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d-Lucky Labeling of Honeycomb Network

Rini Angeline Sahayamary A[1*], Teresa Arockiamary S[2]

[1]Don Bosco Polytechnic College, Chennai-600 012, India
[2]Stella Maris College, Chennai-600 086, India

[*]*Corresponding Author: riniangeline@gmail.com*

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Abstract— Let $f: V(G) \longrightarrow \mathbb{N}$ be a labeling of the vertices of a graph G by positive integers. Let $S(v)$ denote the sum of labels of the neighbors of the vertex v in G. If v is an isolated vertex of G we put $S(v) = 0$. A labeling f is *lucky* if $S(u) \neq S(v)$ for every pair of adjacent vertices u and v. The *lucky number* of a graph G, denoted by $\eta(G)$, is the least positive integer k such that G has a lucky labeling with $\{1,2,\ldots k\}$ as the set of labels. Let $l: V(G) \longrightarrow \{1,2,\ldots k\}$ be a labeling of the vertices of a graph G by positive integers. Define $c(u) = d(u) + \sum_{v\in N(u)} l(v)$ where $d(u)$ denotes the degree of u and $N(u)$ denotes the neighbourhood of u. We define a labeling l as d-lucky if $c(u) \neq c(v)$, for every pair of adjacent vertices u and v in G. The d-lucky number of a graph G, denoted by $\eta_{dl}(G)$, is the least positive integer k such that G has a d-lucky labeling with $\{1,2,\ldots k\}$ as the set of labels. In this paper, we study d-lucky labeling of Honeycomb network and Honeycomb torus network. Further we have obtained the d-lucky number for Honeycomb network and Honeycomb torus network.

Keywords— Colouring, *d*-lucky labeling, Honeycomb network, Honeycomb torus network.

I. Introduction

A *lucky labeling* of a graph G is an assignment of positive integers to the vertices of G such that if $S(v)$ denotes the sum of labels on the neighbors of v, then S is a vertex colouring of G. In other words, G has a lucky labeling if S is a proper colouring of a graph G which satisfies the condition $S(u)\neq S(v)$ whenever u and v are adjacent.

Further as an extension we have *d*-lucky labeling which considers the degree of u along with $S(v)$ as defined in lucky labeling.

In our paper we have obtained the *d*-lucky number for honeycomb network and honeycomb torus network which has *d*-lucky labeling.

II. Related Work

Lucky labeling was introduced by S. Czerwinski et al.[6]. Lucky labeling was well studied for bloom graph and various other graphs. The notion of *d*-lucky labeling was introduced by Mikra Miller et al.[8]. Further the *d-lucky number* was obtained for Hypercube network, Bene's network, Butterfly network, Mesh, Hypertree, X-tree,[8] Cycle of Ladder, *n*-sunlet and Helm graphs.[2].

Basic definitions

We begin with the definition of *d*-lucky labeling.

Definition 1.1. Let l: $V(G) \longrightarrow \{1,2,\ldots k\}$ be a labeling of the vertices of a graph G by positive integers. Define $c(u) = d(u) + \sum_{v\in N(u)} l(v)$ where $d(u)$ denotes the degree of u. We define a labeling l as d-lucky if $c(u) \neq c(v)$, for every pair of adjacent vertices u and v in G. The d-lucky number of a graph G, denoted by $\eta_{dl}(G)$, is the least positive integer k such that G has a d-lucky labeling with $\{1,2,\ldots k\}$ as the set of labels.

III. Methodology

In our study we consider the honeycomb and honeycomb torus network. We observed that the networks satisfies the conditions of lucky labeling and our study was further extended to *d*-lucky labeling. We have also obtained the *d*-lucky number for these networks. The *d*-lucky number was found to be 2. To justify and facilitate our result we have used the method of dividing the honeycomb network and honeycomb torus network into levels above and below the symmetry line. This is well explained in Figure 1. The $4n$ levels of vertices of $HC(n)$ and $HCT(n)$ are labeled as 1 and 2 following a pattern which is explained in the proof of theorem.

