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Special properties of Fibonacci Array Based on Dimension

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Abstract— In this paper the Fibonacci array based on the dimension are defined and analysed. The bordered width of the Fibonacci array is a Fibonacci number is shown. The concept of secondary transformation, linear tandem, diagonal tandem of an array are introduced. The combinatorial properties of the sub arrays are investigated. The Fibonacci array based on tree is represented and also Parikh vector concepts are discussed.

Keywords— Fibonacci array; Parikh Vectors; Secondary Transpose.

I. INTRODUCTION

In the field of DNA based computation, the concepts of repetition, tandem and palindrome are widespread. The biologist believes that the regulation of activity of the gene and other cell processes, palindrome have played an important role. For finding the cancer cell detection, the pattern matching are widely used. The study of fractal geometry, formal languages, number theory and quasicrystals Fibonacci are widely used. The study of two-dimensional (2-D) languages plays an important role in the theory of image analysis. Image analysis is a well-established and active field with numerous applications. In combinatorics Fibonacci sequence plays a vital role and the applications are widely used in computer science, graphics and molecular biology. The combinatorial properties were used on Fibonacci words in 1970 [1,2,6]. The basic definition and the properties studied by Alberto Apostolico, Valentin E, Brimkov [1]. Jansirani N and Rajkumar Dare introduced Fibonacci 2-D structure and studied in the two dimensional direction [4,5]. Recently the number of repetition of the same structure and distinct palindrome occurrences in any Fibonacci sub arrays are investigated by Mahalingam K, Sivasankar M, Krithivasan K [7]. Anna Lee was introduced the secondary symmetric and secondary orthogonal matrices[3]. From 1966, the classical Parikh mapping which maps from words to vectors are introduced and this is the tool for discovering important feature of languages[8]. In this paper, the relationship between the Fibonacci array and dimension are discussed with an examples. The beautiful combinatorial properties of these arrays are discussed and example of the Fibonacci array is given. Through the concept of row reverse(column reverse) of an array, the linear tandems are defined and through the secondary transpose the diagonal tandem are defined. For constructing the Fibonacci array, the tree concepts are introduced and examined through an example. The Parikh vector of the Fibonacci array are introduced and the relation between the Parikh vector of the Fibonacci array and Fibonacci number are derived.

II. BASIC DEFINITIONS AND PRELIMINARIES

An alphabet Σ is a finite non-empty set of symbols or letters. Σ^* denotes the set of all words Σ including the empty word λ. Σ^+ is the set of all non-empty word over Σ. The length of a word $u \in \Sigma^*$ is denoted by $|u|$. A word $u \in \Sigma^*$ is a prefix of the word w if $w = uv$ for some $v \in \Sigma^*$. The reversal of $u = a_1 a_2 \dots a_n$ is defined to be a string $u^R = a_n \dots a_2 a_1$, where $a_i \in \Sigma$ for $1 \le i \le n$. A word u is said to be a palindrome if $u = u^R$. A word w is said to be primitive if $w = u^n$ implies $n = 1$ and $w = u$. For $\Sigma = \{a, b\}$, the sequence $\{f_n\}, n \ge 1$ defined recursively by $f_0 = a$, $f_1 = b$, $f_n = f_{n-1} f_{n-2}$ for $n \ge 2$ is called the sequence of Fibonacci words. The set of all arrays over Σ is denoted by Σ^{**}. The set of all infinite arrays over Σ is denoted by $\Sigma^{\omega\omega}$.

Definition:2.1

An array or a two –dimensional word, $W = [w_{i,j}]_{1 \le i \le m, 1 \le j \le n}$ of size (m, n) over an alphabet Σ is defined as the two dimensional rectangular arrangement of letters from the alphabet Σ:

$$\begin{pmatrix} w_{m,1} & w_{m,2} & \cdots & w_{m,(n-1)} & w_{m,n} \\ w_{(m-1),1} & w_{(m-1),2} & \cdots & w_{(m-1),(n-1)} & w_{(m-1),n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ w_{2,1} & w_{2,2} & \cdots & w_{2,(n-1)} & w_{2,n} \\ w_{1,1} & w_{1,2} & \cdots & w_{1,(n-1)} & w_{1,n} \end{pmatrix}$$

The bottom most row as the first row and the left most column as the first column convention is adopted in this paper.

