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A New Recursive Two Dimensional Pattern On Kolakoski Sequence

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Abstract— An efficient infinite Kolakoski sequence that's not even in any particular order can be generated in two dimensional [2D] array of size (3x3) over a binary alphabet $\Sigma = \{1,2\}$ is introduced and it is denoted by $K^{3c}_{(i,j)}$- (i-blocks, j – positions, $3c$- 3rd column). In this paper first 66 blocks with 100 positions from Kolakoski sequence is considered and 2D arrays are analyzed. Also combinatorial properties of the basis arrays are studied.

Keywords—2D word, Block, Fibonacci, Kolakoski, Palindrome.

I. INTRODUCTION

The equiangular spiral is one of the most interesting forms in nature, a single spiral galaxy may contain a trillion stars. Fibonacci and Kolakoski sequences have enormous number of spirals that could be related into nature of world. The Kolakoski sequence also known as the olderburger-kolakoski sequence and $Kol\ (w)$ is an infinite sequence over a binary alphabet [4] $\Sigma = \{1,2\}$ if it equals the sequence defined by its run lengths, 1 22 11 2 1 22 1 22 11... The main goal is to construct an infinite kolakoski sequence on which we take first 66 blocks (strings) with 100 positions shall be located in 2D array of size (3x3) which could be framed from each string($w^{n}_{(i,j)}$). When the Kolakoski sequence is partitioned as blocks and positions where the larger part divided by the smaller part is equal to the whole part divided by the larger part the proposition is golden ratio [1]. Also we try to prove that properties of 2D arrays are whether accomplished. And we follow fundamentals of languages [1] which contained prefix, suffix, palindrome $w^{p(n)}_{(i,j)}$ and we try to match its nature with 2D arrays which could lead us to get the classical Kolakoski sequence over a binary alphabet $\Sigma = \{1,2\}$.

II. DEFINITIONS AND NOTATIONS

Definition 2.1. We define an infinite sequence $Kol\ (w)$ over a binary alphabet $\Sigma = \{1,2\}$ under two iterating operations σ_0 and σ_1

$$\sigma_0(\text{Even}) = \begin{cases} 1 \to 1 \\ 2 \to 11 \end{cases} \ \& \ \sigma_1(\text{Odd}) = \begin{cases} 1 \to 2 \\ 2 \to 22 \end{cases}$$

Definition 2.2. Here we start an infinite $Kol\ (w)$ sequence with seed value as '1' under two iterating operations σ_0 and σ_1 hence the classical Kolakoski sequence is $Kol\ (w) =$ $1221121221221121122121121221121121221221121221211211221221121221221121121221211221221121221221121212211211221$ . . .

Definition 2.3. A string is a finite sequence of symbols which chosen from a binary alphabet $\Sigma = \{1,2\}$ and is denoted by w, here we define the string as $w^{n}_{(i,j)}$, where n is the number of strings, i, j are blocks and positions respectively in Kolakoski sequence.

Definition 2.4. A substring of leading symbols of a string is a prefix of a string, for an example if $w^{1}_{(3,5)} = \{12211\}$ is a string ($Kol\ (w)$ blocks)[see 2.2] then it has 4 prefixes which are $\{\varphi, 1, 122, 12211\}$, trailing symbols of that string is a suffix of a string, then $\{\varphi, 11, 2211, 12211\}$is the suffix of a string and then sub word of the string is '22'.

Definition 2.5. If a transpose of the string is equal to that same string shall be called as palindrome, for an example

$$w^{p(1)}_{(19,27)} = \{11211\}$$
$$w^{p(2)}_{(46,68)} = \{2112\}$$
$$w^{p(3)}_{(49,72)} = \{1221\}$$

where $p(n)$ is the number of palindrome strings from an infinite Kolakoski sequence.

[*When a palindrome string converted into a 2D array

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by using [2.1] its determinant value would be zero since 1st and 3rd row of palindrome 2D array would be the same since its prefix and suffix are same *]

Definition 2.6. A 2D array over a field F is a rectangular array of scalars each of which is a member of F. 2D arrays are commonly written in box brackets or parenthesis as,

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \text{ or } A=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

III. GENERATING KOLAKOSKI SEQUENCE IN TERMS OF 2D ARRAY OF SIZE (3X3)

The above Kolakoski sequence (See 2.2) could be generated by an algorithm, see [2.1] and let i^{th} iteration of the 2D array reads the number of blocks on sequence, j^{th} iteration reads that the number of positions on sequence.

We can obtain the above Kolakoski sequence by starting with '1' as a seed value and iterating the two substitutions from σ_0 and σ_1, see [2.1]

To generate Kolakoski sequence in 2D array of size (3x3) on every 3rd column of 2D array we take alternatingly σ_0 substitutes letlters on even positions and σ_1 substitutes letters on odd positions, see [2.1] and we denote the 2D array as $K^{n}_{(i,j)}$= $K^{3c}_{(i,j)}$then,

$$K^{3c}_{(i,3)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,6)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,9)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,12)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,15)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,18)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,21)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,24)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,27)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,30)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,33)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,36)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,39)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,42)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,45)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,48)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,51)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,54)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,57)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,60)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

$$K^{3c}_{(i,63)}=\begin{pmatrix} 1 & Even & \\ 2 & Odd & \\ 1 & Even & \end{pmatrix}$$

$$K^{3c}_{(i,66)}=\begin{pmatrix} 2 & Odd & \\ 1 & Even & \\ 2 & Odd & \end{pmatrix}$$

Here, 3rd column of each 2D array shall represent Kolakoski sequence.

