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3-Equitable Prime Cordial Labeling of Standard Splitting Graphs

V. Sharon Philomena[1*], B. Kavya[2]

[1,2]P.G. Department of Mathematics, Women's Christian College, Chennai–600 006..

Corresponding Author: sharonphilo2008@gmail.com Tel.: 18015667977, 2 9087976935

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Abstract— A 3-equitable prime cordial labeling is extension of prime cordial labeling .Splitting graph $S'(G)$ was introduced by Sampath Kumar and Walikar [6]. For a graph G the *splitting graph* S' of G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$. In this paper we prove the splitting graph of cycle C_n, path P_n, bistar $B_{n,n}$ and wheel W_n admits 3-equitable prime cordial labeling.

Keywords—Cordial labeling, 3-equitable Prime cordial labeling, Splitting graph, cycle, path, bistar and wheel.

I. Introduction

Graph labeling deals with assignment of integers to the vertices or edges or both subject to certain conditions. Most graph labeling methods trace their origin to one introduced by Rosa in 1967 [5]. Graph labeling is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and crystallography. In this paper, we consider only finite, simple undirected graphs. We consider a graph $G = (V(G), E(G))$ and we let $|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations and terminology we follow Harary [3] and for number theory we follow Burton. A labeling of a graph G is a mapping that carries vertices and edges into a set of numbers, usually integers**.**Any graph labeling will have the following three common characteristics:

1. A set of numbers from which vertex labels are chosen;
2. A rule that assigns a value to each edge;
3. A condition that this value has to satisfy.

An excellent survey of graph labeling and various types of graph labeling can be found in Gallian.Cordial labeling was introduced by Cahit in 1987 [1]. The concept of *prime cordial labeling* was introduced by Sundaram*et al* [7]. The 3-equitable prime cordial labeling is an extension of prime cordial labeling introduced by Murugesan *et al* [4] in the year 2013.

A 3-*equitable prime cordial labeling* of a graph G with vertex set $V(G)$ and edge set $E(G)$ is a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ such that the induced edge function $f : E(G) \to \{0, 1, 2\}$ defined by $f(e = uv)$ =

- 1 if $gcd(f(u), f(v)) = 1$ and $gcd(f(u) + f(v), f(u) - f(v)) = 1$;
- 2 if $gcd(f(u), f(v)) = 1$ and $gcd(f(u) + f(v), f(u) - f(v)) = 2$;
- 0 otherwise.

satisfies the condition $|e_f(i) - e_f(j)| \le 1$, $0 \le i, j \le 2$, where $e_f(0)$, $e_f(1)$, $e_f(2)$ denote the number of edges with label 0, 1 and 2 respectively under f. A graph which admits 3-equitable prime cordial labeling is called a 3-equitable prime cordial graph.

II. Related Work

Murugesan *et al* [4] introduced 3-equitable prime cordial labeling and proved that standard graphs such as cycle , Path, Star graph and complete graph are 3-equitable prime cordial labeling. SwetaSrivastav and Sangeeta Gupta prove that cycle with one chord, twin chord and the split graph of star are 3-equitable prime cordial graph.

III. Results and Discussion

Theorem 1:
The splitting graph of path admits 3-equitable prime cordial labeling.

Proof:
A trail with distinct vertices $v_1, v_2, \ldots, v_n$ is called path P_n. We obtain splitting graph of path by adding vertices v_1', $v_2', \ldots v_n{}'$ corresponding to the vertices $v_1, v_2, \ldots, v_n$. The total number of vertices and edges are denoted as

$$|V(S'(P_n))| = 2n \quad \text{and} \quad |E(S'(P_n))| = 3(n-1)$$

We define the vertex labeling $f: V(S'(P_n)) \to \{1,2,3 \ldots \ldots 2n\}$ as follows

