

A Study on Influence of Sensitivity Analysis on Normalization Techniques by Applying Equal and Exchange of Weight Metrics

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Abstract - Sensitivity analysis is used to find the robustness of a normalization technique. It is applied in two ways, one by applying equal weights to all the criteria (MSA_EQ) and other by exchanging of weights (MSA_EX). The result obtained for sensitivity analysis when it is conducted with equal weight is described in this paper. The impact of sensitivity analysis on assigning equal weights to criteria is described with different shaded colours for each of the normalization technique. The change in ranking order of the alternatives before and after of sensitivity analysis is described with shaded colour. In this analysis, linear max min normalization has minimum number of altered ranking order for alternatives. In both of these analysis (assigning equal weight to the criteria and exchanging weight of the criteria), selected six normalization techniques maintains different number of alterations in ranking order of alternatives.

Keywords: Multi criteria decision making, sensitivity analysis, TOPSIS, simplified TOPSIS, sFTOPSIS, MCDM Evaluation Metrics, Normalization Techniques, Evaluation of normalization techniques

I. INTRODUCTION

Sensitivity analysis is a fundamental concept to check the effective implementation of quantitative decision models (Senthil et al., 2014). It examines the effect of the changes of a single parameter on the final rankings of the alternatives (Triantaphyllou et al., 1996), (Masuda &Tatsuya, 1990). To compare the normalization techniques two kinds of sensitivity analysis is applied such as one by applying equal weights to all the criteria (M_{SA_EQ}) and other by exchanging of weights (M_{SA_EX}) (Chakraborty & Yeh, 2007). The sensitivity analysis, RCI and rank reversals are such kinds of parameters which analyses the robustness of the MCDM applications.

II. LITERATURE SURVEY

Normalization is a process of converting incommensurable units into dimensionless units. It is an operation to convert different measurement units into standard form for computation (Aydin, 2014), (Yoon & Hwang, 1995).

The most popular normalization techniques which are applied in MCDM are described as follows.

- **Vector normalization (Peter et al., 2016)**
- **Linear Max normalization (Irfan & Tayfun, 2014)**
- **Linear Max-min normalization (Singh & Lyes, 2011)**
- **Linear sum based normalization (Subrata & Chung, 2009)**
- **Gaussian normalization (Rong & Luo, 2004)**
- **Non-monotonic normalization (Maysam et al., 2012)**

The procedure, best features and limitations of these normalization techniques are described as follows.

2.1 Vector Normalization

This normal form divides the performance rating of decision matrix (Aydin, 2014). It converts all attributes into dimensionless measurement unit which simplifies the comparison (Subrata & Chung, 2009). The normalized value r_{ij} is obtained by,

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^n x_{ij}^2}} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2.1)$$

Where, x_{ij} - Original ratings of decision matrix, r_{ij} - Normalized value of the matrix

2.2 Linear Max Normalization

In this technique, the normal value is obtained from individual performance of each attribute with maximum performance rating of the attribute (Zavadskas et al., 2008).

For benefit attributes

$$r_{ij} = \frac{x_{ij}}{x_j^{max}} \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2.2)$$

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^{max}} \quad (2.3)$$

For cost attributes

$$r_{ij} = \frac{x_j^{min}}{x_{ij}} \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2.4)$$

Where, x_{ij} - original ratings of decision matrix, r_{ij} - normalized value of the decision matrix, x_j^{max} - maximum ratings of the alternatives for each criterion C_j , x_j^{min} - minimum ratings of the alternatives for each criterion C_j

2.3 Linear Max-Min Normalization

To normalize the criteria values, it considers ratings of maximum and minimum performance of the attribute (Singh et al., 2011).