Let's take first 3 blocks from above Kolakoski sequence, see [2.2]

1 22 11

Here 1st block is '1' which is represented by an even number σ_0, see [2.1], 2nd block is '22' which is represented by an odd number σ_1, see [2.1] and 3rd block is '11' which is represented by an even number σ_0.

It's clearly shows that, the letters are positioned as alternatingly as even, odd, even then,

Block 1: 1 → Even → 1
Block 2: 2 → Odd → 2
Block 3: 1 → Even → 2

We can represent it in 2D array of size 3x3 as,

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$$K^{3c}_{(2,3)} = \begin{pmatrix} 1 & Even & 1 \\ 2 & Odd & 2 \\ 1 & Even & 2 \end{pmatrix}$$

$$K^{3c}_{(2,3)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Let's take the next 3 blocks from above Kolakoski sequence (See 2.2)

2 1 22

Here 4th t block is '2' which is represented by an Odd number σ_1, see [2.1], 5th block is '1' which is represented by an Even number σ_0, see [2.1] and 6th block is '22' which is represented by an Odd number σ_1, see [2.1].

It clearly shows that, the letters are positioned as alternatively as odd, even, odd then,

Block 4: 2 → Odd → 1
Block 5: 1 → Even → 1
Block 6: 2 → Odd → 2

We can represent it in 2D array of size (3x3) as,

$$K^{3c}_{(4,6)} = \begin{pmatrix} 2 & Odd & 1 \\ 1 & Even & 1 \\ 2 & Odd & 2 \end{pmatrix}$$

$$K^{3c}_{(4,6)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

In the same way, we get 2D arrays as follows for the first 12 blocks with 18 positions from Kolakoski sequence, see [2.2].

$$K^{3c}_{(6,9)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(8,12)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(10,15)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(12,18)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

3.1. Palindrome of the string

Now the next string on kolakoski sequence, see [2.2] is, 11 2 11, see [2.5], then the corresponding 2D array is,

$$K^{3c}_{(15,21)} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$ [*Palindrome 2D array - 1st and 3rd row are same and so 2nd and 3rd column]

For the next blocks and positions we get following 2D arrays respectively,

$$K^{3c}_{(17,24)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(19,27)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(21,30)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(23,33)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(25,36)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(27,39)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(29,42)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(31,45)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Now the next string on kolakoski sequence, see [2.2] is, 2 11 2, see [2.5] and 1 22 1, see [2.5], then the corresponding 2D array is,

$$K^{3c}_{(34,48)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix}$$

$$K^{3c}_{(37,51)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

[*Palindrome 2D array - 1st and 3rd row are same and so 2nd and 3rd column]

For the next block and positions we get following 2D arrays respectively,

$$K^{3c}_{(39,54)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(41,57)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(43,60)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$K^{3c}_{(45,63)} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$K^{3c}_{(47,66)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

From the 3rd column of each 2D array we generate Kolakoski sequence for the first 47 blocks with 66 positions which are,

$Kol\,(w) =$ 1 22 11 2 1 22 11 2 11 2 2 1 2 11 2 1 22 11 2 11 2 1 22 1 22 11 2 1 22 1 2 1 1 2 1 1 22 1 22 11 2 1 22 1 22

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Lemma 1 For every palindrome string $w^{p(n)}_{(i,j)}$, see [2.5] from an infinite Kolakoski sequence, see [2.2] over a binary alphabet $\Sigma = \{1,2\}$, see [2.1], every 2D array of size (3x3) is singular.

Proof: A 2D array of size (3x3) is generated from the palindrome string $w^{p(n)}_{(i,j)}$, see [3.1] over a binary alphabet $\Sigma = \{1,2\}$, see [2.1] in which any two rows and two columns are same because of similarity of prefix and suffix, see [2.4] in every palindrome string, see [2.5] hence determinant value of a 2D array of size (3x3) is zero for every palindrome string under two iterating operations σ_0 and σ_1,see [2.1].

Lemma 2 For every non-palindrome string from an infinite Kolakoski sequence, see[2.2] over a binary alphabet $\Sigma = \{1,2\}$, see [2.1], determinant value of a 2D array of size (3x3) are exactly -3 and 3 alternatively (Invertible).

Proof: Non-Palindrome string in which prefix and suffix are different from each other so that any two row of the 2D array of size (3x3) should not be same under two iterating operations σ_0 and σ_1, see [2.1] hence the determinant value of the 2D array of size (3x3) will not be equal to zero (Invertible) and it will be exactly -3 and 3 alternatively for all the 2D arrays under non-palindrome string.

