IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Special Issue, 5, March 2019 E-ISSN: 2347-2693

# Dom-chromatic Number of certain graphs

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DOI: https://doi.org/10.26438/ijcse/v7si5.198202 | Available online at: www.ijcseonline.org

***Abstract--*** For a given χ-coloring of a graph $(V, E, \psi_G)$ , a dominating set $S \subseteq V(G)$ is said to be dom-colouring set if it contains at least one vertex from each colour class of G. The dom-chromatic number is the minimal cardinality taken over all dom-colouring sets and is denoted by $\gamma_{dc}(G)$. In this paper we have obtained the dom-chromatic number of various types of graphs like star graphs, windmill graphs, ladder graphs, comb graphs and for cycles.

***Keywords:*** Dominating set, Dom-colouring set, Star graphs, Windmill graphs, Ladder graphs, Comb graphs

## I. INTRODUCTION

In graph theory, coloring and dominating are two important areas which have been extensively studied. Domination has many other applications in the real world. In 1958, domination was formalized as a theoretical area in graph theory by C. Berge [1]. He referred to the domination number as the introduction co-efficient of external stability and denoted it as $\beta(G)$. In 1962, Ore was the first to use the term "domination" for undirected graphs and he denoted the domination number by $\delta(G)$ and also he introduced the concepts of minimal and minimum dominating sets of vertices in a graph[2]. In 1977, Cockayne and Hedetniemi was introduced the accepted notation $\gamma(G)$ to denote the domination number[3]. The book by Haynes, Hedetniemi and Slater depicts the application of the concept of domination in dominating queens, sets of representatives, school bus routing, computer communication networks, *(r, d)*-configurations, radio stations, social network theory, landing surveying, kernels of games etc. [7, 8]

## II. PRELIMINARIES

**Definition 2.1** [4, 9] *Let $G(V, E, \psi_G)$ be a graph. A subset $D \subseteq V$ of a graph G is called the dominating set, in which each vertex of G, is either in D or adjacent to some vertex in D*

**Illustration**

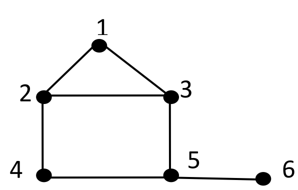


**Figure 1:** Graph *G* with 6 vertices and 7 edges

In the above graph {1, 4, 6} and {1, 5} are some dominating sets.

**Definition 2.2** *The minimum size of a dominating set of vertices of a graph G is called the domination number of G. It is denoted by $\gamma(G)$.*
*For the graph in Figure* 1, *the set* {1, 5} *is the minimal dominating set and hence the domination number* is 2.
The dominating set problem aims in finding the minimum dominating set of a graph *G* and such a set is called the $\gamma$-set of *G*.

**Definition 2.3** [5, 6] *A coloring of a graph G is an assignment of colors to its vertices so that no two adjacent vertices have the same color. The set of all vertices with any one color is independent and is called a color class. A graph which uses k- colours is called k- colouring. The chromatic number $\chi(G)$ is defined as the minimum k for which G has a k-coloring.*

**Definition 2.4** *For a given $\chi$-coloring of a graph $G(V, E, \psi_G)$ , a dominating set $S \subseteq V(G)$ is said to be dom-colouring set if it contains at least one vertex of each colour class of G.*

In Figure 1, colour the vertices 1 and 6 with colour red, the vertices 2 and 5 with colour blue and the vertices 3 and 4 with colour green. Thus the chromatic number of that graph is 3 and the graph uses colours red, blue and green. The colour classes of Figure 1 is given by {1, 6}, {2, 5} and {3, 4} whose vertices are coloured with red, blue and green respectively.

Some of the dominating sets of the graph in Figure 1 is {1, 5}, {1, 4, 5} and {1, 3, 5, 6} of which the set {1, 5} is minimal. But this set does not contain at least one vertex from all colour class. Thus it is not a dom-colouring set. But the sets {1, 4, 5} and {1, 3, 5, 6} has at least one vertex from each colour class. Hence they form the dom-colouring sets of that graph.

**Definition 2.5** *The dom-chromatic number is the minimal cardinality taken over all dom-colouring sets and is denoted by* $\gamma_{dc}(G)$.

For the graph in Figure 1, {1, 4, 5} is the minimal dom-colouring set and thus the dom-chromatic number of that graph is $\gamma_{dc}(G) = 3$.

