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Generalized Measure of Fuzzy Entropy in Various Parameter

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Abstract- In the present paper, the fuzzy entropy measures are obtained by existing literatures. A generalized parametric fuzzy entropy in various parameter is defined. Fuzziness is one of the pandemic attributes of human thinking and objectives things whereas fuzzy set theory is one of the effective tools of researching and processing fuzzy phenomena in real world's problems. Taking this fact into contemplate, we have introduced and investigated new generalized measure of fuzzy entropy based upon real parameters, studied their fundamental and desirable properties and presented these measures through graph.

***Keywords*:** Fuzzy set, Fuzzy entropy, parametric fuzzy entropy.

I. INTRODUCTION

Uncertainty and fuzziness are the basic nature of human and thinking and of many real world objectives. Fuzziness is found in our decision, in our language and in the way we process information. The main use of information is to remove uncertainty and fuzziness. A measure of fuzziness which is often used and cited in the literature of fuzzy information is entropy first mentioned by Zadeh. Shannon's entropy measures the average uncertainty in bits associated with the prediction of outcomes in a random experiments. De Luca and Termini introduced some requirements which capture our intuitive comprehension of the degree of fuzziness and consequently developed a measure of fuzzy entropy which corresponds to Shannon's probabilistic entropy ,given by

$$H(A) = -\sum\left[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\right]$$

We measure information supplied by the amount of probabilistic uncertainty removed in an experiment and the measure of uncertainty removed is also called as a measure of information while measure of fuzziness is the measure of vagueness and ambiguity of uncertainties. In this paper, some important properties of fuzzy entropy are given.

II. MAIN RESULT

We have proposed two new generalized information measure for a fuzzy distribution { $\mu_A(x_1)$ $\mu_A(x_2)$ $\mu_A(x_3)$ ……….. $\mu_A(x_n)$, $0<\mu_A(x_i)<1$ }and studied their essential and other properties .

The proposed measure are

1.Generalised fuzzy entropy involving parameters are $\alpha,\beta,\gamma,m,and,a$

We propose the generalized fuzzy entropy depending upon parameters $\alpha,\beta,\gamma,m,and,a$ as given by following mathematical expression

$$H^{m,a}_{(\alpha,\beta,\gamma)}(A) = \frac{1}{a-\alpha}\sum_{i=1}^{n}\log\frac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a}(x_i)+(1-\mu_A(x_i))^{\alpha+m\beta+(1-m)\gamma-a}}{\mu_A^{m\beta+(1-m)\gamma}(x_i)+(1-\mu_A(x_i))^{m\beta+(1-m)\gamma}} \quad\text{.....................(1)}$$

where

$$m\in R,\beta\ge 0,\gamma\ge 0,\alpha+m\beta+(1-m)\gamma-a\ge 0, m\beta+(1-m)\gamma\ge 1$$

When $\alpha\to a,$ (1) becomes

$$H^{m,a}_{(\alpha,\beta,\gamma)}(A) = -\sum_{i=1}^{n}\frac{\mu_A^{m\beta+(1-m)\gamma}(x_i)\log\mu_A(x_i)+(1-\mu_A(x_i))^{m\beta+(1-m)\gamma}\log(1-\mu_A(x_i))}{\mu_A^{m\beta+(1-m)\gamma}(x_i)+(1-\mu_A(x_i))^{m\beta+(1-m)\gamma}} \quad\text{……………(2)}$$

If β =1, γ =1, then

$$H_{(\alpha,\beta,\gamma)}^{m,a}(A) = -\sum_{i=1}^{n} \mu_A(x_i)\log \mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))$$

Which is fuzzy entropy introduced by De Luca and Termini. Thus we see that the measure introduced in eq^n(1) is generalised measure of fuzzy entropy.

Next we study the properties of generalized measure of fuzzy entropy-

We see that

1. $H_{(\alpha,\beta,\gamma)}^{m,a}(A) \geq 0$

2. $\dfrac{\partial^2 H_{(\alpha,\beta,\gamma)}^{m,a}(A)}{\partial \mu_A^2(x_i)} < 0$

because $H_{(\alpha,\beta,\gamma)}^{m,a}(A) = \dfrac{1}{a-\alpha}\sum_{i=1}^{n}\log \dfrac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a}(x_i)+(1-\mu_A(x_i))^{\alpha+m\beta+(1-m)\gamma-a}}{\mu_A^{m\beta+(1-m)\gamma}(x_i)+(1-\mu_A(x_i))^{m\beta+(1-m)\gamma}}$

so $\dfrac{\partial H_{(\alpha,\beta,\gamma)}^{m,a}}{\partial \mu_A(x_i)} = \dfrac{1}{a-\alpha}\left[\dfrac{(\alpha+m\beta+(1-m)\gamma-a)\left\{\mu_A^{\alpha+m\beta+(1-m)\gamma-a-1}(x_i)-(1-\mu_A(x_i))^{\alpha+m\beta+(1-m)\gamma-a-1}\right\}}{\mu_A^{\alpha+m\beta+(1-m)\gamma-a}(x_i)-(1-\mu_A(x_i))^{\alpha+m\beta+(1-m)\gamma-a}}\right]$-