Definition:2.2
A subarray of an array W is also an array which is a part of W (formed by the intersection of certain consecutive rows and certain consecutive columns of W).

Definition:2.3
Any rectangular subarray of W is a block. A block $X = W[0 \ldots k, 0 \ldots l], 0 \leq k < m$ and $0 \leq l < n$ is a prefix of W, while a block $X = W[k \ldots m-1, l \ldots n-1], 0 \leq k < m$ and $0 \leq l < n$ is a suffix of W.

Definition:2.4
An array W is primitive if it cannot be partitioned into non-overlapping replicas of some block X,

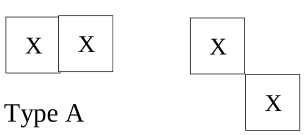
$i.e.$, if setting $W = \begin{bmatrix} X & \ldots & X \\ \ldots & \ldots & \ldots \\ X & \ldots & X \end{bmatrix}$, where W has k rows and l columns, implies $k = 1, l = 1$.

Definition:2.5

An array $W = \begin{array}{|ccc|c|} \hline X & \ldots & X & X' \\ \ldots & \ldots & \ldots & \ldots \\ X & \ldots & X & X' \\ \hline X'' & \ldots & X'' & X''' \\ \hline \end{array}$

is a two dimensional repetition if there are at least two X-blocks along both dimensions.

Definition:2.6
The tandem in W is a configuration consisting of two occurrences of a same primitive blocks X that touch each other with one side or corner.



Type A Type B

Definition:2.7
An array $A = (a_{ij})$ of size (m, n) over an alphabet $\sum$, the transpose of the array is defined as $(a_{ij})^T = (a_{ji})$ for all $1 \leq i \leq m$, $1 \leq j \leq n$.

Definition:2.8
The reverse image of W denoted by W^R is defined as

$$W^R = [w_{m-i+1, n-j+1}], 1 \leq i \leq m, 1 \leq j \leq n.$$

An array W is said to be palindrome if $W = W^R$.

Definition:2.9
The sequence of Fibonacci arrays $f_{m,n} (m, n \geq 0)$ is defined as follows:
For $m \geq 1$ and $n \geq 2$(with column-wise reduction)

$$f_{m,n} = f_{m,n-2} \cdot f_{m,n-1} = [f_{m,n-2}, f_{m,n-1}]$$

and for $m \geq 2$ and $n \geq 1$(with row-wise reduction)

$$f_{m,n} = f_{m-2,n} \ominus f_{m-1,n} = \begin{bmatrix} f_{m-2,n} \\ f_{m-1,n} \end{bmatrix}$$

and so on recursively until all the entries are one of $\{f_{0,0}, f_{0,1}, f_{1,0}, f_{1,1}\}$.

Remark:2.10
In either way reduction (row to column or column to row), the order and the element of the array will be same.

Example:2.11

$$f_{3,2} = f_{3,0} \cdot f_{3,1} = [f_{3,0} f_{3,1}] = \begin{bmatrix} f_{1,0} & f_{1,1} \\ f_{0,0} & f_{0,1} \\ f_{1,0} & f_{1,1} \end{bmatrix}$$

$$f_{3,2} = f_{1,2} \ominus f_{2,2} = \begin{bmatrix} f_{1,2} \\ f_{2,2} \end{bmatrix} = \begin{bmatrix} f_{1,0} & f_{1,1} \\ f_{0,0} & f_{0,1} \\ f_{1,0} & f_{1,1} \end{bmatrix}.$$

Definition:2.12
The Fibonacci number are the sequence of numbers $\{F_n\}_{n=1}^{\infty}$ defined by the linear recurrence equation $F_n = F_{n-1} + F_{n-2}$, with $F_1 = F_2 = 1$.

Definition:2.13
If a prefix and a suffix of an array are equal then the array is said to be bordered.

Definition:2.14
If the first and the last row (column) of an array are equal then the array is said to be row (column) bordered of width one.