IV. Results and Discussion

1. Honeycomb network

In recent times, studies on honeycomb network has become an important one. D Antony Xavier , R C Thivyarathi [4], P. Sivagami, Indra Rajasingh, Sharmila Mary Arul [9] have worked on Hexagonal and Honeycomb networks. The number of vertices and edges of $HC(r)$ is $6r^2$ and $9r^2-3r$ respectively.

Theorem 1: The *n*-dimensional Honeycomb network is *d*-lucky for $n \geq 2$ and $\eta_{dl}[HC(n)] = 2$.

***Proof*:** In the course of proof for our convenience we divide the Honeycomb network into levels l_0 and l_i, $i \geq 1$. See Figure 1. To facilitate labeling of vertices, we have considered l_i as upper level and l_{-i} as lower level. Let us denote the vertices in level l_i, $i \geq 0$ as *l*(*i*,*j*), $1 \leq j \leq 2n$. For every two levels in the upper as well as lower above and below lo, the number of vertices to be labeled are 2*n*-*i*.

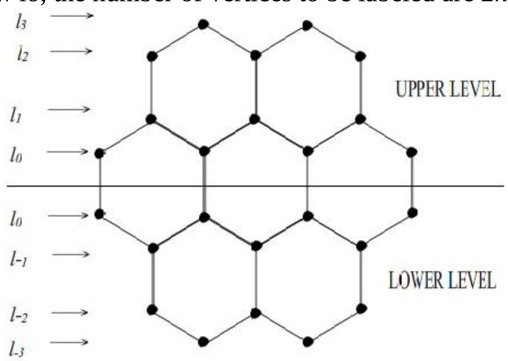

Figure 1:*HC*(2)

Case 1: When *n* is even

There are 4*n* levels of vertices to be labeled. Level l_0 has 2*n* number of vertices, l_1 and l_2 has 2*n*-1 vertices. Similarly for every two levels we will have one vertex less upto 2*n* levels. The same pattern follows for the vertices in lower levels.

Label the vertices of l_0 in upper level as 2. The vertices in l_1 are labeled as 2. Now there are 2*n*-2 levels of vertices to be labeled. Next label the vertices in l_2 as 1. Vertices in l_3 are assigned the label 2. Vertices in l_4 are labeled as 2. Vertices in l_5 are also assigned the same value 2. The vertices in l_6 are labeled as 1. Vertices in l_7 are labeled as 2.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and *l*(2,*j*). Degrees of vertices in l_1 are 3. Labels of l_0, l_2 and l_3 are 2,1 and 2 respectively. We begin finding $c(u)$ with *l*(1,*j*). Neighbours of *l*(1,*j*) are *l*(2,*j*), *l*(0,*j*) and *l*(0,*j*+1). Take *l*(1,1) as *u*. So $c(u) = d(u) + \sum_{v \in N(u)} l(v) = 8$. Take *l*(2,1) as *v*. $c(v) = d(v) + \sum_{u \in N(v)} l(u) = 5$. Now we take vertices in level 2*n*-2 and 2*n*-1 which is of degree 2. Take *l*(7,1) as *u*. Neighbours of *l*(7,1) are *l*(6,1) and *l*(6,2). So $c(u) = d(u) + \sum_{v \in N(u)} l(v) = 4$. Take *l*(6,1) as *v*. so $c(v) = d(v) + \sum_{u \in N(v)} l(u) = 6$. If we take *l*(6,2) as *v*, then $c(v) = d(u) + \sum_{v \in N(u)} l(v) = 9$. In both cases of $c(u) = 4$, $c(v) = 6$ and $c(v) = 9$, $c(u) \neq c(v)$. A similar argument holds for the vertices in upper levels for every pair of adjacent vertices u and v.