For benefit attributes,

$$r_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}} \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2.5)$$

For cost attributes,

$$r_{ij} = \frac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}} \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (2.6)$$

Where, x_{ij} - original ratings of decision matrix, r_{ij} - normalized value of the decision matrix, x_j^{max} - maximum ratings of the alternatives for each criterion C_j , x_j^{min} - minimum ratings of the alternatives for each criterion C_j

2.4 Linear Sum Based Normalization

In this technique, normal value is obtained from individual performance of each attribute with performance rating of all attribute (Yoon & Hwang, 1995)

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^n x_j} \quad \text{for } j = 1, 2, \dots, n \quad (2.7)$$

Where, x_{ij} - original ratings of decision matrix, r_{ij} - normalized value of the decision matrix, x_j - performance rating for each alternative

2.5 Gaussian Normalization

The normalized value r_{ij} is obtained by

$$r_{ij} = \frac{a_{ij} - a_i'}{\sqrt{\sum_{j=1}^n (a_{ij} - a_i')^2}} \quad \text{for } i=1,2,3,\dots,m \text{ for } j=1,2,3,\dots,n \quad (2.8)$$

Where, a_{ij} stands for the rating for each alternative 'i' based on the criteria j, a_i' stands for the average rating of alternative 'i'.

2.6 Non-Monotonic Normalization

The stepwise procedure for Non-monotonic normalization (Shih et al., 2007), (Maysam et al., 2012) is described as follows.

$$Z = \frac{x_{ij} - x_j^0}{\sigma_j} \quad (2.9)$$

Where, x_j^0 - most favorable value, σ_j - standard deviation of alternative with respect to the j^{th} attribute

The most popular normalization techniques which are applied in MCDM are described. To compare the

III. Influence of Sensitivity Analysis Metric (M_{SA_EQ} and M_{SA_EX}) in normalization techniques

3.1 Sensitivity analysis – When assigning equal weight to the criteria (M_{SA_EQ})

The result obtained for sensitivity analysis when it is conducted with equal weight is described with different shaded colours for each of the normalization technique. The number of changes that occurred in ranking order of alternatives for vector normalization is three (03) and it is indicted by lighter orange colour, linear max normalization is eight (08) and it is indicted by orange colour, linear max min normalization is two (02) and it is indicted by lighter red colour, linear sum based normalization is four (04) and it is indicted by lighter blue colour, Gaussian normalization is seven (07) and it is indicted by lighter purple colour and non monotonic normalization is four (04) and it is indicted by lighter olive green colour. In this analysis, linear max min normalization has minimum number of altered ranking order for alternatives.

3.2 Sensitivity analysis – When exchanging weight of the criteria (M_{SA_EX})

The Sensitivity analysis is conducted with exchange of each criterion weight. The changes in the ranking order of the alternative is described in Table 3. The number of changes that occurred in ranking order of alternatives for vector normalization is four (04), linear max normalization is six (06), linear max min normalization is two (02), linear sum based normalization is six (06), Gaussian normalization is nine (09) and non monotonic normalization is four (04). In this analysis also linear max min normalization retains minimum number of altered ranking order for alternatives.

In both of these analysis (assigning equal weight to the criteria and exchanging weight of the criteria), all these six normalization techniques maintains different number of alterations in ranking order of alternatives. The sensitivity analysis metrics (M_{SA_EQ} and M_{SA_EX}) obtained for these six normalization techniques is described in Table 2.

Table 4 Sensitivity Analysis metric (M_{SA_EQ} and M_{SA_EX}) for normalization techniques

Sensitivity Analysis metric (M_{SA_EQ} and M_{SA_EX})	Normalization Techniques					
	N1	N2	N3	N4	N5	N6
Equal weight (%) (M_{SA_EQ})	21.43	57.14	14.29	28.57	50.00	28.57
Exchange of Weight (%) (M_{SA_EX})	28.57	42.86	14.29	42.86	64.29	28.57

- N1* Vector Normalization *N4* Linear Sum based Normalization
- N2* Linear Max Normalization *N5* Gaussian Normalization
- N3* Linear Max Min Normalization *N6* Non Monotonic Normalization

In this analysis, linear max min normalization provides lesser value compared to other normalization techniques. The metric values (M_{SA_EQ} and M_{SA_EX}) obtained for these normalization techniques depicted as a graph which is shown in Figure 1.