Definition:2.15
If the first and the last r rows (columns) of an array is said to be row (column) bordered of width r.

III. DIMENSION OF FIBONACCI ARRAY

In this section we give the definition of the dimension of the Fibonacci array, secondary transpose, linear tandem and diagonal tandem of an array and discuss the basic properties.

Definition:3.1
The dimension of the Fibonacci array is a number of rows and columns of the reduced (concatenation) array.
Example:3.2

$$W_{3\times4} = \begin{bmatrix} f_{11}\, f_{10}\, f_{11}\, f_{11}\, f_{10} \\ f_{01}\, f_{00}\, f_{01}\, f_{01}\, f_{00} \\ f_{11}\, f_{10}\, f_{11}\, f_{11}\, f_{10} \end{bmatrix}_{3\times5}$$

Definition:3.3

An array $A = (a_{ij})$ of size (m, n) over an alphabet $\sum$, the secondary transpose of the array is defined as
$A^S = (a_{ij})^S = a_{n-j+1, m-i+1}$, for all $1 \leq i \leq m$, $1 \leq j \leq n$.

Definiton:3.4
The linear tandem (LT) in W is a configuration consisting of the occurrences of a primitive blocks X and $(X_R)^R$(denotes the row reversal of X) or $(X_c)^R$ (denotes the column reversal of X) that touch each other with one side.

X	$(\boldsymbol{X_R})^{\boldsymbol{R}}\boldsymbol{or}\ (\boldsymbol{X_c})^{\boldsymbol{R}}$

Definiton:3.5
The diagonal tandem (DT) in W is a configuration consisting of the occurrences of a primitive blocks X and $(X^T)^S or\ (X^S)^T$ that touch each other in diagonal.

X	
	$(\boldsymbol{X^T})^{\boldsymbol{S}}\boldsymbol{or}\ (\boldsymbol{X^S})^{\boldsymbol{T}}$

Theorem:3.6
Let $\mathrm{M}_{(m,n)}$ be an array, then the dimension of a Fibonacci array is a Fibonacci number.

$$dim(\mathrm{M}_{(m,n)}) = \mathrm{F}(m+1), \mathrm{F}(n+1).$$

Proof:
Proof of this theorem is by mathematical induction method. The changes of the dimensions are occurred in $m, n \geq 4$. So to prove this is true for $m = 4, n = 5$

$$M_{4\times5} = \begin{bmatrix} f_{01} & f_{00} & f_{01} & f_{00} & f_{01} & f_{01} & f_{00} & f_{01} \\ f_{11} & f_{10} & f_{11} & f_{10} & f_{11} & f_{11} & f_{10} & f_{11} \\ f_{11} & f_{10} & f_{11} & f_{10} & f_{11} & f_{11} & f_{10} & f_{11} \\ f_{01} & f_{00} & f_{01} & f_{00} & f_{01} & f_{01} & f_{00} & f_{01} \\ f_{01} & f_{10} & f_{11} & f_{10} & f_{11} & f_{11} & f_{10} & f_{11} \end{bmatrix}_{5\times8}$$

Then the dimension of the array is (5,8).
$F(4) = 5$ and $F(5) = 8$. Hence this is true for $m = 4, n = 5$. Now assume this is true for $t, s \in \mathbb{N}$ (set of all positive integer).
Then, we to prove this is true for $m = t+1, n = s+1$.

$$F(t+1), F(s+1) = F(t) + F(t-1), \mathrm{F}(s) + F(s-1)$$

Where $F(t), F(t-1), \mathrm{F}(s), F(s-1) \in \mathcal{F}$ (set of all Fibonacci number). Hence the theorem is true for all $m, n \in \mathbb{N}$.