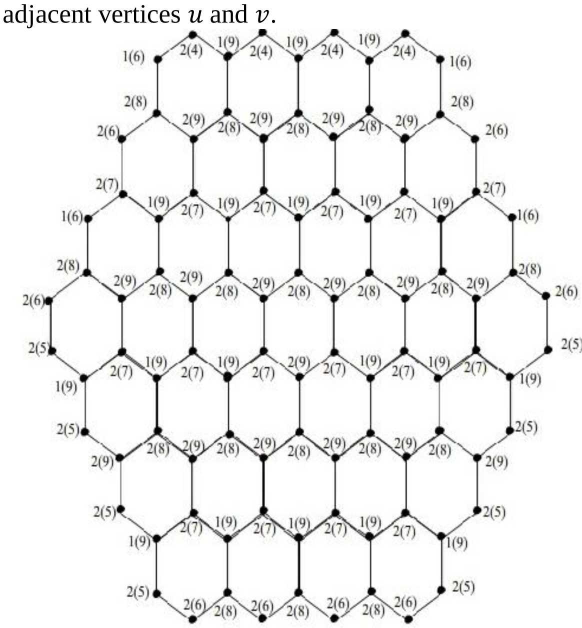

Figure 2: *HC*(4)

Label the vertices of l_0 in lower level as 2. Now we begin to label the lower levels l_{-i}. In level l_{-1} the vertices are labeled as 1. Label the vertices in l_{-2} as 2 and label l_{-3} vertices as 2. Label l_{-4} vertices as 2. Label l_{-5} vertices as 1. Label l_{-6} vertices as 2. Label l_{-7} vertices as 2.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and *l*(2,*j*). Degrees of vertices in l_{-1} are 3. Labels of l_0, l_{-2} and l_{-3} are 2,2 and 2 respectively. We begin finding $c(u)$ with *l*(-1,*j*). Neighbours of *l*(1,*j*) are *l*(-2,*j*), *l*(0,*j*) and *l*(0,*j*+1). Take *l*(-1,1) as *u*. $c(u) = d(u) + \sum_{v \in N(u)} l(v) = 9$. Take *l*(-2,1) as *v*. $c(v) = d(v) + \sum_{u \in N(v)} l(u) = 5$.

Now we take vertices in level 2*n*-2 and 2*n*-1 in lower level. Take *l*(-7,1) as *u*. Neighbours of *l*(-7,1) are *l*(-6,1) and *l*(-6,2). So $c(u) = d(u) + \sum_{v \in N(u)} l(v) = 6$. Take *l*(-6,1) as *v*. so $c(v) = d(v) + \sum_{u \in N(v)} l(u) = 5$. If we take *l*(-6,2) as *v*, then $c(v) = d(v) + \sum_{u \in N(v)} l(u) = 8$. In both cases of $c(u) = 6$, $c(v) = 5$ and $c(v) = 8$, $c(u) \neq c(v)$ for every pair of adjacent vertices u and v. Figure 2 gives the *d*-lucky labeling for *n* even with $c(u)$ in paranthesis.

Case 2: When *n* is odd.

Label the vertices of l_0 in upper level as 2. Label the vertices in l_1 as 2. Next label the vertices in l_2 as 1. Vertices in l_3 are labeled as 2. Vertices in l_4 are labeled as 2. Vertices in l_5 are labeled as 1 and 2 alternatively.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and $l(2,j)$. Degrees of vertices in l_1 are 3. Labels of l_0, l_2 and l_3 are 2,1 and 2 respectively. We begin finding $c(u)$ with $l(1,j)$. Neighbours of $l(1,j)$ are $l(2,j)$, $l(0,j)$ and $l(0,j+1)$. Take $l(1,1)$ as u. $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 8$. Take $l(2,1)$ as v. $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 5$, $c(u) \neq c(v)$ for every pair of adjacent vertices u and v.