Case 1: when i is even

$$f(v_i) = (2i-1) 1 \le i \le n$$
$$f(v_i') = 2i \quad 1 \le i \le n$$

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Subcase 1: when $i \equiv 2(mod\ 6)$ then define

$$f(v_i') = 2i + 2$$

Case 2: when i is odd

$$f(v_i) = 2i \quad 1 \le i \le n$$

$$f(v_i') = (2i - 1) \quad 1 \le i \le n$$

Subcase 1: when $i \equiv 3(mod\ 6)$ then define

$$f(v_i) = 2i - 2$$

Using the above equation the edge labels are computed. We define the edge labeling as $g: E(S'(P_n)) \rightarrow \{0,1,2\}$

$$g(v_i v_{i+1}) = 1,$$

$$g(v_i' v_{i+1}) \text{ and } g(v_i v'_{i+1}) = 2,$$

$$g(v_i v_{i+1}') \text{ and } g(v'_i v_{i+1}) = 0$$

The number of edges with label '0', '1', '2' are

$$e_f(0) = e_f(1) = e_f(2) = n - 1$$

Thus the condition $|e_f(i) - e_f(j)| \le 1$, for all $0 \le i, j \le 2$ is satisfied.

Hence $S'(P_n)$ admits 3-equitable prime cordial labeling.

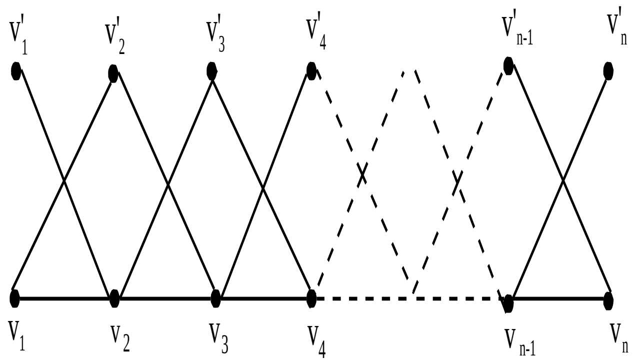

Figure 1: Splitting graph of path P_n

Theorem 2:
The splitting graph of cycle for $n \ge 3$ admits 3-equitable prime cordial labeling.

Proof:
Let C_n be the cycle with vertices $u_1, u_2, \ldots, u_n$ .We obtain splitting graph of cycle by adding vertices $v_1, v_2, \ldots, v_n$ corresponding to the vertices $u_1, u_2, \ldots, u_n$. The total number of vertices and edges are denoted as

$$|V(S'(C_n))| = 2n \quad \text{and} \quad |E(S'(C_n))| = 3n$$

We define the vertex labeling as $f: V(S'(C_n)) \rightarrow \{1,2, \ldots .2n\}$

When $n = 3$ the labeling of C_3 is given as follows. $f(u_i) = i \quad for \quad 1 \le i \le 3$

$$f(v_i) = 2i + 1 \quad for \quad i \equiv 1(mod\ 2)$$

$$f(v_i) = 2i \quad for \quad i \equiv 0(mod\ 2)$$

$$f(v_i) = 2i \quad for \quad i = n$$

When $n \ge 4$ the labeling is given as follows

Case 1: When i is odd

$$f(u_i) = 2i - 1 \quad for \quad 1 \le i \le n$$

$$f(v_i) = 2i + 1 \quad for \quad 1 \le i \le n$$

Case 2: When i is even

$$f(u_i) = 2i - 2 \quad for \quad 1 \le i \le n$$

$$f(v_i) = 2i \quad for \quad 1 \le i \le n$$

Subcase : $i \equiv 4(mod\ 6)$

$$f(u_i) = 2i$$

$$f(v_i) = 2i - 2$$

Where $10 \le i \le n$ (when n is odd) and $4 \le i \le n$ (when n is even)

We define the vertex labeling as $g: E(S'(C_n)) \rightarrow \{0,1,2\}$ .Using the above equation the edge labels are computed as follows .