## III. RESULTS AND DISCUSSION

### 3. DOM-CHROMATIC NUMBER OF STAR GRAPHS

**Definition 3.1** *A star graph is a complete bipartite graph* $K_{m,n}$ *in which the value of m is constant and is* 1.

**Theorem 3.1** *For all star graphs G denoted by* $K_{1,n}, n \geq 1$,

i. $\gamma_{dc}(G) = 2$
ii. $\gamma(G) \neq \gamma_{dc}(G)$

***Proof:*** Let $G$ be any star graph $K_{1,n}, n \geq 1$. Let the vertices of the partite sets be labeled as $\{v_1\}$ which belongs to the first partite set and $\{v_2, v_3, \ldots, v_n\}$ which belongs to the second partite set. Since $K_{1,n}$ is a complete bipartite graph, $v_1$ is adjacent to all the vertices $v_2, v_3, \ldots v_n$. Hence the vertex $v_1$ forms the minimum dominating set $D$ of $G$ (i.e) $D = \{v_1\}$. Therefore $\gamma(K_{1,n}) = 1$. But $D$ does not contain atleast one vertex from each colour class. So have to include another vertex from the other partite set. Hence $D = \{v_1, v_i\},\ i \geq 2$ is the minimum dominating set which contains at least one vertex from each colour class. Thus $\gamma_{dc}(K_{1,n}) = 2$. Since $\gamma(K_{1,n}) = 1$ and $\gamma_{dc}(K_{1,n}) = 2$, we have $\gamma(K_{1,n}) \neq \gamma_{dc}(K_{1,n})$ for all $n$. See Figure 2.

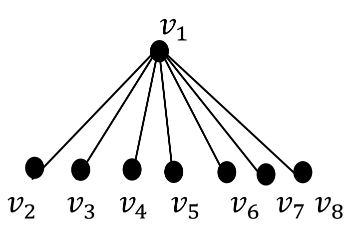


**Figure 2:** $\gamma(K_{1,7}) = 1$; $\gamma_{dc}(K_{1,7}) = 2$

### 4. DOM-CHROMATIC NUMBER OF COMB GRAPHS

**Definition 4.1** [12] *A graph obtained by attaching a single pendant edge to each vertex of a path* $P_p$ *is called a comb graph and is denoted by* $P_p^+$. *It consists of* $2p$ *vertices and* $2p-1$ *edges.*

**Theorem 4.1**[4] *For any path* $P_p$ *with* $p \geq 4$ *vertices,* $\gamma_{dc}(P_p) = \gamma(P_p) = \left\lceil \frac{p}{3} \right\rceil$.

**Theorem 4.2** *Let G be a comb graph* $P_p^+$. *Then* $\gamma_{dc}(G) = p$ *for all* $p$.

***Proof:*** Let $G$ be any comb graph $P_p^+$ with $2p$ vertices where $p \geq 2$. Let the vertices of $G$ be labeled as $\{v_1, v_2, \ldots v_p, u_1, u_2, \ldots u_p\}$ where $v_1, v_2, \ldots v_p$ represent the vertices of the path $P_p$ and $u_1, u_2, \ldots u_p$ represent the vertices of the pendant edges. See Figure 3.

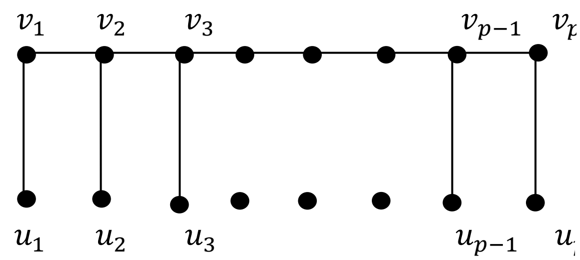


Figure 3: Comb graph $P_p^+$

Now colour the vertices of the path with colours 1 and 2 alternatively from left to right and the vertices of the pendant edges with colours 2 and 1 alternatively from left to right. By Theorem 4.1, $\gamma_{dc}(P_p) = \gamma(P_p) = \left\lceil \frac{p}{3} \right\rceil$. Inorder to find the minimum dominating set of the comb graph $P_p^+$, we have to include the pendant vertices of $P_p^+$ which are not adjacent to the vertices that are already taken in the dominating set of the path $P_p$. Here two cases arise.

