 = d + t$, $d_1 = d - t$, where d is the average diameter of the tube. By inspection of Fig-1, the simple geometrical Pythagorean’s theorem may be utilized to find the length of each bar $L = \sqrt{b^2 + h^2}$.

Therefore, weight becomes $WT = 2\rho\pi dt\sqrt{b^2 + h^2}$

Let P_1 and P_2 be the reaction forces along the bar 1 and bar 2 respectively. Considering the equilibrium condition at loading point, the following equations are obtained $P_1 \cos \theta + P_2 \cos \theta = 2P$

$$P_1 \sin\left(\frac{\pi}{2} - \theta\right) + P_2 \sin\left(\frac{\pi}{2} - \theta\right) = 0$$

Solving above two equations we get,

$P_1 = P_2$ and the axial force on each bar is

$$P_1 = P_2 = P_A = \frac{P\sqrt{b^2 + h^2}}{h} \text{ as } \cos \theta = \frac{h}{\sqrt{h^2 + b^2}}$$

The stress on each bar is $\sigma = \frac{P_A}{A} = \frac{P\sqrt{b^2 + h^2}}{d\pi t}$,

Then the crisp single objective two bar truss mathematical model is formed with weight as objective function subject to stress constraints as follows

$$\text{Minimize } WT(d, h, y) = 2\rho\pi d y$$

$$\begin{aligned} \text{Subject to } & \frac{Pyh^{-1}}{d\pi t} \leq \sigma_0; \\ & b^2 y^{-2} + h^2 y^{-2} \leq 1; \\ & d, h, y > 0; \end{aligned} \tag{1}$$

A. FUZZY MATHEMATICAL MODEL OF TWO BAR TRUSS

In the traditional structural model, we have seen that, the density of the metal is constant. In practice, many cases, thickness of the truss bar, applied force and stresses (compressive and tensile) are flexible in nature. For this reason, we shall fuzzify those. Then the crisp model (1) is transformed into fuzzy model as follows

$$\text{Minimize } WT(d, h, y) = 2\rho\tilde{d}\pi y$$

$$\begin{aligned} \text{Subject to } & \frac{\tilde{P}yh^{-1}}{d\tilde{t}} \leq \tilde{\sigma}_0; \\ & b^2 y^{-2} + h^2 y^{-2} \leq 1; \\ & d, h, y > 0; \end{aligned} \tag{2}$$

where \tilde{P}, \tilde{t} and $\tilde{\sigma}_0$ are fuzzy in nature.

III. PRILIMINARIES

Definition: A fuzzy number is a fuzzy set like $u : R \rightarrow I = [0,1]$ satisfying:

1. u is upper semi-continuous,
2. $u(x) = 0$ outside some interval $[a,b]$,
3. There are real numbers $a; b$ such that $a \leq b \leq c \leq d$ and
 - a. $u(x)$ is monotonically increases on $[a,b]$;
 - b. $u(x)$ is monotonically decreases on $[c,d]$;
 - c. $u(x) = 1; b \leq x \leq c$.

The membership function u can be expressed as

$$u(x) = \begin{cases} u_L(x), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ u_R(x), & c \leq x \leq d \\ 0, & otherwise, \end{cases}$$

Where $u_L : [a,b] \rightarrow [0,1]$ and $u_R : [c,d] \rightarrow [0,1]$ are left and right membership functions of fuzzy number u . An equivalent parametric form is also given as follows.

Definition: A fuzzy number u in parametric form is a pair (\underline{u}, \bar{u}) of function $\underline{u}(r), \bar{u}(r), 0 \leq r \leq 1$ which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded monotonically increasing left continuous function;
2. $\bar{u}(r)$ is a bounded monotonically decreasing left continuous function;
3. $\underline{u}(r), \bar{u}(r), 0 \leq r \leq 1$.