Case 1: When n is odd

$$g(u_i v_i) = 0, \quad when\ i \equiv 0(mod 2)$$

$$g(u_i v_{i+2}) = 0, \quad when\ i \equiv 0(mod 2)$$

$$g(u_i v_i) = 1, \ when \ i = n$$

$$g(u_i u_{i+1}) = 1, \ when \ 1 \le i \le n$$

$$g(u_1 v_{i-1}) = 1, \ when \ i = n$$

$$g(u_i v_i) = 2, \ when \ i \equiv 1(mod\ 2)$$

$$g(u_i v_{i+2}) = 2, \ when \ i \equiv 1(mod\ 2)$$

$$g(u_1 v_n) = 2, \quad when\ i = n$$

Case 2: when n is even

$$g(u_i v_i) = 0 \quad when\ i \equiv 0(mod\ 2)$$

$$g(u_i u_{i+1}) = 1 \quad when \ 1 \le i \le n$$

$$g(u_i v_i) = 2 \quad when \ i \equiv 1(mod\ 2)$$

The number of edges with label '0', '1', '2' are

$$e_f(0) = e_f(1) = e_f(2) = n$$

Thus the condition $|e_f(i) - e_f(j)| \le 1$, for all $0 \le i, j \le 2$ is satisfied.

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Hence $S'(C_n)$ admits 3-equitable prime cordial labeling.

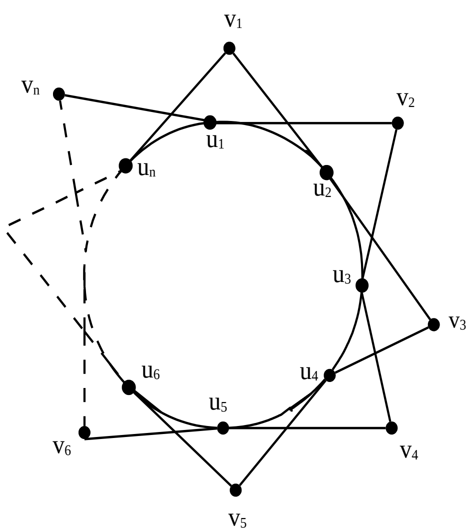

Figure 2: splitting graph of cycle C_n

Theorem 3:
The graph $S'(B_{n,n})$ admit 3-equitable prime cordial labeling.

Proof:
The bistar $B_{n,n}$ is a graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge with vertex set $\{u, v, u_i,\ v_i : 1 \le i \le n\}$ where u_i, v_i are pendant vertices. We obtain splitting graph of bistar by adding vertices $u',\ v',\ {u_i}',\ {v_i}'$ corresponding to the vertices $u,\ v,\ u_i,\ v_i$ where $1 \le i \le n$. The total number of vertices and edges are denoted as $|V(S'(B_{n,n}))|$ =4(n+1) and $|E(S'(B_{n,n}))|$ = 6n+3

We define the vertex labeling as $f: V(S'(B_{n,n})) \rightarrow \{1,2,3 \ldots \ldots 4(n+1)\}$ as follows

Let p_1 be the highest prime number < 4(n +1)

$$f(u) = 2\ ,\ f(u') = 1\ ,\quad f(v) = p_1$$

$$f(v') = 4(n+1)$$

$$f(u_i) = 2i + 1\ for\ 1 \le i \le n$$

$$f({u_i}') = 2i + 2\ for\ 1 \le i \le n$$

$$f(v_i) = u'_n + 2i\ for\ 1 \le i \le n$$

$f({v_i}') = u_n + 2i\ for\ 1 \le i \le n$ (except p_1)

We define the edge labeling as $g: E(S'(B_{n,n})) \rightarrow \{0,1,2\}$. Using the above equation the edge labels are computedas follows

$$g(uv') = 0\ , g(uv) = 1\ ,\ g(u'v) = 2$$

$$g(u{u'}_i) = 0\ \ for\ 1 \le i \le n$$
$$g(v'v_i) = 0 for\ 1 \le i \le n$$

$$g(uu'_i) = 1\ \ for\ 1 \le i \le n$$
$$g(v'v_i) = 1\ \ for\ 1 \le i \le n$$
$$g(u'u_i) = 2\ \ for\ 1 \le i \le n$$
$$g(vv'_i) = 2\ \ for\ 1 \le i \le n$$

The number of edges with label '0', '1', '2' are

$$e_f(0) = e_f(1) = e_f(2) = 2n + 1$$

Thus the condition $|e_f(i) - e_f(j)| \le 1$, for all $0 \le i,\ j \le 2$ is satisfied. Hence $S'(B_{n,n})$ admits 3-equitable prime cordial labeling.