**Case 1:** When $p$ is a multiple of 3.

By Theorem 4.1 $\gamma_{dc}(P_p) = \gamma(P_p) = \frac{p}{3}$. The minimum dominating and the dom-colour set of $P_p$ is $\left\{3i-1, 1 \leq i \leq \frac{p}{3}\right\}$. Let $D' = \left\{u_{3m-2}, u_{3m} \,/\, 1 \leq m \leq \frac{p}{3}\right\}$ be the minimum dominating set of the pendant vertices of $P_p^+$. Hence the minimum dominating set of $P_p$ and $D'$ form the dom-colouring set of $P_p^+$. That is it contains atleast one vertex of each colour class of the given graph. Hence $\gamma_{dc}(P_p^+) = \gamma_{dc}(P_p) + |D'| = \frac{p}{3} + 2\left(\frac{p}{3}\right) = p$.

**Case 2:** $p$ is not a multiple of 3.

Consider the largest vertex induced subgraph $G'$ of $P_p^+$ of order $3i, i \geq 1$ with vertices $V = \{v_1, v_2, \ldots, v_2 \,/ i \geq 1\}$. Let the number of vertices of $G'$ be $2(3i), i \geq 1$. Let $D = D' \cup D''$ be the minimum dominating set of $P_p^+$ where $D' =$

 

$\left\{v_{3m-1}, u_{3m-2},\ u_{3m}\ /\ 1 \le m \le \left\lfloor \frac{p}{3} \right\rfloor\right\}$ is the minimum dominating set of $G'$ and $D''$ is the minimum dominating set of $G'' = G(V) - G'(V')$.. Here two cases arise to find $D''$.

**Subcase (i)** $p \equiv 1(mod\ 3)$

Let $D'' = \{\, v_p\}$ be the minimum dominating set of $G''$. Therefore the minimum dominating set of ${P_p}^+$ is $D = D' \cup D'' = \left\{v_{3m-1}, u_{3m-2},\ u_{3m}\ /\ 1 \le m \le \left\lfloor \frac{p}{3} \right\rfloor\right\} \cup \{v_p\}$ form a dom-colouring set. Hence $\gamma_{dc}({P_p}^+) = |D| = |D' \cup D''| = 3\left\lfloor \frac{p}{3} \right\rfloor + 1 = p$. See Figure 4.

**Subcase (ii) )** $p \equiv 2(mod\ 3)$

Let $D'' = \{\, v_{p-1},\ u_p\}$ be the minimum dominating set of $G''$. Therefore the minimum dominating set of ${P_p}^+$ is $D = D' \cup D'' = \left\{v_{3m-1}, u_{3m-2},\ u_{3m}\ /\ 1 \le m \le \left\lfloor \frac{p}{3} \right\rfloor\right\} \cup \{\, v_{p-1},\ u_p\}$ form a dom-colouring set. Hence $\gamma_{dc}({P_p}^+) = |D| = |D' \cup D''| = 3\left\lfloor \frac{p}{3} \right\rfloor + 2 = p$.

**Illustration**

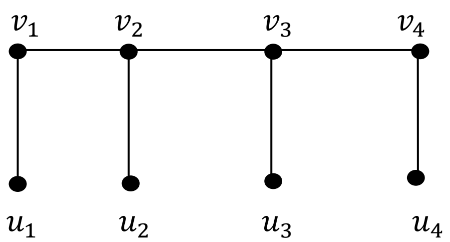


Figure 4: Comb graph $P_4^+$

The dominating set of the graph in Figure 4 is $\{\, v_2, u_1, u_3, v_4\}$ which is also equal to its minimum dom-colouring set and hence $\gamma_{dc}(G) = p = 4$.

## 5. DOM-CHROMATIC NUMBER OF LADDER GRAPHS

**Definition 5.1 [10]** *A ladder graph denoted by $L_n$ is a planar undirected graph with 2n number of vertices and 3n-2 edges. The ladder graph can be obtained as the cartesian product of 2 path graphs, one of which has only one edge. Ladder graphs are symbolically written as* $L_n = P_n \times P_2$.