Theorem: 3.7
Let $F_{m\times n}$ be a Fibonacci array, then the bordered width of the column array is $F(n-1)$. Similarly the bordered width of the row array is $F(m-1)$.
Proof:
The proof is by the concept of mathematical induction of positive integers. To prove this theorem is true for $m = 3, n = 4$

$$F_{3\times4} = \begin{bmatrix} \boldsymbol{f_{10}} & \boldsymbol{f_{11}} & \boldsymbol{f_{11}} & \boldsymbol{f_{10}} & \boldsymbol{f_{11}} \\ f_{11} & f_{01} & f_{01} & f_{00} & f_{01} \\ \boldsymbol{f_{10}} & \boldsymbol{f_{11}} & \boldsymbol{f_{11}} & \boldsymbol{f_{10}} & \boldsymbol{f_{11}} \end{bmatrix}_{3\times5}$$

Here $F(2) = 1$. Hence the border width of the row array is 1.

$$F_{3\times4} = \begin{bmatrix} \boldsymbol{f_{10}} & \boldsymbol{f_{11}} & \boldsymbol{f_{11}} & \boldsymbol{f_{10}} & \boldsymbol{f_{11}} \\ f_{11} & f_{01} & f_{01} & f_{00} & f_{01} \\ \boldsymbol{f_{10}} & \boldsymbol{f_{11}} & \boldsymbol{f_{11}} & \boldsymbol{f_{10}} & \boldsymbol{f_{11}} \end{bmatrix}_{3\times5}$$

Here, $F(3) = 2$. Hence the bordered width of the column array is 2. Similarly, we can prove for (m, n) arrays.

Lemma:3.8
Given a Fibonacci array $F_{m\times n}$,
(a) $F_{m\times n}$ cannot be begin or end with two consecutive occurrences of a same symbol.
(b) $F_{m\times n}$ cannot contain three consecutive occurrences of a same symbol.
Proof:
The proof of the lemma follows from the construction of the array itself.

Lemma:3.9
Let $F_{m\times n}$ be a Fibonacci array over the alphabet $\{a, b, c, d\}$. Then $F_{n\times n}$ cannot contain a subarray of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^i, \begin{pmatrix} b & a \\ d & c \end{pmatrix}^i, \begin{pmatrix} c & d \\ a & b \end{pmatrix}^i, \begin{pmatrix} d & c \\ b & a \end{pmatrix}^i$ with $i \geq 2$,
$f_{00} = a, f_{01} = b, f_{10} = c, f_{11} = d$.
Proof:
The proof of the lemma from considering the distinct subarrays of dimension 2×2.

Properties:3.10
Given a Fibonacci array $F_{m\times n}$,
(a) if (m, n) is even, then
(i). LT and DT are occurred.
(ii). The subarray $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is occurred in all the edges.
(b) if (m, n) is odd, then
(i). Type A and Type B tandems are occurred.
(ii). The subarray (d) is occurred in all the edges.
(c) if m is odd, n is even then
(i). LT and Type A tandems are occurred.
(ii). The subarray (c) is occurred in $w_{1,1}, w_{m,1}$ and the subarray (d) is occurred in $w_{1,n}, w_{m,n}$.
(d) if m is even, n is odd then
(i). LT , Type A tandems are occurred.
(ii). The subarray (d) is occurred in $w_{1,1}, w_{1,n}$ and the subarray (b) is occurred in $w_{m,1}, w_{m,n}$.

Remark:3.11
For a Fibonacci array $F_{3\times3}$, the distinct subgroup complexity are

$(a), (b), (c), (d), (a \quad b), (b \quad a), (c \quad d), (d \quad c), \begin{pmatrix} a \\ c \end{pmatrix},$

$\begin{pmatrix} c \\ a \end{pmatrix}, \begin{pmatrix} d \\ b \end{pmatrix}, \begin{pmatrix} b \\ d \end{pmatrix}, (d \quad c \quad d), (c \quad d \quad c), (d \quad d \quad c),$

$(c \quad d \quad d), (b \quad a \quad b), (a \quad b \quad a), (b \quad b \quad a),$

$(a \quad b \quad b), \begin{pmatrix} d \\ b \\ d \end{pmatrix}, \begin{pmatrix} b \\ d \\ b \end{pmatrix}, \begin{pmatrix} d \\ d \\ b \end{pmatrix}, \begin{pmatrix} b \\ d \\ d \end{pmatrix}, \begin{pmatrix} c \\ a \\ c \end{pmatrix}, \begin{pmatrix} a \\ c \\ a \end{pmatrix}, \begin{pmatrix} c \\ c \\ a \end{pmatrix}, \begin{pmatrix} a \\ c \\ c \end{pmatrix},$