Corollary: In each 2D arrays of size (3x3) under the string $w^{n}_{(i,j)}$ the element a_{23} has a recurrence relation with $\{f_n\}_{n\geq 1}$.

Proof: Since we take three positions consistently to generate $K^{3c}_{(i,j)}$ (3x3) $\forall\ j = 3,6,9,12,15,18\ldots$ the i^{th} block could be achieved with the seed values $f_0 = 1, f_1 = 1$. If $j = 3n$ then $= \frac{j}{f_3/f_2}$. So that for each value of $n = 1,2,3,4,5,6,\ldots$ we get, $i = 2,4,6,8,10,12,\ldots$repectively.

[*Since we had three palindrome 2D arrays, see [2.5] inside 66 blocks of Kolakoski sequence, the recurrence relation with $\{f_n\}_{n\geq 1}$ will be nearly 1.5].

A 2D array of size (3x3) under two iterating operations σ_0 and σ_1, see [2.1] in which the elements a_{13} and a_{33} are 1 & 2 respectively except for the 2D array that has been constructed from the palindrome string, see [2.5]. And all the 1st and 2nd column of each 2D array including under palindrome string are 1 2 1, 2 1 2 (alternatively), even odd even (2 1 2), odd even odd (1 2 1) (alternatively), respectively. So that all the elements in 2D array of size (3x3) are predictable except a_{23} where the relation between block (i) and position (j) are exactly 1.5 (Nearly Golden Ratio) which is possible only on a_{23} among all the elements in 2D array of size (3x3). Hence it has recurrence relation with$\{f_n\}_{n\geq 1}$.

IV. COMBINATORIAL PROPERTIES OF THE 2D ARRAYS

4.1. Trace of 2D arrays

If $K^{n}_{(i,j)} = [a_{ij}]$ where blocks and positions are related with $\{f_n\}_{n\geq 1}$ then,

(i) $tr\left(K^{n}_{(i,j)}\right) = tr(\overline{K^{n}_{(i,j)}})$

(ii)$tr\left(K^{n}_{(i,j)} + K^{n}_{(i,j)}\right) = tr\left(K^{n}_{(i,j)}\right) + tr(K^{n}_{(i,j)})$

(iii)$tr\left(K^{n}_{(i,j)}\overline{K^{n}_{(i,j)}}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2$

4.2. Determinant of 2D arrays

Example: (i) If $K^{1}_{(2,3)} = \begin{pmatrix}1&2&1\\2&1&2\\1&2&2\end{pmatrix}$, see [3] then $\overline{K^{1}_{(2,3)}}) = \begin{pmatrix}1&2&1\\2&1&2\\1&2&2\end{pmatrix}$

Here $tr\ (K^{1}_{(2,3)}) = tr(\overline{K^{1}_{(2,3)}})$

(ii) If $K^{2}_{(4,6)} = \begin{pmatrix}2&1&1\\1&2&1\\2&1&2\end{pmatrix}$, see [3] and

$$K^{3}_{(6,9)} = \begin{pmatrix}1&2&1\\2&1&2\\1&2&2\end{pmatrix}$$

Then, $K^{2}_{(4,6)} + K^{3}_{(6,9)} = \begin{pmatrix}3&3&2\\3&3&3\\3&3&4\end{pmatrix} = tr\ (K^{2}_{(4,6)} + K^{3}_{(6,9)})$

$= tr\ (K^{2}_{(4,6)}) + tr\ (K^{3}_{(6,9)})$

(iii) If $K^{4}_{(8,12)} = \begin{pmatrix}2&1&1\\1&2&2\\2&1&2\end{pmatrix}$, see [3] and

$\overline{K^{4}_{(8,12)}}) = \begin{pmatrix}2&1&2\\1&2&1\\1&2&2\end{pmatrix}$

Hence, $tr\ (K^{4}_{(8,12)}\overline{K^{4}_{(8,12)}}) = tr\begin{pmatrix}6&6&7\\6&9&8\\7&8&9\end{pmatrix} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2$

(iv) As we concluded in lemma 2 every 2D array of size (3x3) over the binary alphabet $\Sigma = \{1,2\}$ under every non-palindrome string from an infinite kolakoski sequence, determinant value is alternatively -3 and 3. It means that every 2D array of size (3x3) where any two rows and columns are interchanged with post coming 2D array. Then if any two rows or columns of a 2D array are interchanged, then the sign of the value of determinant has been changed is showed.

V. CONCLUSION

An Infinite Kolakoski sequence can be generated from $K^{n}_{(i,j)} = \sum_{i\geq 2,\ j>2}^{m,n} K^{3c}_{(i,j)}$ where $K^{n}_{(i,j)}$ represents that the total number of 2D arrays is shown. The nature of 2D arrays could be lead us to generate an infinite Kolakoski sequence which is not in order, over the binary alphabet $\Sigma = \{1,2\}$ is shown. Also it has significant association with the Fibonacci sequence$\{f_n\}_{n\geq 1}$. Our future work focuses to obtain more combinatorial properties in this pattern.

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