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Liar's Domination in Sun Graphs

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Abstract—Liar's dominating set is one that identifies an intruder's location even if one device in the neighborhood of the intruder vertex becomes faulty, that is, any one device in the neighborhood of the intruder vertex can misidentify any vertex in its closed neighborhood as the location of the intruder. The liar's domination number is the minimum cardinality of a liar's dominating set. In this paper, we determine the liar's domination number for sun graphs, sun let graphs, line graphs of sun let graphs and wheel graphs.

Keywords— Domination, Liar's domination, Sun graphs, Sun let graphs, Wheel graphs

I. INTRODUCTION

The theory of domination occurs in number of problems and has been the nucleus of research activity in graph theory. The communication network problem is a motivation for all other problems in domination. Each node in that communication network needs to communicate with all the nodes in order to be efficient. The other applications of domination includes facility location problem, land surveying problem and many more.

Liar's Domination was introduced by Slater [18] in the year 2009. Liar's domination arises in modelling protection devices where one device may be faulty. A graph G is modelled in a way that every vertex is the possible location for the intruder to enter. By placing protection devices at a vertex say v, the intruder can be detected at any vertex in its closed neighbourhood $N[v]$. Also the location of the intruder in $N[v]$ can be identified. In this scenario, there is a possibility of misidentifying the vertex as the intruder or may not report the intruder in its closed neighbourhood $N[v]$. Liar's dominating set can identify the location of the intruder exactly even when one device becomes faulty.

Given a graph $G(V, E)$, for any vertex $v \in V$, we denote the open neighbourhoods of v in G by $N(v) = \{x \in V(G)/(x,v) \in E(G)\}$ and the closed neighbourhoods as $N[v] = N(v) \cup v$. A set S of vertices of a graph $G = (V, E)$ is a dominating set of G if every vertex in $V(G)-S$ is adjacent to some vertex of S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. For an integer $k \geq 1$, a dominating set $D \subseteq V$ is a k-tuple dominating set if $|N[v] \cap D| \geq k$ for all $v \in V$. The minimum cardinality of a k-tuple dominating set is called the k-tuple domination number of G and is denoted by $\gamma_k(G)$ [5].

As defined by Slater [17], a dominating set $S \subset V(G)$ is a liar's dominating set if for any vertex $v \in V(G)$ if all or all but one of the vertices in $N[v] \cap S$ report v as the intruder location, and at most one vertex w in $N[v] \cap S$ either reports a vertex $x \in N[w]$ or fails to report any vertex, then the vertex v can be correctly identified as the intruder vertex. In other words, if an intruder is at any vertex v, then the protection devices outside of $N[v]$ are assumed to not report any intruder, one vertex $w \in N[v] \cap S$ can report nothing or any vertex in $N[w]$ as the intruder vertex, every other element of $N[v] \cap S$ will correctly report vertex v as the intruder location, and v will be correctly identified as the intruder vertex.

A vertex set $L \subseteq V(G)$ is a Liar's Dominating Set (LDS) if and only if (1) L double dominates every $v \in V(G)$ and (2) for every pair u, v of distinct vertices we have $|(N[u] \cup N[v]) \cap L| \geq 3$. The minimum cardinality of a liar's dominating set for graph G is called the liar's domination number and is denoted $\gamma_L(G)$. In order to detect an intruder in any graph, a dominating set is needed. But since any one device can fail to detect the intruder, a double dominating set is required. Since every triple dominating set is a liar's dominating set, liar's dominating sets lie between double dominating sets and triple dominating sets [17].

Slater [18] has showed that for general graphs the liar's dominating set problem is NP-*hard* and has given a lower bound for trees. Roden and Slater [17] proved that even for bipartite graphs the problem is NP-*hard*. Panda and Paul [10, 13] have proved that the problem is NP-*hard* for split graphs and chordal graphs and later they proposed a linear time algorithm for proper interval graphs. Liar dominating set for circulant networks was given by Paul Manuel [9]. B. S. Panda et al. [14] studied the problem for bounded degree graphs and p-claw free graphs. Alimadadi et al. [1] provided the characterization of graphs and trees for which liar's domination number is $|V|$ and $|V|-1$ respectively. In this paper, we determine the minimum liar's domination number for sun graphs, sun let graphs, line graphs of sun let graphs and wheel graphs.

II. LIAR'S DOMINATING SET FOR WHEEL GRAPHS

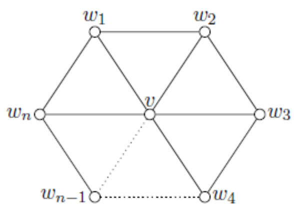

Figure 1: W_n

For any integer $n \geq 4$, the wheel graph W_n is the $(n+1)$-vertex graph obtained by joining a vertex v to each of the n vertices $\{w_1, w_2,...,w_n\}$ of the cycle graph C_n [20]. In W_n, $d(v) = n$ and $d(w_i) = 3$, $1 \leq i \leq n$.