**Remark 5.1** *For convenience we label the vertices of the paths of* $L_n$ *as* $P_1\!: v_2,\ v_4, \ldots\ v_{2n}$ *and* $P_2\!: v_1,\ v_3, \ldots\ v_{2n-1}$. *See Figure 5.*

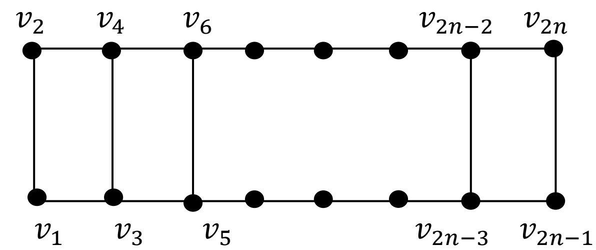


Figure 5: Ladder Graph $L_8$

**Theorem 5.1** *Let G be a ladder graph $L_n$ with 2n vertices and 3n-2 edges. Then* $\gamma_{dc}(L_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & when\ n\ is\ odd \\ \left\lceil \frac{n}{2} \right\rceil + 1 & when\ n\ is\ even \end{cases}$

***Proof:*** Let $G$ be a ladder graph $L_n$ with $2n$ vertices and $3n$-2 edges. Let $P_1\!: v_2,\ v_4,\ v_6\ \ldots\ v_{2n}$ and $P_2\!: v_1,\ v_3,\ v_5\ \ldots\ v_{2n-1}$ be the two paths of $L_n$. Colour the vertices in the path $P_1$ of $L_n$ with colours 1 and 2 alternatively and the vertices in the path $P_2$ of $L_n$ with colours 2 and 1 alternatively from left to right. The following cases arise in finding the dominating set of $L_n$.

**Case 1:** $n = 4k - 1$, where $k$ is any integer.
The dominating set $D = \{v_{8i-7},\ v_{8i-2}\}\, for\ i = 1,2 \ldots \left\lceil \frac{n}{4} \right\rceil$ is a minimum dominating set containing at least one vertex from each colour class.
∴ The cardinality of $D = \gamma_{dc}(L_n) = \left\lceil \frac{n}{4} \right\rceil + \left\lceil \frac{n}{4} \right\rceil$

$$= \left\lceil \frac{4k-1}{4} \right\rceil + \left\lceil \frac{4k-1}{4} \right\rceil$$

$$= \left\lceil k - \frac{1}{4} \right\rceil + \left\lceil k - \frac{1}{4} \right\rceil$$
$$= k + k = 2k$$
$$= 2\left(\frac{n+1}{4}\right) = \frac{n+1}{2} = \frac{n}{2} + \frac{1}{2} = \left\lceil \frac{n}{2} \right\rceil,$$
(since $n$ is odd)

**Case 2:** $n = 4k$ , where $k$ is any integer.
The dominating set $D = \left\{v_{8i-7},\ v_{8i-2}\ for\ i = 1, 2 \ldots \left\lceil \frac{n}{4} \right\rceil\right\} \cup \{v_{2n-1}\}$ is a minimum dominating set containing at least one vertex from each colour class.
∴ The cardinality of $D = \gamma_{dc}(L_n) = \left\lceil \frac{n}{2} \right\rceil + 1$

**Case 3:** $n = 4k + 1$, where $k$ is any integer.
The dominating set $D = \{v_1\} \cup \{v_{8i-2},\ v_{8i+1}\}\, for\ i = 1, 2 \ldots \left\lceil \frac{n}{4} \right\rceil$ is a minimum dominating set containing at least one vertex from each colour class.
∴ The cardinality of $D = \gamma_{dc}(L_n) = 1 + \left\lceil \frac{n}{4} \right\rceil + \left\lceil \frac{n}{4} \right\rceil$
$$= 1 + \left\lceil \frac{4k+1}{4} \right\rceil + \left\lceil \frac{4k+1}{4} \right\rceil = 1 + \left\lceil k + \frac{1}{4} \right\rceil + \left\lceil k + \frac{1}{4} \right\rceil$$

 

$= 1 + k + k = 1 + 2k = 1 + 2\left(\frac{n-1}{4}\right) = 1 + \frac{n-1}{2}$

$= 1 + \frac{n}{2} - \frac{1}{2} = \frac{n}{2} + \frac{1}{2} = \left\lceil \frac{n}{2} \right\rceil$ (since $n$ is odd)

**Case 4:** $n = 4k + 2$, where $k$ is any integer.

The dominating set $D = \{v_1\} \cup \left\{v_{8i-2},\ v_{8i+1}\ for\ i = 1, 2 \ldots \left\lceil \frac{n}{4} \right\rceil\right\} \cup \{v_{2n}\}$ is a minimum dominating set containing at least one vertex from each colour class.

$\therefore$ The cardinality of $D = \gamma_{dc}(L_n) = \left\lceil \frac{n}{2} \right\rceil + 1$.