$\begin{pmatrix} d & c & d \\ b & a & b \end{pmatrix}, \begin{pmatrix} c & d & c \\ a & b & a \end{pmatrix}, \begin{pmatrix} d & d & c \\ b & b & a \end{pmatrix}, \begin{pmatrix} c & d & d \\ a & b & b \end{pmatrix},$

$\begin{pmatrix} b & a & b \\ d & c & d \end{pmatrix}, \begin{pmatrix} a & b & a \\ c & d & c \end{pmatrix}, \begin{pmatrix} b & b & a \\ d & d & c \end{pmatrix}, \begin{pmatrix} a & b & b \\ c & d & d \end{pmatrix},$

$\begin{pmatrix} d & c \\ b & a \\ d & c \end{pmatrix}, \begin{pmatrix} b & a \\ d & c \\ b & a \end{pmatrix}, \begin{pmatrix} d & c \\ d & c \\ b & a \end{pmatrix}, \begin{pmatrix} b & a \\ d & c \\ d & c \end{pmatrix}, \begin{pmatrix} c & d \\ a & b \\ c & d \end{pmatrix},$

$\begin{pmatrix} a & b \\ c & d \\ a & b \end{pmatrix}, \begin{pmatrix} a & b \\ c & d \\ c & d \end{pmatrix}, \begin{pmatrix} c & d \\ c & d \\ a & b \end{pmatrix}, \begin{pmatrix} d & d \\ b & b \\ d & d \end{pmatrix}, \begin{pmatrix} b & b \\ d & d \\ b & b \end{pmatrix},$

$\begin{pmatrix} d & d \\ d & d \\ b & b \end{pmatrix}, \begin{pmatrix} a & b & b \\ c & d & d \\ c & d & d \end{pmatrix},$

$\begin{pmatrix} d & c & d \\ b & a & b \\ d & c & d \end{pmatrix}, \begin{pmatrix} d & d & c \\ b & b & a \\ d & d & c \end{pmatrix}, \begin{pmatrix} a & b & a \\ c & d & c \\ a & b & a \end{pmatrix}, \begin{pmatrix} b & a & b \\ d & c & d \\ b & a & b \end{pmatrix},$

$\begin{pmatrix} b & a & b \\ d & c & d \\ d & c & d \end{pmatrix}, \begin{pmatrix} d & c & d \\ d & c & d \\ b & a & b \end{pmatrix}, \begin{pmatrix} c & d & c \\ a & b & a \\ c & d & c \end{pmatrix}, \begin{pmatrix} c & d & c \\ c & d & c \\ a & b & a \end{pmatrix},$

$\begin{pmatrix} a & b & a \\ c & d & c \\ c & d & c \end{pmatrix}, \begin{pmatrix} c & d & d \\ a & b & b \\ c & d & d \end{pmatrix}, \begin{pmatrix} a & b & b \\ c & d & d \\ a & b & b \end{pmatrix}, \begin{pmatrix} c & d & d \\ c & d & d \\ a & b & b \end{pmatrix}.$

IV. TREES PARIKH VECTORS IN FIBONACCI ARRAY

In this section, we use basic definition, notation and results of graph theory for representing Fibonacci array in terms of tree. Any Fibonacci array of size (m, n) can be drive from the concept of tree, which is shown through an example. The Parikh vector of a Fibonacci array are discussed and the number calculated through the Fibonacci number.