The trapezoidal fuzzy number $u = (x_0, y_0, \delta, \beta)$ with two defuzzifier δ, β and left fuzziness $\delta > 0$ and right $\beta > 0$ is a fuzzy set where the membership function serves as

$$u(x) = \begin{cases} \frac{1}{\delta}(x - x_0 + \delta), & x_0 - \delta \leq x \leq x_0 \\ 1, & x \in [x_0, y_0] \\ \frac{1}{\beta}(y_0 - x + \beta), & y_0 \leq x \leq y_0 + \beta \\ 0, & otherwise \end{cases}$$

and its parametric form is $\underline{u}(r) = x_0 - \delta + \delta r, \bar{u}(r) = y_0 + \beta - \beta r$

Definition: Let $\tilde{A} \in F(X), \forall \alpha \in [0,1]$, written as $(\tilde{A})_\alpha = A_\alpha = \{x \in X, \tilde{A}(x) \geq \alpha\}$. A_α is said to be an α -cut set \tilde{A} . The lower and upper bounds of any α -cut set A_α are represented by $A_\alpha^L = \inf \{x \in X, \tilde{A}(x) \geq \alpha\}$ and $A_\alpha^U = \sup \{x \in X, \tilde{A}(x) \geq \alpha\}$ and we suppose that both are finite.

IV. YAGER'S METHOD FOR RANKING OF TRAPEZOIDAL FUZZY NUMBERS

A special version of the linear ranking function was first proposed by Yager [18] below:

$$R(\tilde{a}) = \frac{1}{2} \int_0^1 (A_\alpha^L + A_\alpha^U) d\alpha. \tag{3}$$

For trapezoidal fuzzy numbers $\tilde{a} = (x_0, y_0, \delta, \beta)$, we have:

$$R(\tilde{a}) = \frac{2x_0 + 2y_0 - \delta + \beta}{4}.$$

Therefore for any two trapezoidal fuzzy numbers \tilde{a} and \tilde{b} , we define the ranking of \tilde{a} and \tilde{b} as:

1. $R(\tilde{a}) \geq R(\tilde{b})$ If and only if $\tilde{a} \geq \tilde{b}$;
2. $R(\tilde{a}) > R(\tilde{b})$ If and only if $\tilde{a} > \tilde{b}$;
3. $R(\tilde{a}) = R(\tilde{b})$ If and only if $\tilde{a} = \tilde{b}$.

Remark

For tow arbitrary trapezoidal fuzzy numbers \tilde{a} and \tilde{b} , we have $R(k\tilde{a} + \tilde{b}) = kR(\tilde{a}) + R(\tilde{b})$.

V. A NEW APPROACH FOR RANKING OF TRAPEZOIDAL FUZZY NUMBERS

For an arbitrary trapezoidal fuzzy number $u = (x_0, y_0, \delta, \beta)$ with parametric form $u = (\underline{u}(r), \bar{u}(r))$, we defined the magnitude of the trapezoidal fuzzy number u as

$$\begin{aligned} \text{Mag}(u) &= \frac{1}{2} \left(\int_0^1 (\underline{u}(r) + \bar{u}(r) + x_0 + y_0) r dr \right), \\ &= \frac{1}{12} (6x_0 + 6y_0 - \delta + \beta). \end{aligned} \tag{4}$$

Therefore, for any two trapezoidal fuzzy numbers u and $v \in E$, we define the $\text{Mag}(\cdot)$ on E as a ranking of u and v :

1. $\text{Mag}(u) > \text{Mag}(v)$ if and only if $u > v$;
2. $\text{Mag}(u) < \text{Mag}(v)$ if and only if $u < v$;
3. $\text{Mag}(u) = \text{Mag}(v)$ if and only if $u = v$.

Remark 4.1 For two arbitrary trapezoidal fuzzy numbers u and v , we have

$$\text{Mag}(u + v) = \text{Mag}(u) + \text{Mag}(v).$$

Examples

Here, we take some illustrative example to show the ability of proposed method.

Let us consider three trapezoidal fuzzy numbers $A = (0.4, 0.6, 0.4, 0.2)$, $B = (0.5, 0.5, 0.3, 0.4)$ and $C = (0.6, 0.7, 0.5, 0.1)$

Table 1

TrFN	Yager's Method	Method of Magnitude
A	0.45	0.4833
B	0.525	0.5083
C	0.55	0.6167
Result	$A < B < C$	$A < B < C$

VI. A NEW APPROACH TO FPGP BY RANKING OF TRAPEZOIDAL FUZZY NUMBERS

In this section, we define the fuzzy posynomial geometric programming (FPGP) problem and propose a method for this problem.