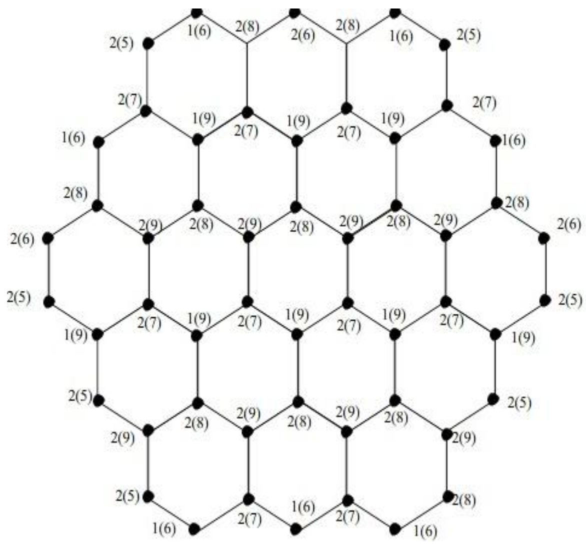

Figure 3:HC(3)

Label the vertices of l_0 in lower level as 2. Now we begin to label the lower levels l_{-i}. In level l_{-1} the vertices are labeled as 1. Label the vertices in l_{-2} as 2 and label l_{-3} vertices as 2. Label l_{-4} vertices as 2. Label l_{-5} vertices as 1.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices u and v in G. To prove our claim, we consider the vertices in $l(1,j)$ and $l(2,j)$. Degrees of vertices in l_{-1} are 3. Labels of l_0, l_{-2} and l_{-3} are 2,2 and 2 respectively. We begin finding $c(u)$ with $l(-1,j)$. Neighbours of $l(1,j)$ are $l(-2,j)$, $l(0,j)$ and $l(0,j+1)$. Take $l(-1,1)$ as u. $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 9$. Take $l(-2,1)$ as v. $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 5$. Therefore all the vertices in the lower levels satisfies the condition $c(u) \neq c(v)$, for every pair of adjacent vertices u and v. Figure 3 represents the d-lucky labeling for n odd with $c(u)$ in the paranthesis.

Hence n-dimensional Honeycomb network admits d-lucky labeling and $\eta_{dl}[HC(n)] = 2$.

2. Honeycomb Torus network

Honeycomb Torus network is obtained by adding layer of hexagons around the boundary of $HC(r\text{-}1)$, with wraparound edges. The number of vertices and edges of $HC(r)$ are $6r^2$ and $9r^2$ respectively. Including wraparound edges the Honeycomb Torus network becomes 3-regular. D Antony Xavier , R C Thivyarathi [5] had studied about the proper lucky number of torus network. Honeycomb Torus was studied by P. Sivagami, Indra Rajasingh, Sharmila Mary Arul[9] and Indra Rajasigh, S. Teresa Arockiamary[7]. We denote the honeycomb torus as $HCT(n)$. See Figure 4.

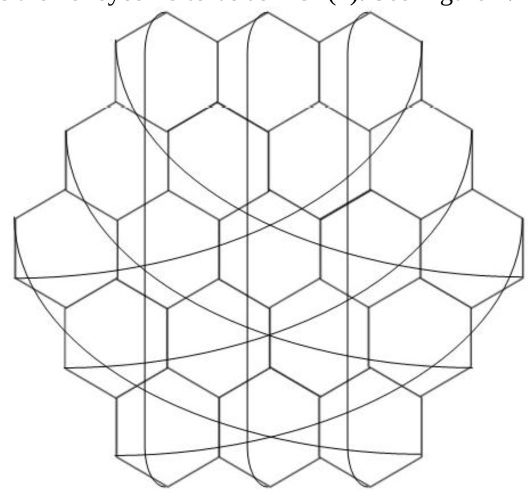
Figure 4:*HCT*(3)

Theorem 2. The n-dimensional Honeycomb torus network admits d-lucky labeling and $\eta_{dl}[HCT(n)] = 2$.

Proof : To label the vertices, we divide them as upper level l_i and lower level l_{-i} as in Figure 1.