Example:4.1

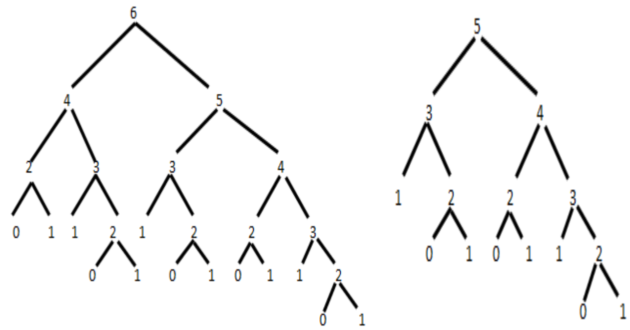


Figure 1. Structure of Fibonacci

6×5	1	0	1	0	1	1	0	1
0	01	00	01	00	01	01	00	01
1	11	10	11	10	11	11	10	11
1	11	10	11	10	11	11	10	11
0	01	00	01	00	01	01	00	01
1	11	10	11	10	11	11	10	11
1	11	10	11	10	11	11	10	11
0	01	00	01	00	01	01	00	01
1	11	10	11	10	11	11	10	11
0	01	00	01	00	01	01	00	01
1	11	10	11	10	11	11	10	11
1	11	10	11	10	11	11	10	11
0	01	00	01	00	01	01	00	01
1	11	10	11	10	11	11	10	11

Figure 2. Fibonacci array entries from Figure 1

6×5								
	b	a	b	a	b	b	a	b
	d	c	d	c	d	d	c	d
	d	c	d	c	d	d	c	d
	b	a	b	a	b	b	a	b
	d	c	d	c	d	d	c	d
	d	c	d	c	d	d	c	d
	b	a	b	a	b	b	a	b
	d	c	d	c	d	d	c	d
	b	a	b	a	b	b	a	b
	d	c	d	c	d	d	c	d
	d	c	d	c	d	d	c	d
	b	a	b	a	b	b	a	b
	d	c	d	c	d	d	c	d

Figure 3. Fibonacci array entries of Figure 2 over Σ

Definition:4.2

A word u is called a subword (also called scattered subword) of a word w, if there exist words $x_1 \dots . x_n$ and $y_0 \dots y_n$, such that $u = y_0 x_1 y_1 \dots x_n y_n$. The number of occurrences of the word u as a subword of the word w is denoted by $|w|_u$. In particular, if u is a symbol in the alphabet, then $|w|_u$ equals the number of occurrences of the symbol u in w.

Remark:4.3

Two occurrences of a subword are considered different if they differ by at least one position of some letter.

Definition:4.4

The Parikh vector of a word $w \in \sum^*$of length n is $\left(|w|_{a_1}, |w|_{a_2} \dots |w|_{a_n}\right)$, where $|w|_{a_n}$ is the number of occurrences of a_n in the word w.

Definition:4.5

Let $F_{m\times n}$ be a Fibonacci array over $\sum$, then the Parikh vector of $F_{m\times n}$ is $\{\, |F_{m\times n}|_{a_1}, |F_{m\times n}|_{a_2}, \dots |F_{m\times n}|_{a_i}\}$, where $|F_{m\times n}|_{a_i}$, is the number of occurrences of a_i in the Fibonacci array of size (m, n).

Definition:4.6

Let $\sum_k = \{a_1 < a_2 < \cdots < a_k\}$ be an ordered alphabet. The Parikh matrix mapping is the monoid morphism $\psi_k : \sum_k^* \longrightarrow \mathcal{M}_{k+1}$(The set of all triangle matrices of dimension $(k+1)$), defined by the condition: $\psi_k(\lambda) = I_{k+1}$, $(k+1) \times (k+1)$ unit matrix, and if $\psi_k(a_q) = (m_{i,j})_{1\le i,j\le(k+1)}$, $m_{i,i} = 1$, $m_{q,q+1} = 1$, all other elements of the matrix $\psi_k(a_q)$ are $0, 1 \le q \le k$.

Theorem:4.7

The Given a Fibonacci array $F_{m\times n}$,

(a) if (m, n) is even, then

(i). The number of rows and column of a is $F(m-1), F(n-1)$, the number of column of b is $F(n)$ and the number of rows of c is $F(m)$.

(ii). The number of occurrences of a is
$|F_{m\times n}|_a = F(m-1) \times F(n-1)$,
the number of occurrences of b is
$|F_{m\times n}|_b = F(m-1) \times F(n)$,
the number of occurrences of c is
$|F_{m\times n}|_c = F(m) \times F(n-1)$ and
the number of occurrences of d is
$|F_{m\times n}|_d = F(m) \times F(n)$.