Table 2 Sensitivity analysis on normalization techniques – when assigning equal weight to criteria

ALTERNATIVE		Sensitivity Analysis (Assigning equal weight to all the criteria)																							
		Vector Normalization				Linear Max Normalization				Linear Max Min Normalization				Linear Sum Based Normalization				Gaussian Normalization				Non Monotonic Normalization			
		BEFORE		AFTER		BEFORE		AFTER		BEFORE		AFTER		BEFORE		AFTER		BEFORE		AFTER		BEFORE		AFTER	
		RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK

A ₁	0.1 16 6	14	0.1 32 7	14	0.8 12 9	2	0.7 97 6	4	0.8 35 4	1	0.8 63 5	2	1	1	0.5 96 1	2	0.6 75 1	9	0.6 34 3	7	0	14	0	14
A ₂	0.5 15 2	13	0.5 13 6	13	0.8 13 2	1	0.7 98	1	0.8 35 3	2	0.8 63 6	1	0.5 97 7	2	1	1	0.6 75 2	8	0.6 34 1	9	0.0 75 9	13	0.0 75 6	13

Table 3 Sensitivity analysis on normalization techniques – when exchanging weight of criteria

Alternative	Sensitivity Analysis (Exchange of Weight)																							
	Vector Normalization				Linear Max Normalization				Linear Max Min Normalization				Linear Sum Based Normalization				Gaussian Normalization				Non Monotonic Normalization			
	Before		After		Before		After		Before		After		Before		After		Before		After		Before		After	
	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK	RCC	RANK
A ₁	0.1 16 6	14	0.1 18 4	14	0.8 12 9	2	0.8 10 1	4	0.8 35 4	1	0.8 37 5	1	1	1	1	1	0.6 75 1	9	0.6 75 1	10	0	14	0.0 75 9	13
A ₂	0.5 15 2	13	0.6 21	12	0.8 13 2	1	0.8 13 1	1	0.8 35 3	2	0.8 35 5	3	0.5 97 7	2	0.6 01 1	2	0.6 75 2	8	0.6 75 3	8	0.0 75 9	13	0	14

N1 Vector Normalization *N2* Linear Max Normalization
N3 Linear Max Min Normalization *N4* Linear Sum based Normalization
N5 Gaussian Normalization *N6* Non Monotonic Normalization

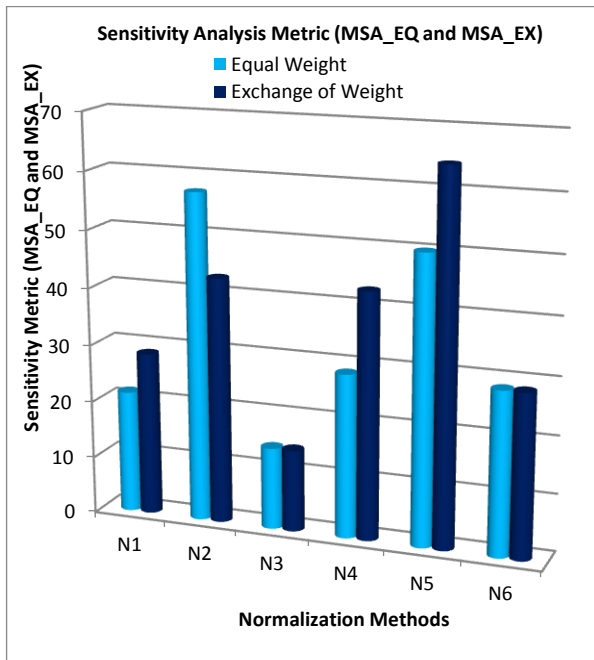


Figure 1 Sensitivity Analysis Metric (M_{SA_EQ} and M_{SA_EX}) for normalization techniques

The graph indicates that linear max min normalization has lowest value for M_{SA_EQ} and M_{SA_EX} . It has lowest number of ranking order changes compared to other normalization techniques.

IV. CONCLUSION

Metrics are designed for GFTOPSIS from the evaluation parameters of MCDM techniques. Better normalization technique and better weight method is identified based on the results of GFTOPSIS evaluation metrics. These metrics designed based on evaluation parameters such as sensitivity analysis, rank reversal, repeated ranking and repeated-rank occurrence which thoroughly checks different ranking properties. Hence the identified normalization technique and weight method definitely improves the robustness of ranking. It has been validated with results of sFTOPSIS.

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N1 Vector Normalization *N4* Linear Sum based Normalization

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