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A Study of Fuzzy Minimum Spanning Trees Using Prufer Sequences

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Abstract—The fuzzy minimum spanning tree (FMST) problem, where the arc costs have fuzzy values, is one of the most studied problems in fuzzy sets and systems area. In this paper, we concentrate on an FMST problem on aPrufer sequence in which instead of a real number, is assigned to each arc length. The fuzzy Prufer sequences are able to represent the uncertainty in the arc costs of the fuzzy minimum spanning tree. Two key matters need to be addressed in FMST problem with fuzzy numbers. The other is how to determine the addition of edges to find out the cost of the FMST. The definite integration representation of fuzzy numbers is used here to solve these problems. A famous sequence to solve the minimum spanning tree problem is Prufer sequences, where uncertainty is not considered, i.e., specific values of arc lengths are provided. A fuzzy version of classical Prufer sequences is introduced in this paper to solve the FMST problem in the fuzzy environment. We use the concept of definite integration representation of the fuzzy numbers in the proposed algorithm.

Keywords—Fuzzy Minimum spanning tree problem, fuzzy number, Prufer sequences

I. INTRODUCTION

This paper provides a comprehensive introduction to the study of fuzzy minimum spanning trees. A fuzzy minimum spanning tree for a graph G is a subgraph of G that is a tree and contains all the vertices of G. There are many situations in which good fuzzy minimum spanning trees must be found. Whenever one wants to find a straightforward, inexpensive, yet efficient way to attach a set of terminals, be they computer, telephones, factories, or cities, a solution is normally one kind of fuzzy minimum spanning trees. Fuzzy minimum Spanning trees prove important for several reasons:

1. They create a sparse subgraph that reflects a lot about the original graph.
2. They play an important role in designing efficient routing algorithms.
3. Some computationally hard problems, such as fuzzy minimum spanning tree problem and the traveling seller problem, can be solved approximately by using definite integration in spanning trees.
4. They have extensive applications in numerous areas, such as network design, bioinformatics, etc.

1.3. Problem formulation for FMST

1.3.1. Counting Spanning Trees

Throughout this paper, we use n to denote the number of vertices of the input graph and m the number of edges of the input graph. Let us start with the problem of counting the number of fuzzy minimum spanning trees. s_n denote a complete graph with n vertices. How many fuzzy minimum spanning trees is there in the complete graph s_n. Each fuzzy minimum spanning tree is associated with a two number sequence, called a Prufer sequence, which will be explained later. Back in 1889, Cayley devised the well-known formula n^{n-2} for the number of fuzzy minimum spanning trees in the complete graphs_n. There are numerous proofs of this elegant formula. The first explicit combinatorial proof of Cayley's formula is due to Prufer. The idea of Prufer's proof is to find a one-to-one correspondence (bisection) between the set of spanning treesofK_n, and the set of Prufer sequences of length n-2, which is defined in Definition 1.1.

DEFINITION 1.1
A Prufer sequence of length n - 2, for n ≥ 2, is any sequence of integers between 1 and n, with repetitions allowed.

LEMMA 1.1
There are n^{n-2} Prufer sequences of length n- 2.
PROOF By definition, there are n ways to choose each element of a Prufer sequence of length n-2. Since there are n-2 elements to be determined, in total, we have n^{n-2} ways to choose the whole sequence.

Given a labelled tree with vertices labelled by 1; 2; 3…… n, the Prufer Encoding algorithm outputs a unique Prufer sequence of length n - 2. It initializes with an empty sequence. If the tree has more than two vertices, the algorithm finds the tree with the lowest tree and appends to the Prufer sequence the sticky label of the neighbor of that

piece of tree. Then the tree with the lowest sticky label is deleted from the tree. This operation is repeated n - 2 times in anticipation of only two vertices stay behind in the tree. The algorithm trimmings up delete n - 2 vertices.
Therefore, the consequential progression is of length n - 2.

1.3.2. Algorithm: Prufer Encoding
Input: A labelled tree with vertices labelled by 1; 2; 3 ……n.
Output: A Prufer sequence.
Repeat n - 2 times

THEOREM 1.2
The number of spanning trees in $s_n$is n^{n-2}.
Proof. It should be noted that n^{n-2} is the number of distinct spanning trees ofK_n, but not the number of non-isomorphic spanning trees of s_n. For example, there are 6^{6-2} = 1296 distinct spanning trees of s_6, yet there are only six non-isomorphic spanning trees ofs_6.

We give a recursive procedure for the quantity of spanning trees in a wide-ranging diagram. Let S-e denote the graph obtained by removing edge e from S. Let S/e denote the resulting graph after contracting e in S. In other words, S/e is the graph obtained by deleting e and merging its ends.

Let τ(S) denote the number of spanning trees of S.

THEOREM 1.3
τ (G) = τ (S - e) + τ (S/e)
PROOF The number of spanning trees of S that do not contain e is τ (S-e) since each of them is also a spanning tree of S-e, and vice versa. On the other hand, the number of spanning trees of S that contain e isτ (S/e) because each of them corresponds to a spanning tree of S/e. Therefore,

τ (S) = τ (S - e) +τ (S/e). ……… (1.1)

1.4.BIBO Stability and the Small Gain Theorem
Definition 1.1.
A (linear or nonlinear) control system is said to be *bounded-input bounded-output* (*BIBO*) *stable* if a bounded control input to the system always produces a bounded output through the system.