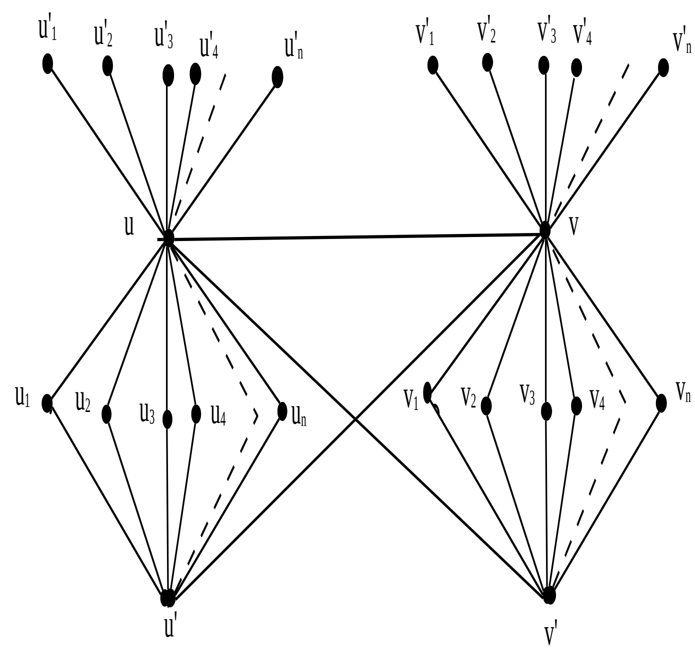

Figure 3: Splitting graph of bistar$B_{n,n}$

Theorem 4:
The graph $S'(W_n)$ for $n \le 9$ admits 3-equitable prime cordial labeling.

Proof:

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A wheel graph W_n is obtained from a cycle C_n by adding a new vertex and joining it to all the vertices of the cycle by an edge. The apex vertex is denoted as w and the vertices as $u_1, u_2, \ldots\ldots, u_n$ .We obtain splitting graph of wheel by adding vertices $u_1', u_2', \ldots u_n{}'$ corresponding to the vertices and w' corresponding to the apex vertex. The total number of vertices and edges are denoted as $|V(S'(W_n))|$ = 2(n+1) and $|E(S'(W_n))|$ = 6n

We define the vertex labeling as $f: V(S'(W_n)) \rightarrow \{1,2 \ldots \ldots 2(n+1)\}$ as follows

$$f(w) = 2$$

$$f(w') = 1$$

$$f(u_n') = 20 \ for \ n \le 9$$

Case 1: $i \equiv 1(mod\ 2)$

we define $1 \le k \le n$ then the vertex labeling is given as

$$f(u_i) = 3k, f(u_i') = 6k$$

Case 2: $i \equiv 0(mod\ 2)$

$$f(u_i) = 2i + 1 \text{except the multiples of 3)}$$

$$f(u_i') = 2(f(u_i)) \text{ (except the multiples of 6)}$$

The remaining vertices of u_i and u_i' assign distinct odd and even numbers respectively

Using the above equation the edge labels are computed. We define the edge labeling as $g: E(S'(B_{n,n})) \rightarrow \{0,1,2\}$

$$g(wu_i') = 0$$

$$g(u_i w) = 1$$

$$g(u_i w')\ and\ \ g(u_i u_{i+1}) = 2$$

The number of edges with label '0', '1', '2' are

$$e_f(0) = e_f(1) = e_f(2) = 6n$$

Thus the condition $|\ e_f(i) - e_f(j)\ | \le 1$, for all $0 \le\ i, j\ \le 2$ is satisfied. Hence $S'(W_n)$ admits 3-equitable prime cordial labeling.