Case 1: When n is even

There are $4n$ levels of vertices to be labeled. Level l_0 has $2n$ number of vertices, l_1 and l_2 has $2n$-1 vertices similarly for every two levels we will have one vertex reducing upto $2n$ levels. The same pattern follows for the vertices in lower levels.

Label the vertices of l_0 in upper level as 2. The vertices in l_1 are labeled as 2. Next label the vertices in l_2 as 1. Vertices in l_3 are labeled as 2. Vertices in l_4 are labeled as 2. Vertices in l_5 are labeled as 2. The vertices in l_6 are labeled as 1. Vertices in l_7 are labeled as 2.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices u and v in G. To prove our claim, we consider the vertices in $l(1,j)$ and $l(2,j)$. Degrees of vertices in l_1 are 3. Labels of l_0, l_2 and l_3 are 2,1 and 2 respectively. We begin finding $c(u)$ with $l(1,j)$. Neighbours of $l(1,j)$ are $l(2,j)$, $l(0,j)$ and $l(0,j+1)$. Take $l(1,1)$ as u. So $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 8$. Take $l(2,1)$ as v. $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 5$. Now we take vertices in level $2n$-2 and $2n$-1 which is of degree 2. Take $l(7,1)$ as u. Neighbours of $l(7,1)$ are $l(6,1)$ and $l(6,2)$. So $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 4$. Take $l(6,1)$ as v. so $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 6$. If we take $l(6,2)$ as v, then $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 9$. In both cases of $c(u) = 4$, $c(v) = 6$ and $c(v) = 9$. Thus $c(u) \neq c(v)$, for every pair of adjacent vertices u and v in the upper level.

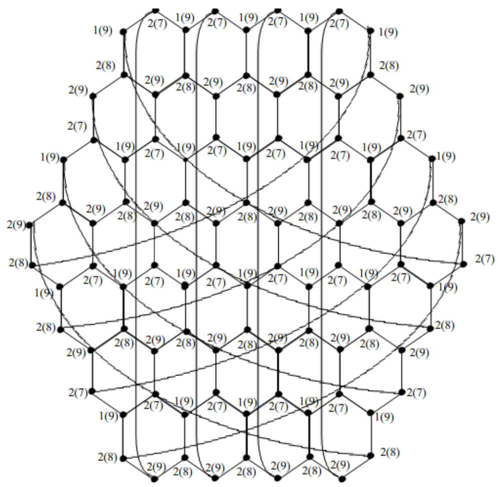

Figure 5:*HCT*(4)

Label the vertices of l_0 in lower level as 2. Now we begin to label the lower levels l_{-i}. In level l_{-1} the vertices are labeled as 1. Label the vertices in l_{-2} as 2 and label l_{-3} vertices as 2. Label l_{-4} vertices as 2. Label l_{-5} vertices as 1. Label l_{-6} vertices as 2. Label l_{-7} vertices as 2.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and *l*(2,*j*). Degrees of vertices in l_{-1} are 3. Labels of l_0, l_{-2} and l_{-3} are 2,2 and 2 respectively. We begin finding $c(u)$ with *l*(-1,*j*). Neighbours of *l*(1,*j*) are *l*(-2,*j*), *l*(0,*j*) and *l*(0,*j*+1). Take *l*(-1,1) as *u*. $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 9$. Take *l*(-2,1) as *v*. $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 5$.

Now we take vertices in level 2*n*-2 and 2*n*-1 in lower level. Take *l*(-7,1) as *u*. Neighbours of *l*(-7,1) are *l*(-6,1) and *l*(-6,2). So $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 6$. Take *l*(-6,1) as *v*. so $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 5$. If we take *l*(-6,2) as *v*, then $c(v) = d(u) + \sum_{v\in N(u)} l(v) = 8$. In both cases of $c(u) = 6$, $c(v) = 5$ and $c(v) = 8$. Thus $c(u) \neq c(v)$ for every pair of adjacent vertices u and v.
The Figure 5 represents the *d*-lucky labeling for *n* even with $c(u)$ in the paranthesis.