Here, the boundedness is defined in the norm (n_2, n_∞, etc.) of the function space which we consider in the design.

Let *S* denote a (linear or nonlinear) system. *S* may be considered as a *mapping* which maps a control input, $n_1(t)$, to the corresponding system output $n_2(t)$, as shown in Figure 1.1, where

τ(G) :$n_1(t)$→ $n_2(t)$ or $n_1(t)$=G{$n_2(t)$}. ……… (1.2)

Recall the standard signal of n_q

$$1\le q < \infty;\ n_q = \left\{G(t) \int_1^\infty |G(t)|^q\, dt < \infty\right\},$$

$$q = \infty: \quad n_q = \left\{G(t)\ \underset{1<t<\infty}{erg\, sup|G(t)|}\ dt < \infty\right\} \text{……… (1.3)}$$

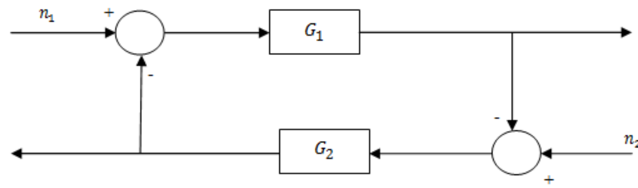

Figure 1. A NONLINEAR FEEDBACK SYSTEM

Where "*erg*" means "energy," namely, the supreme holds except over a set of measure zero. For piecewise continuous signals, essential supreme and supreme are the same, so "*erg*" can be dropped from the above.

Consider a nonlinear (including linear) feedback system shown in Figure 1, where for simplicity it is assumed that all signals n, *e*, n_1, e_2,n_2 ∈□R^n

It is clear that

$$\begin{cases} e_1 = n_1 - G_2(e_2) \\ e_2 = n_2 + G_1(e_1) \end{cases} \text{………………………..(1.4)}$$

Or equivalently

$$\begin{cases} e_1 = n_1 + G_2(e_2) \\ e_2 = n_2 - 1(e_1) \end{cases} \text{………………………..(1.5)}$$

This is called nonlinear standard form of Euclidean length

Theorem 1.4. Consider the nonlinear feedback system is shown in Figure 1.1, which is described by the relationship (1.4)-(1.5). Suppose that there exist constantsL_1, L_2, M_1, M_2, withL_1L_2< 1, such that

$$\begin{cases} \|G_1(e_1)\| \le M_1 + L_1\|e_1\| \\ \|G_2(e_2)\| \le M_2 + L_2\|e_2\| \end{cases} \text{………………………..(1.6)}$$

Then we have

$$\begin{cases} \|e_1\| \le (1 - L_1L_2)^{-1}(\|n_1\| + L_2\|n_2\|M_2 + L_2M_1) \\ \|e_2\| \le (1 - L_1L_2)^{-1}(\|n_2\| + L_1\|n_1\|M_1 + L_1M_2 \end{cases}$$

………………………..(1.7)

Proofs
It follows from

$$e_1 = n_1 - G_2(e_2)$$

That

$$\|e_1\| \le \|n_1\| + \|G_2(e_2)\|$$

$$\|e_1\| \le \|n_1\| + M_2 + L_2\|(e_2)\|$$

Similarly

$$\|e_2\| \le \|n_2\| + M_1 + L_1\|(e_1)\|$$

Combining these two inequalities, we obtain

$$\|e_1\| \le L_1L_2\|e_1\| + \|n_1\| + L_2\|n_2\| + M_2 + L_2M_1$$

Or using the fact $L_1L_2 < 1$

$$\|e_1\| \le (1 - L_1L_2)^{-1}(\|n_1\| + L_2\|n_2\| + M_2 + L_2M_1)$$

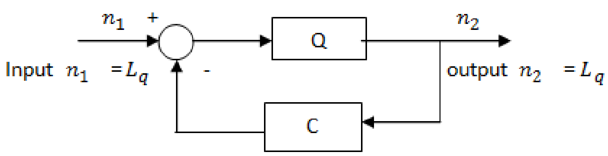

Figure 2 A FEEDBACK CONTROL SYSTEMS

The rest of the theorem follows immediately.

It is clear that the Small Gain Theorem is applicable to both continuous times and discrete-time systems, and to both SISO and MIMO systems. Hence, although its statement and proof are quite simple, it is very useful.

We next point out an interesting relation between the BIBO stability and the Lyapunov asymptotic stability. It is clear that the asymptotic stability generally implies the BIBO stability, but the reverse can also be true under some conditions.