## 6. DOM-CHROMATIC NUMBER OF WINDMILL GRAPHS

**Definition 6.1** [11] *The windmill graph $W_n^{(m)}$ is a graph obtained by taking m copies of the complete graph $K_n$ with a vertex in common.*

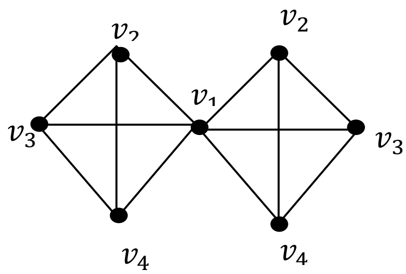


Figure 6: Windmill graph $W_4^{(2)}$

In Figure 6, the vertex $v_1$ adjacent to all other vertices is coloured with colour 1 and the vertices $v_2, v_3, v_4$ in the copy are coloured with colours 2, 3, 4.

**Theorem 6.1** *For all windmill graphs $W_n^{(m)}$, $\gamma_{dc}\left(W_n^{(m)}\right) = n$.*

***Proof:*** Let $G$ be a windmill graph $W_n^{(m)}$ with $nm - (m - 1)$ vertices. Let the vertices be labeled as $v_1, v_2, \ldots, v_n$ for each $m$ copies of the complete graph. Let $v_1$ be adjacent to all the vertices of $G$. Let this vertex be coloured with colour 1. Consider one copy of the $m$ complete graphs of $W_n^{(m)}$. Since the vertices of this copy are connected to every other vertex present in the same copy, colour each such vertex $v_2, v_3, \ldots, v_n$ with distinct $n - 1$ colours (leaving the vertex $v_1$ that is already coloured). The same colouring is repeated for the remaining $m - 1$ copies of $W_n^{(m)}$. Clearly $D = \{v_1\}$ is the minimum dominating set of $G$. But this does not contain at least one vertex from each colour class. So we have to include the remaining $n - 1$ vertices of any one copy of the complete graph. Thus we get $D = \{v_1\} \cup \{v_2, v_3, \ldots, v_n\}$ which is the minimum dominating set which contains at least one vertex from each colour class. Hence $\gamma_{dc}\left(W_n^{(m)}\right) =$ Cardinality of $D = 1 + (n - 1) = n$.

## 7. DOM-CHROMATIC NUMBER OF CYCLE GRAPHS

**Definition 7.1** *A closed walk $v_1, \ldots\ldots v_n, v_1$ in which $n \geq 3$ and $v_1, v_1, \ldots\ldots v_n$ are distinct is called a cycle of length n. The cycle consisting of 'n' vertices is denoted by $C_n$.*

**Remark 7.1** *For convenience we label the vertices of $C_n$ as* 1, 2, *.....,n in the clockwise direction.*

**Theorem 7.1** *Let G be a cycle $C_n$, of length* $n > 5$. *Then* $\gamma_{dc}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

***Proof:*** Let $C_n$ be labeled as $v_1, v_2, \ldots, v_n$ in the clockwise direction.

**Case 1: When *n* is odd**

Colour the first vertex of the cycle with colour *a*. Choose $\frac{n-3}{2}$ number of vertices in both clockwise and anticlockwise direction and colour the vertices alternatively with colours *b* and *a*. The remaining last two vertices are coloured with either colours *c* and *a* or *c* and *b* in the clockwise direction. That is, if the vertices adjacent to the last two vertices are coloured *a* then use colours *c* and *b*, or of the vertices adjacent to the last two vertices are coloured *b*, then use colours *c* and *a*.

Since each vertex in a cycle is adjacent to exactly two vertices, the vertices in a cycle are considered in sets of three to form the minimum dominating and dom-colouring sets. Here 3 cases arise.

**Subcase (i)** When $n \equiv 0(mod\ 3)$ we have $n = 3k,\ k \geq 3$.

The set $D = \{v_2,\ v_5,\ v_8,\ v_{11}, \ldots v_{n-1}\}$. (i.e) $D = \left\{v_{3i-1}, 1 \leq i \leq \frac{n}{3}\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class. Hence $\gamma_{dc}(C_n) = \frac{n}{3}$.

**Subcase (ii)** When $n \equiv 1(mod\ 3)$ we have $n = 3k + 1$, $k \geq 2$ and $n$ is odd.

The set $D = \left\{v_2,\ v_5,\ v_8,\ v_{11}, \ldots,\ v_{\frac{n-3}{2}}\right\} \cup \left\{v_{\frac{n+1}{2}}\right\} \cup \left\{v_{\frac{n+5}{2}},\ v_{\frac{n+11}{2}},\ v_{\frac{n+17}{2}}, \ldots, (n-1)\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class.