(b) if (m, n) is odd, then

(i). The number of rows and column of d is $F(m), F(n)$, the number of column of c is $F(n-1)$ and the number of rows of b is $F(m-1)$.

(ii). The number of occurrences of a is
$|F_{m\times n}|_a = F(m-1) \times F(n-1)$,
the number of occurrences of b is
$|F_{m\times n}|_b = F(m-1) \times F(n)$,
the number of occurrences of c is
$|F_{m\times n}|_c = F(m) \times F(n-1)$ and
the number of occurrences of d is
$|F_{m\times n}|_d = F(m) \times F(n)$.

(c) if m is odd, n is even then

(i). The number of rows and column of c is $F(m), F(n-1)$, the number of column of d is $F(n)$ and the number of rows of a is $F(m-1)$.

(ii). The number of occurrences of a is
$|F_{m\times n}|_a = F(m-1) \times F(n-1)$,
the number of occurrences of b is
$|F_{m\times n}|_b = F(m-1) \times F(n)$,
the number of occurrences of c is
$|F_{m\times n}|_c = F(m) \times F(n-1)$ and
the number of occurrences of d is
$|F_{m\times n}|_d = F(m) \times F(n)$.

(d) if m is even, n is odd then

(i). The number of rows and column of b is $F(m-1), F(n)$, the number of column of a is $F(n-1)$ and the number of rows of d is $F(m)$.

(ii). The number of occurrences of a is
$|F_{m\times n}|_a = F(m-1) \times F(n-1)$,
the number of occurrences of b is
$|F_{m\times n}|_b = F(m-1) \times F(n)$,
the number of occurrences of c is
$|F_{m\times n}|_c = F(m) \times F(n-1)$ and
the number of occurrences of d is
$|F_{m\times n}|_d = F(m) \times F(n)$.

Proof:

The proof is immediate from the construction of Fibonacci array, because the concatenation is based on either row concatenation or column concatenation.

Theorem:4.8

The number of occurrences of the letters in an array obtained through the concept of Parikh matrices and the concepts of Fibonacci numbers is the same by considering array of size 3×3 is shown. This result can be generalized an array of any size.

Proof:

$$\psi_4(a) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \psi_4(b) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\psi_4(c) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \psi_4(d) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$M_{3\times3}=\begin{pmatrix} d & b & d\\ b & a & b\\ d & b & d\end{pmatrix}$$

$$\psi_4(dbd)=\psi_4(d)\psi_4(b)\psi_4(d)=\begin{pmatrix}1&0&0&0&0\\0&1&1&0&0\\0&0&1&0&0\\0&0&0&1&2\\0&0&0&0&1\end{pmatrix}$$

$$\psi_4(bab)=\psi_4(b)\psi_4(a)\psi_4(b)=\begin{pmatrix}1&1&0&0&0\\0&1&1&0&0\\0&0&1&2&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}$$

Row Parikh of $M=\begin{pmatrix}1&1&0&0&0\\0&1&2&0&0\\0&0&1&2&0\\0&0&0&1&4\\0&0&0&0&1\end{pmatrix}$, from this The number of occurrences of $a=1$, $b=2$, $c=2$, $d=4$.

In Fibonacci, if (m,n) is odd, then

(i). The number of occurrences of rows and column of d is $F(m), F(n), = 2, 2$

the number of occurrences of column of c is $F(n-1) = 1$

the number of occurrences of rows of b is $F(m-1) = 1$

(ii). The number of occurrences of a is

$$|F_{m\times n}|_a = F(m-1)\times F(n-1) = 1$$

the number of occurrences of b is

$$|F_{m\times n}|_b = F(m-1)\times F(n) = 2$$

the number of occurrences of c is

$$|F_{m\times n}|_c = F(m)\times F(n-1) = 2$$

the number of occurrences of d is

$$|F_{m\times n}|_d = F(m)\times F(n)=4.$$

Hence, the theorem is proved.

V. Conclusion and Future Scope

Combinatorial properties of two dimensional word are studied for the Fibonacci array. A new approach from graph theory to Fibonacci array is introduced and Parikh vectors concept for the Fibonacci is discussed. The future work focusses on more complexity properties on Fibonacci array.

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