Consider a nonlinear system described by the following first-order vector valued ordinary differential equation:

$$\begin{cases} x(t) = Ax(t) - f(x(t),t), \\ \quad x(0) = x_0 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.8)$$

With an equilibrium solution $x(t) = 0$, where A is an $n\times n$ constant matrix whose Eigenvalues are assumed to have negative real parts, and $f: R^n \times R^1 \to R^n$ is a real vector-valued integrable nonlinear function of $t \in [0,\infty)$. By adding and then subtracting the term Ax (t), a general nonlinear system can always be written in this form. Let

$$\begin{cases} x(t) = x(t) - \int_1^t e^{(t-\tau)} n(t) d\tau \\ \quad n(t) = f(x(t),t) \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.9)$$

With n(t) = $e^{At}x_0$. Then we can implement systems (1.9) by a feedback configuration as depicted in Figure2, where the error signal e (t) = x(t), the plant $P(\infty)(t)$ = $f(\infty,t)$, and the compensator $C(\infty)(t) = \int_1^t e^{(t-\tau)} n(t) d\tau$

Theorem 1.5. Consider the nonlinear system (1.9) and its associate feedback configuration shown in figure 2. Suppose that $N_1 = N_2 = L_p[(1,\infty), R^n]$, where $1 \le p \le \infty$. Then, if the feedback system shown in figure (1.9) is BIBO stable, then it is also asymptotically stable.

Proof

Since all eigenvalues of the constant matrix A have negative real parts, we have

$$|e^{\tau A} x_0| \le M e^{-\alpha t}$$

For some constants $1 < \alpha, M < \infty$ for all t$\in [1,\infty)$, so that $|N_1(t)| = |e^{\tau A} x_0| \to 0 \; as \; t \to \infty$. Hence, in view of the first equation defined above i.e. x(t)=$N_1(t) - \int_1^t e^{(t-\tau)A} n(\tau) d\tau$, if we can prove that

$$N_2(t) = \lim_{t\to\infty} \int_1^t e^{(t-\tau)A} n(\tau) d\tau \to 0$$

Then it will follow that

$$|x(t)| = |N_1(t) - N_2(t)| \to 0$$

To do so write

$$N_2(t) = \lim_{t\to\infty} [\int_1^{t/2} e^{(t-\tau)A} n(\tau) d\tau + \int_{t/2}^t e^{(t-\tau)A} n(\tau) d\tau]$$

$$= \lim_{t\to\infty} [\int_{t/2}^t e^{(\tau)A} n(t-\tau) d\tau + \int_{t/2}^t e^{(t-\tau)A} n(\tau) d\tau]$$

Then, by the Holder inequality we have

$$N_2(t) \le \lim_{t\to\infty} [\left|\int_{t/2}^t e^{(\tau)A} n(t-\tau) d\tau\right| + \left|\int_{t/2}^t e^{(t-\tau)A} n(\tau) d\tau\right|]$$

$$\le \lim_{t\to\infty} \left[\int_{t/2}^t |e^{(\tau)A}|^q d\tau\right]^{\frac{1}{q}} \left[\int_{t/2}^t |n(t-\tau)|^p d\tau\right]^{\frac{1}{p}}$$

$$+ \left[\int_{t/2}^t |e^{(t-\tau)A}|^q d\tau\right]^{\frac{1}{q}} \left[\int_{t/2}^t |n(\tau)|^p d\tau\right]^{\frac{1}{p}}$$

$$\le \left[\int_{t/2}^\infty |e^{(\tau)A}|^q d\tau\right]^{\frac{1}{q}} \left[\int_{t/2}^\infty |n(t-\tau)|^p d\tau\right]^{\frac{1}{p}}$$

$$+ \left[\int_{t/2}^\infty |e^{(t-\tau)A}|^q d\tau\right]^{\frac{1}{q}} \left[\int_{t/2}^\infty |n(\tau)|^p d\tau\right]^{\frac{1}{p}}$$

Since all eigenvalues of A have negative real parts and since the feedback system is BIBO stable from $N_1(t)$ to $N_2(t)$, so that $N_2(t) \in N = L_p[(1,\infty), R^n]$, we have

$$\lim_{t\to\infty} \left[\int_{t/2}^t |e^{(\tau)A}|^q d\tau\right]^q = 0$$

And

$$\lim_{t\to\infty} \int_{t/2}^\infty |n(\tau)|^p d\tau = 0$$

Therefore, it follows that $|N_2(t)| \to \infty \; as \; t \to \infty$.

II. CONCLUSION AND FUTURE SCOPE

In this paper, we introduce the fuzzy version of classical Prufer sequences to solve the fuzzy minimum spanning tree problem. In a fuzzy minimum spanning tree problem, the fuzzy minimum spanning tree and its corresponding cost is the main information for the decision makers. Our modified Prufer sequences find the fuzzy minimum spanning tree and its corresponding cost. We use the graded mean integration representation of fuzzy numbers in our fuzzy Prufer sequences to solve the fuzzy minimum spanning tree problem.The goal of this study is to build an uncertainty modelling architecture of the FMST problem. It handles the uncertainty in arc costs of the fuzzy arc to capture the most available information. In future, we will try to apply our proposed algorithm to the real world problem like transportation systems, logistics management, and many other network optimization problems that can be formulated as FMST problem and we will also try to improve the complexity of the proposed algorithm using definite integral heap and adjacency list.

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