Hence $\gamma_{dc}(C_n) = \frac{k}{2} + 1 + \frac{k}{2} = \frac{2k}{2} + 1 = k + 1$

$= \frac{n-1}{3} + 1 = \frac{n+2}{3} = \frac{n}{3} + \frac{2}{3} = \left\lceil \frac{n}{3} \right\rceil$.

**Subcase (iii)** When $n \equiv 2(mod\ 3)$ we have $n = 3k + 2$, $k \geq 2$ and $n$ is odd.

 

The set $D = \left\{v_1,\ v_4,\ v_7,\ v_{10}, \dots ,\ v_{\frac{n-3}{2}}\right\} \cup \left\{v_{\frac{n+1}{2}}\right\} \cup \left\{v_{\frac{n+7}{2}},\ v_{\frac{n+13}{2}},\ v_{\frac{n+19}{2}}, \dots , (n-2)\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class.

Hence $\gamma_{dc}(C_n) = \frac{k+1}{2} + 1 + \frac{k-1}{2} = \frac{2k}{2} + 1 = k+1$
$= \frac{n-2}{3} + 1 = \frac{n+1}{3} = \frac{n}{3} + \frac{1}{3} = \left\lceil \frac{n}{3} \right\rceil$.

**Case 2: When *n* is even**

Colour the first vertex of the cycle with colour $a$. Choose $\frac{n-2}{2}$ number of vertices in both clockwise and anticlockwise direction and colour the vertices alternatively with colours *b* and *a*. The remaining last vertex $\frac{n}{2} + 1$ is coloured with either colours *b* or *a*. That is, if the vertices adjacent to $\frac{n}{2} + 1$ is coloured with colour $a$ then $\frac{n}{2} + 1^{th}$ vertex is coloured with *b* and vice versa.

Since each vertex in a cycle is adjacent to exactly two vertices, the vertices in a cycle are considered in sets of three to form the minimum dominating and dom-colouring sets. Here 3 cases arise.

**Subcase (i)** When $n \equiv 0\ (mod\ 3)$ we have $n = 3k,\ k \geq 2$ and *n* is even.

The set $D = \{v_1,\ v_4,\ v_7,\ v_{10}, \dots\ v_{n-2}\}$. (i.e) $D = \left\{v_{3i-2}, 1 \leq i \leq \frac{n}{3}\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class. Hence $\gamma_{dc}(C_n) = \frac{n}{3}$.

**Subcase (ii)** When $n \equiv 1 (mod\ 3)$ we have $n = 3k+1$, $k \geq 2$ and *n* is even.

The set $D = \left\{v_1,\ v_4,\ v_7,\ v_{10}, \dots\ v_{\frac{n}{2}-1}\right\} \cup \left\{v_{\frac{n}{2}+1}\right\} \cup \left\{v_{\frac{n}{2}+3},\ v_{\frac{n}{2}+6},\ v_{\frac{n}{2}+9}, \dots , (n-2)\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class.

Hence $\gamma_{dc}(C_n) = \frac{k+1}{2} + 1 + \frac{k-1}{2} = \frac{2k}{2} + 1 = k+1$
$= \frac{n-2}{3} + 1 = \frac{n+1}{3} = \frac{n}{3} + \frac{1}{3} = \left\lceil \frac{n}{3} \right\rceil$.

**Subcase (iii)** When $n \equiv 2 (mod\ 3)$ we have $n = 3k+2$, $k \geq 2$ and *n* is even.

The set $D = \left\{v_1,\ v_4,\ v_7,\ v_{10}, \dots ,\ v_{\frac{n}{2}}\right\} \cup \left\{v_{\frac{n}{2}+2},\ v_{\frac{n}{2}+5},\ v_{\frac{n}{2}+8}, \dots , (n-2)\right\}$ is the minimum dominating set of $C_n$. Clearly $D$ contains at least one vertex from each colour class.

Hence $\gamma_{dc}(C_n) = \frac{k}{2} + 1 + \frac{k}{2} = \frac{2k}{2} + 1 = k+1$
$= \frac{n-1}{3} + 1 = \frac{n+2}{3} = \frac{n}{3} + \frac{2}{3} = \left\lceil \frac{n}{3} \right\rceil$.

## IV. CONCLUSION

The concepts on domination number and dom- chromatic number has been discussed in this paper. The results have been extended to various types of graphs like Star graphs, Windmill graphs, Ladder graphs, Comb graphs and for Cycles.

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