$\dfrac{1}{a-\alpha}\left[\dfrac{(m\beta+(1-m)\gamma)\left\{\mu_A^{m\beta+(1-m)\gamma-1}(x_i)-(1-\mu_A(x_i))^{m\beta+(1-m)\gamma-1}\right\}}{\mu_A^{m\beta+(1-m)\gamma}(x_i)-(1-\mu_A(x_i))^{m\beta+(1-m)\gamma}}\right]$

At $\mu_A(x_i) = \dfrac{1}{2}$,

$$\frac{\partial^2 H_{(\alpha,\beta,\gamma)}^{m,\alpha}}{\partial \mu_A^2(x_i)} = -4[\alpha - a + 2(m\beta+(1-m)\gamma - 1]$$

$$\frac{\partial^2 H_{(\alpha,\beta,\gamma)}^{m,\alpha}}{\partial \mu_A^2(x_i)} = -4[\alpha + m\beta+(1-m)\gamma - a + m\beta + (1-m)\gamma - 1]$$

Which is less than zero because $\alpha + m\beta + (1-m)\gamma - a \geq 0; m\beta + (1-m)\gamma - 1 \geq 0$

So, $H_{(\alpha,\beta,\gamma)}^{m,a}(A)$ is a concave function.

3. $H_{(\alpha,\beta,\gamma)}^{m,a}(A)$ doesnot change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$

4. $H_{(\alpha,\beta,\gamma)}^{m,a}(A)$ is an increasing function of $\mu_A(x_i)$ for $0 \leq \mu_A(x_i) \leq \dfrac{1}{2}$

i.e. $\left\{H_{(\alpha,\beta,\gamma)}^{m,a}(A) / \mu_A(x_i) = 0\right\} = 0$ and $\left\{H_{(\alpha,\beta,\gamma)}^{m,a}(A) \Big/ \mu_A(x_i) = \dfrac{1}{2}\right\} = n\log 2 > 0$

5 $H_{(\alpha,\beta,\gamma)}^{m,a}(A)$ is decreasing function of $\mu_A(x_i)$ for $\dfrac{1}{2} \leq \mu_A(x_i) \leq 1$

i.e. $\left\{H_{(\alpha,\beta,\gamma)}^{m,a}(A) \Big/ \mu_A(x_i) = \dfrac{1}{2}\right\} = n\log 2$ and $\left\{H_{(\alpha,\beta,\gamma)}^{m,a}(A) / \mu_A(x_i) = 1\right\} = 0$

6. $H_{(\alpha,\beta,\gamma)}^{m,a}(A)$=0 for $\mu_A(x_i)$=0 or 1.

7. Differentiating with respect to $\mu_A(x_i)$ and then putting $\dfrac{\partial H_{(\alpha,\beta,\gamma)}^{m,a}}{\partial \mu_A(x_i)} = 0,$

We get $\mu_A(x_i) = \frac{1}{2}$

Again at $\mu_A(x_i) = \frac{1}{2}$,

We have $\frac{\partial^2 H^{m,\alpha}_{(\alpha,\beta,\gamma)}}{\partial \mu_A^2(x_i)} = -4[\alpha - a + 2(m\beta + (1-m)\gamma - 1] < 0$

Thus we see that maximum value of fuzzy entropy exists at $\mu_A(x_i) = \frac{1}{2}$

If we denote the maximum value by f(n) =n log 2
Further f' (n) =log 2>0
This shows that the maximum value of the generalized fuzzy entropy is an increasing function of $\mu_A(x_i)$.

8. Differentiating (1) with respect to α ,we get

$$\frac{dH}{d\alpha} = \frac{1}{(a-\alpha)^2}\sum_{i=1}^{n}\log\left(\frac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a} + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}}{\mu_A^{m\beta+(1-m)\gamma} + (1-\mu_A)^{m\beta+(1-m)\gamma}}\right) +$$
$$\frac{1}{a-\alpha}\sum_{i=1}^{n}\frac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a}\log\mu_A + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}\log(1-\mu_A)}{\mu_A^{\alpha+m\beta+(1-m)\gamma-a} + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}}$$

$$(a-\alpha)^2\frac{dH}{d\alpha} = \sum_{i=1}^{n}\log\left(\frac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a} + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}}{\mu_A^{m\beta+(1-m)\gamma} + (1-\mu_A)^{m\beta+(1-m)\gamma}}\right) +$$
$$\sum_{i=1}^{n}\frac{\mu_A^{\alpha+m\beta+(1-m)\gamma-a}\log\mu_A^{a-\alpha} + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}\log(1-\mu_A)^{a-\alpha}}{\mu_A^{\alpha+m\beta+(1-m)\gamma-a} + (1-\mu_A)^{\alpha+m\beta+(1-m)\gamma-a}}$$

(i) When $\alpha > a$

Then $\frac{dH}{d\alpha} \leq 0$ which shows that H (A) is monotonically decreasing function of $\alpha > a$.