Wheel graphs are planar graphs and so they have a unique planar embedding. There are n^2-3n+3 cycles in W_n and it is always Hamiltonian. The wheel W_n supplied a counter example to a conjecture of Paul Erdos on Ramsey theory [4].

Theorem 2.1. [18] For a cycle C_n we have $\gamma_L(C_n) = \left\lceil\frac{3n}{4}\right\rceil$.

Theorem 2.2. *Let W_n be a wheel graph with $n \geq 4$ then* $\gamma_L(W_n) = \left\lceil\frac{n}{2}\right\rceil + 1$.

Proof. Let L include w_i's, i odd, $1 \leq i \leq n-1$; so that $|N[w_i]\cap L| = 2$, i even and $|N[v]\cap L| \geq 2$. But in order to make $|N[w_i] \cap L| \geq 2$, i odd, v should be included in L and it is sufficient since each w_i's, i odd are double dominated and each w_i's, i even are triple dominated. Therefore L becomes a liar's dominating set. Also since $\gamma_L(C_n) = \left\lceil\frac{3n}{4}\right\rceil > \frac{n}{2}$, $\gamma_L(W_n) = \left\lceil\frac{n}{2}\right\rceil + 1$. □

III. LIAR'S DOMINATING SET FOR SUN GRAPHS

A sun is a chordal graph G on $2n$ vertices for some $n \geq 3$ whose vertex set can be partitioned into two sets $W = \{w_1,...,w_n\}$, $U = \{u_1,...,u_n\}$, such that W is independent and for each i and j, w_j is adjacent to u_i if and only if $i = j$ or $i \equiv j + 1 \pmod n$. A complete sun is a sun G in which $G(U)$ is a complete graph [2]. Let us call this complete sun graph as 1-CS_n. In general an N−complete sun graph is obtained by replacing each of the outer edge of $(N-1)$-CS_n by K_3 which introduce a vertex for each outer edge. See Figure 4.

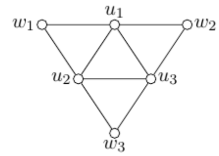

Figure 2: 1-CS_3

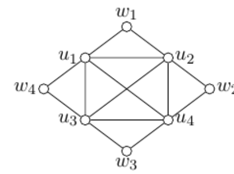

Figure 3: 1-CS_4

Theorem 3.1. *Let 1-CS_n be a complete sun graph with $n \geq 3$ then γ_L(1-CS_n) = n.*

Proof. Since $d(w_i) = 2$ and $N(w_i)$'s are u_i's, in order to satisfy $|(N[w_i]\cap L| \geq 2$, L should have $N(w_i)$ or one vertex from $N(w_i)$ and w_i. Let $L = \{u_i|1 \leq i \leq n\}$. It is clear that $|N[u] \cap L| \geq 2$ for all u. Also, since all u_i's are in L and u_i's form a complete graph, $|(N[u_i] \cup N[u_j]) \cap L| \geq 3$ and $|(N[u_i] \cup N[w_j]) \cap L| \geq 3$. Moreover since $|N(w_i) \cap N(w_j)| \leq 1$, $|(N[w_i] \cup N[w_j]) \cap L| \geq 3$. Hence L is a liar's dominating set. Therefore, $\gamma_L(1-CS_n) = n$. □

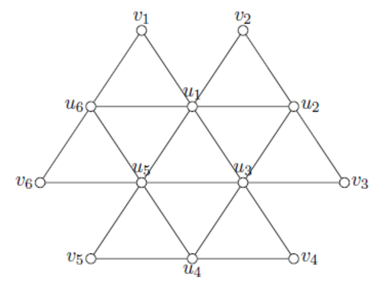

Figure 4: 2-CS_3

Starting at any vertex, the inner vertices of 2-CS_n are labeled as u_1, u_2,... in clockwise direction. The new vertices introduced by the outer edge of 1-CS_n to form 2-CS_n are labelled as v_j's such that v_j is adjacent to u_i if and only if $i = j$ or $i \equiv j + 1 \pmod n$.

Theorem 3.2. *Let 2-CS_n be a complete 2-sun graph with $n \geq 3$ then γ_L(2-CS_n) = $2n$.*

Proof. Let $L = \{u_i,|1 \leq i \leq 2n\}$ be a liar's dominating set. It is obvious that $|(N[u] \cap L| \geq 2$ for all u. Also since each u_i is adjacent with u_{i-1} and u_{i+1}, $|(N[u_i] \cup N[u_j]) \cap L| \geq 3$ and $|(N[u_i] \cup N[w_j]) \cap L| \geq 3$. Moreover, $|N(w_i) \cap N(w_j)| \leq 1$, $|(N[w_i] \cup N[w_j]) \cap L| \geq 3$. Therefore L is a liar's dominating set. Hence, γ_L(2-CS_n) = $2n$. □

In view of theorem 3.1 and 3.2, we have the following result.