Definition: Let $F(R)$ be the set of all trapezoidal fuzzy numbers. The model

$$\begin{aligned} \text{Min } \tilde{g}_0(x) \\ \text{subject to } \tilde{g}_i(x) \leq 1, \quad 1 \leq i \leq p; \\ x \geq 0 \end{aligned} \tag{5}$$

where $x = (x_1, x_2, \dots, x_m)^T$ is an m -dimensional variable vector, $\tilde{g}_i(x) = \sum_{k=1}^j \tilde{c}_{ik} \prod_{l=1}^m x_l^{\beta_{ikl}}$ ($1 \leq i \leq p$) is a fuzzy posynomial function of x , $\tilde{g}_{ik} > 0$ and $\tilde{1} \in F(R)$ and γ_{ikl} an arbitrary real number is called an FPGP problem [10]. Using ranking method, any FPGP can be reduced to a posynomial geometric programming (PGP).

Definition 6.2 Let a PGP be

$$\begin{aligned} \text{Minimum } g_0(x) \\ \text{subject to } g_i(x) \leq 1, 1 \leq i \leq p, x > 0 \end{aligned} \tag{6}$$

where $g_i(x) = \sum_{k=1}^{j_i} c_{ik} \prod_{k=1}^m x_l^{r_{ikl}}$ ($0 \leq i \leq p$) is a posynomial, x is an m -dimensional vector, $c_{ik} > 0$, $b_i > 0$, r_{ikl} an arbitrary real number, then its dual programming is denoted

$$\text{Maximum } d(w) = \prod_{i=0}^p \prod_{k=1}^{j_i} \left(\frac{\tilde{c}_{ik}}{w_{ik}} \right)^{w_{ik}} \prod_{i=1}^p w_{i0}^{w_{i0}} \tag{7}$$

$$\begin{aligned} \text{subject to } \sum_{k=1}^{j_0} w_{0k} &= 1, \\ \sum_{i=0}^p \sum_{k=1}^{j_i} \beta_{ikl} w_{ik} &= 0, \quad 1 \leq l \leq m, \quad w > 0. \end{aligned}$$

Here $W = (w_{01}, w_{02}, \dots, w_{0j_0}, \dots, w_{p1}, \dots, w_{pj_p})$, $w_{i0} = \sum_{k=1}^{j_i} w_{ik}$ $i = 0, \dots, p$ and $w_{00} = \sum_{k=1}^{j_0} w_{0k} = 1$.

Now we can solve the FPGP problem by the following steps:

Step 1: we can change the FPGP into PGP by using a new approaching method.

Step 2: Using dual method for solving PGP.

VII. SOLUTION OF FUZZY TWO BAR TRUSS MATHEMATICAL MODEL USING FUZZY POSYNOMIAL GEOMETRIC PROGRAMMING TECHNIQUE

The fuzzy mathematical model of two bar truss is

$$\begin{aligned} \text{Minimize } WT(d, h, y) &= 2\rho\tilde{d}\pi y \\ \text{Subject to } \frac{\tilde{P}yh^{-1}}{d\pi\tilde{t}} &\leq \tilde{\sigma}_0; \\ b^2y^{-2} + h^2y^{-2} &\leq 1; \\ d, h, y &> 0; \end{aligned} \tag{8}$$

where \tilde{P}, \tilde{t} and $\tilde{\sigma}_0$ are trapezoidal fuzzy number.