(ii) When $\alpha < a$

$$\left(\frac{dH}{d\alpha}\right)_{\alpha=0} < 0$$

Also, $\left(\frac{dH}{d\alpha}\right)_{\alpha=a} = 0$

By this, we see that $\frac{dH}{d\alpha}$ is negative for 0< $\alpha < a$

Hence from (i) and (ii), $H^{m,a}_{(\alpha,\beta,\gamma)}(A)$ is monotonically decreasing function of α ,for $\alpha \geq 0$

Thus fuzzy entropy defined by (i) satisfies all properties.

Next,we have computed different values of $H_1(A)$ for different values of parameters given in (1) and presented the generalized measure graphically.

Case I - Since in (1) $m \in R, \beta \geq 0, \gamma \geq 0, \alpha + m\beta + (1-m)\gamma - a \geq 0, m\beta + (1-m)\gamma \geq 1$,so suppose

$m\beta + (1-m)\gamma = 2,$

$\Rightarrow \alpha - a + 2 \geq 0, \Rightarrow \alpha \geq a - 2$

If $a = 2,$ then $\alpha \geq 0$

Taking different values of α ,we get different value of $H_1(A)$,

$$\alpha = 1 \qquad H_1(A) = -\log(\mu_A^2(x_i) + (1-\mu_A(x_i))^2)$$

$$\alpha = 2 \qquad H_1(A) = -\left(\frac{\mu_A^2(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))^2\log(1-\mu_A(x_i))}{\mu_A^2(x_i) + (1-\mu_A(x_i))^2}\right)$$

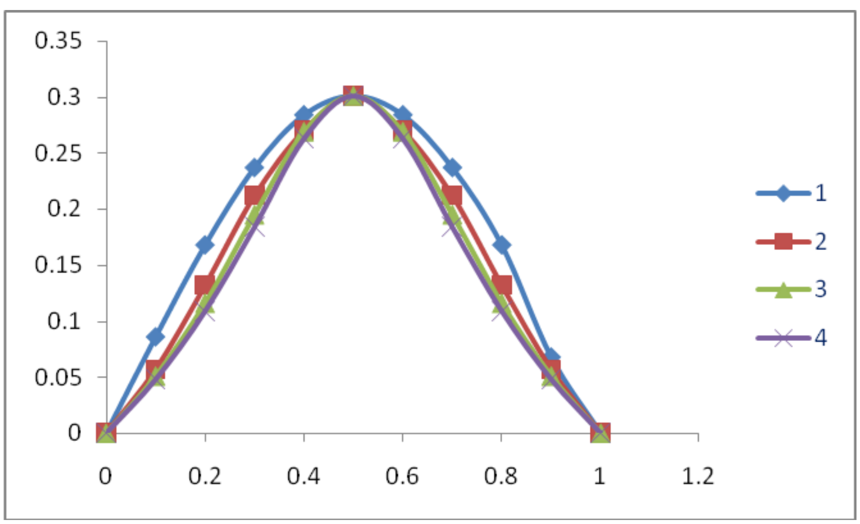
$$\alpha = 3 \qquad H_1(A) = -\log\left(\frac{\mu_A^3(x_i) + (1-\mu_A(x_i))^3}{\mu_A^2(x_i) + (1-\mu_A(x_i))^2}\right)$$

$$\alpha = 4 \qquad H_1(A) = -\log\left(\frac{\mu_A^4(x_i) + (1-\mu_A(x_i))^4}{\mu_A^2(x_i) + (1-\mu_A(x_i))^2}\right)$$

we have compiled the values of $H_1(A)$ for $\alpha \geq 0$ in Table(1) and presented the fuzzy entropy in Fig.(1) which clearly shows that the fuzzy entropy is a concave function.

Table(1)

α	$\mu_A(x_i)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	$H_1(A)$	0	0.086	0.168	0.237	0.284	0.301	0.284	0.237	0.168	0.086	0
2	$H_1(A)$	0	0.057	0.132	0.212	0.27	0.301	0.27	0.212	0.132	0.057	0
3	$H_1(A)$	0	0.051	0.116	0.195	0.269	0.301	0.269	0.195	0.116	0.051	0
4	$H_1(A)$	0	0.048	0.109	0.184	0.263	0.301	0.263	0.184	0.109	0.048	0



Where horizontal axis denotes value of $H_i(A)$ and vertical axis denotes value of $\mu_A(x_i)$.

III. CONCLUSION

Indeed we introduced new measure of fuzzy entropy in different parameters. This is a generalization of fuzzy entropy which were introduced by De-Luca & Termini [2] logarithmic entropy. This measure of entropy also satisfies different properties of entropy which have been proved by us and have been shown in table-1 and graph.

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