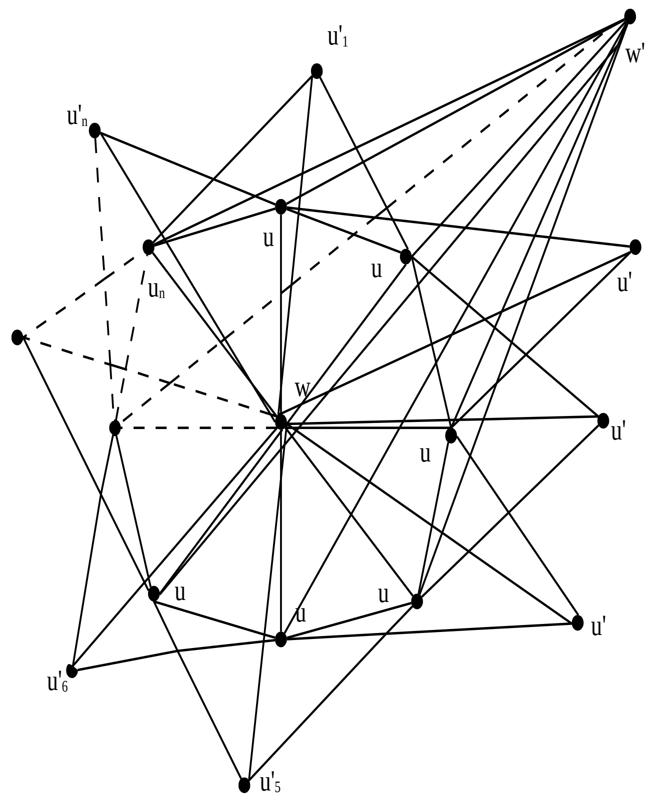

Figure 4: Splitting graph of wheel W_n

IV. Conclusion and Future Scope

By using a property from number theory,we proved 3-equitable prime cordial graph for some standard splitting graphs. To investigate similar results for other family as well in the context of different graph labeling problems is an open research area.

References

[1] Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, ArsCombinatoria, Vol 23, pp. 201-207, 1987.

[2] J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 16, 2017.

[3] F. Harary Graph Theory, Addition-Wesley, Reading Mass, 1972.

[4] S.Murugesan et al "*3-equitable Prime Cordial labeling of graphs*", Internatinal journal of Applied Information Systems, volume 5July 2013.

[5] Rosa A, (1966) "On certain valuation of the vertices of a Graph theory of graphs" Int.Symposium Rome, Gordon and Breach, N Y Dound, Paris pp. 349-355.

[6] E.Sampathkumar and H.B. Walikar, *On Splitting Graph of a Graph*, J.Karnatak Univ. Sci., vol 25(13)(1980), pp 13-16.

[7] M.Sundaram, R.Ponraj and S.Somasundaram, *Prime cordial labeling of graphs*, Journal of Indian Academy of Mathematics, vol 27(2005), pp 373-390

Authors Profile

Mrs.V. Sharon Philomena pursed Bachelor of Science from University of Madras, Chennai in 2002 and Master of Science from University of Madras in year 2004. She is currently pursuing Ph.D in Graph labeling. and working as Assistant Professor in PG Department of Mathematics, University of Madras, Chennai since 2010. She is a life member of Anna Periyar All India Mathematical Society. She has published more than 15 research papers in reputed International Journals including IJAER, International Journal of Computing algorithms. Organized conferences including UGC sponsored National workshops. She has received UGC –MRP a grant of 5 lakhs on Graph matching towards Women related Cancer. She has 15 years of teaching experience .

Ms KavyaB completed her Bachelor of Science from University ofMadras, Chennai 2017 and Pursuing Master of Science from University of Madras, Chennai.

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