Case 2:When *n* is odd.
Label the vertices of l_0 in upper level as 2. Label the vertices in l_1 as 2. Next label the vertices in l_2 as 1. Vertices in l_3 are labeled as 2. Vertices in l_4 are labeled as 2. Vertices in l_5 are labeled as 2.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and *l*(2,*j*). Degrees of vertices in l_1 are 3. Labels of l_0, l_2 and l_3 are 2,1 and 2 respectively. We begin finding $c(u)$ with *l*(1,*j*). Neighbours of *l*(1,*j*) are *l*(2,*j*), *l*(0,*j*) and *l*(0,*j*+1). Take *l*(1,1) as *u*. $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 8$. Take *l*(2,1) as *v*. $c(v) = d(u) + \sum_{v\in N(u)} l(v) = 9$. Thus $c(u) \neq c(v)$. Since the degrees of level 5 is 3. $c(u)$ =8 $c(v)$ =9. Thus $c(u) \neq c(v)$. All the vertices in upper levels satisfies the condition for every pair of adjacent vertices u and v.

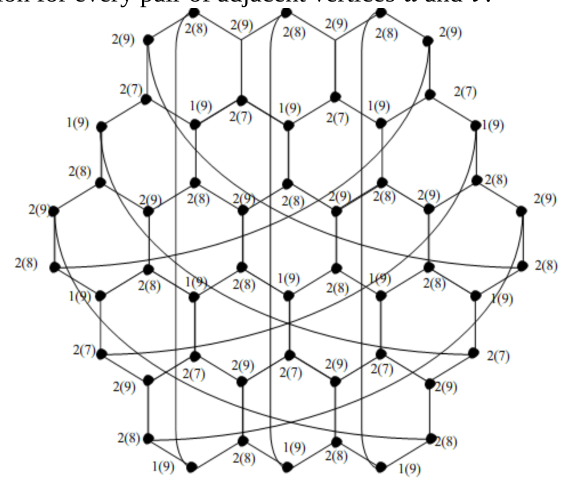

Figure 6:*HCT*(3)

Label the vertices of l_0 in lower level as 2. Now we begin to label the lower levels l_{-i}. In level l_{-1} the vertices are labeled as 1. Label the vertices in l_{-2} as 2 and label l_{-3} vertices as 2. Label l_{-4} vertices as 2. Label l_{-5} vertices as 1.

Claim: $c(u) \neq c(v)$ for every pair of adjacent vertices *u* and *v* in *G*. To prove our claim, we consider the vertices in *l*(1,*j*) and *l*(2,*j*). Degrees of vertices in l_{-1} are 3. Labels of l_0, l_{-2} and l_{-3} are 2,2 and 2 respectively. We begin finding $c(u)$ with *l*(-1,*j*). Neighbours of *l*(1,*j*) are *l*(-2,*j*), *l*(0,*j*) and *l*(0,*j*+1). Take *l*(-1,1) as *u*. $c(u) = d(u) + \sum_{v\in N(u)} l(v) = 9$. Take *l*(-2,1) as *v*. $c(v) = d(v) + \sum_{u\in N(v)} l(u) = 7$. Thus $c(u) \neq c(v)$. Therefore all the vertices in the lower levels satisfies the condition for every pair of adjacent vertices u and v.igure 6 represents the *d*-lucky labeling for *n* odd with $c(u)$ in paranthesis.

Hence *n*-dimensional Honeycomb torus network admits *d*-lucky labeling and $\eta_{dl}[HCT(n)] = 2$.

V. CONCLUSION AND FUTURE SCOPE

The *d*-lucky number has been obtained for Honeycomb network and Honeycomb Torus network and we have proved that $\eta_{dl}[G] = 2$. Further our study is extended to Hexagonal network.

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