Applying Geometric Programming Technique, the dual programming of the problem (8) is

$$\begin{aligned} \text{maximum } g(w) &= \left(\frac{2\rho\tilde{t}}{w_{01}} \right)^{w_{01}} \left(\frac{\tilde{P}}{\pi\tilde{t}\tilde{\sigma}_0} \right)^{w_{11}} \left(\frac{b^2}{w_{21}} \right)^{w_{21}} \\ &\times \left(\frac{1}{w_{22}} \right)^{w_{22}} (w_{21} + w_{22})^{(w_{21} + w_{22})} \end{aligned}$$

$$\begin{aligned} \text{subject to } w_{01} &= 1 \quad ; \quad 1.w_{01} + w_{11} + (-2).w_{21} + (-2).w_{22} = 0 \\ 0.w_{01} + (-1).w_{11} + 0.w_{21} + 2.w_{22} &= 0 \\ 1.w_{01} + (-1).w_{11} + 0.w_{21} + 0.w_{22} &= 0; \quad w_{01}, w_{11}, w_{21}, w_{22} > 0 \end{aligned}$$

This is a system of four linear equation with four unknowns. Solving we get the optimal values as follows

$$w_{01}^* = 1, w_{11}^* = 1, w_{21}^* = 0.5 \text{ and } w_{22}^* = 0.5$$

From primal dual relation we get

$$2\rho d \pi y \tilde{t} = w_{01} g^*(w)$$

$$\frac{\tilde{P}}{\pi\tilde{t}\tilde{\sigma}_0} y d^{-1} h^{-1} = \frac{w_{11}}{w_{11}}$$

$$b^2 y^{-2} = \frac{w_{21}}{w_{21} + w_{22}} \quad \text{and} \quad h^2 y^{-2} = \frac{w_{22}}{w_{21} + w_{22}}$$

So the dual objective value is given by

$$\begin{aligned} g^*(w) &= \left(\frac{2\rho\tilde{t}}{w_{01}} \right)^{w_{01}} \left(\frac{\tilde{P}}{\pi\tilde{t}\tilde{\sigma}_0} \right)^{w_{11}} \left(\frac{b^2}{w_{21}} \right)^{w_{21}} \left(\frac{1}{w_{21}} \right)^{w_{22}} \\ &\times (w_{21} + w_{22})^{(w_{21} + w_{22})} \end{aligned}$$

$$y^* = \sqrt{\frac{b^2(w_{21} + w_{22})}{w_{21}}}, \quad h^* = \sqrt{\frac{b^2 w_{22}}{w_{21}}}$$

$$d^* = \frac{\tilde{P}}{\pi\tilde{t}\tilde{\sigma}_0} \times \sqrt{\frac{b^2(w_{21} + w_{22})}{w_{21}}} \times \sqrt{\frac{w_{21}}{b^2 w_{22}}}$$

VIII. NUMERICAL ILLUSTRATION

The input data for the structural optimization problem (2) is given as follows:

$P = (P_{x_0}, P_{y_0}, P_{\delta}, P_{\beta}) = (30500, 32500, 2500, 1500)$ lbs. The two bars are identical, having a cross section with wall thickness $\tilde{t} = (t_{x_0}, t_{y_0}, t_{\delta}, t_{\beta}) = (0.075, 0.09, 0.015, 0.03)$ in.

The distance between the supports is $2b = 60$ in. The material properties are: density $\rho = 0.3$ lbs / in³ and permissible stress

$$\sigma_0 = (\sigma_{0x_0}, \sigma_{0y_0}, \sigma_{0\delta}, \sigma_{0\beta}) = (57500, 59500, 2500, 1500) \text{ psi.}$$

Now determine the mean diameter d of the bars and height of the structure model subject to minimum weight of the structure.

Table 2

Method	GP	Yager's method	Proposed Method
$WT^* (lbs)$	19.74	19.3133	19.29112
Diameter $d^* (in)$	2.47487345	2.798887	2.919572
Height $h^* (in)$	30	30	30
$y^* (in)$	42.426402	42.426402	42.426402

IX. CONCLUSION

We used the new approaching method to an FPGP problem. In particular, we emphasize that the obtained objective function is better by using the method than by Yager's method. Based on the obtained results in the last section, we conclude that using the approaching method is useful to solve an FPGP problem. We have solved fuzzy two bar truss structural model with fuzzy parameter by geometric programming techniques. This technique can be applied to solve the different decision making problems in other engineering and management sciences.

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