Theorem 3.3. *Let N-CS_n be a complete N-sun graph with $n \geq 3$ then* $\gamma_L(N-CS_n) = \frac{n}{2}(2^N)$.

IV. LIAR'S DOMINATING SET FOR SUN LET GRAPHS

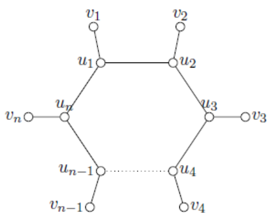

Figure 5: S_n

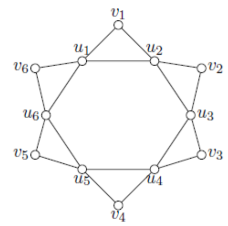

Figure 6: $L(S_5)$

The n-sun let graph on $2n$ vertices is obtained by attaching n pendant edges to the cycle C_n and is denoted by S_n [20]. Starting at any vertex in the cycle C_n, label the vertices as u_1, u_2,..., u_n in the clockwise direction. The corresponding pendant vertex are labelled as v_i for each u_i. The line graph of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G, and if u, $v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G. Here we denote the line graph of sun let graphs by $L(S_n)$ [20]. The graph of $L(S_n)$ and the middle graph of a cycle graph $M(C_n)$ look similar.

Theorem 4.1. *Let S_n be a sun let graph with $n \geq 3$, then $\gamma_L(S_n) = 2n$.*

Proof. Since v_i's are pendant vertices, both v_i and u_i should be in L. Hence, $\gamma_L(S_n) = 2n$.

Theorem 4.2. [3] Let $M(C_n)$ be the middle graph of a cycle graph, where C_n is the cycle graph of order n. Then $\gamma_{LR}(M(C_n)) \leq n$.

Theorem 4.3. *Let $L(S_n)$ be a line graph of a sun let graph with $n \geq 3$, then $\gamma_L(L(S_n)) = n$.*

Proof. Since each v_i is adjacent to u_i and u_{i+1}, it is clear that either (u_i, v_i) or (u_i, u_{i+1}) or (v_i, u_{i+1}) should be in L. Without loss of generality let $L = \{u_i | 1 \leq i \leq n\}$ such that $\gamma_L(L(S_n)) = n$. □

V. CONCLUSION

Liar's domination can be used to determine the information about the location of an intruder provided that one detector becomes faulty. In this paper we provided the liar's dominating number for sun graphs, sun let graphs and wheel graphs. We extend this result for more classes of graphs in our future work.

REFERENCES

[1] A. Alimadadi, M. Chellali, and Doost Ali Mojdeh, "*Liar's dominating sets in graphs*" Discrete Applied Mathematics, Vol. 211, pp. 204-210, 2016.

[2] A. Brandstädt, V. B. Le and J. P. Spinrad, "*Graph Classes: A Survey*", PA: SIAM, Philadelphia, pp. 112, 1987.

[3] D. D Durgan, F. N. Altundag, *Liar's Domination in Graphs*, Bulletin of International Mathematical Virtual Institute, Vol. 7, pp. 407-15, 2017.

[4] R. J. Faudree, M. D. Brendan, "*A conjecture of Erdös and the Ramsey number r(W)*", J. Combinatorial Math., and Combinatorial Comput., Vol. 13: pp. 23-31, 1993.

[5] F. Harary and T.W. Haynes, "*Double domination in graphs*", Ars Combin., Vol. 55, pp. 201-213, 2000.

[6] M. Jakovac, S. Klavazar, "*Vertex-, Edge-, and Total-Colorings of Sierpinski-like Graphs*", Discrete Mathematics Vol. 309, pp. 1548-1556, 2009.

[7] S. Klavzar, "*Coloring Sierpinski Graphs and Sierpinski Gasket Graphs*" Taiwanese J. Math. Vol. 12, pp. 513-522, 2008.

[8] S. Klavzar and U. Milutinovic, "*Graphs S(n; k) and a variant of the Tower of Hanoi problem*", Czechoslovak Math. J., Vol. 47, pp. 95-104, 1997.

[9] P. Manuel, "*Location and Liar Domination of Circulant Networks*", Ars Combinatoria Waterloo then Winnipeg, Vol. 101, pp. 309-320, 2011.

[10] B.S. Panda and S. Paul, "*Liar's domination in graphs: Complexity and algorithm*", Discrete Applied Mathematics, Vol. 161, pp. 1085-1092, 2013.

[11] B. S. Panda and S. Paul, "*Connected Liar's domination in graphs: Complexity and algorithms*", Discrete Mathematics, Algorithms and Applications, Vol. 4, pp. 1-16, 2013.

[12] B. S. Panda and S. Paul, "*Hardness results and approximation algorithm for total liar's domination in graphs*", Journal of Combinatorial Optimization, Vol. 27, pp. 643-662, 2014.

[13] B. S. Panda and D. Pradhan, "*A linear time algorithm to compute a minimum restrained dominating set in proper interval graphs*", Discrete Mathematics, Algorithms and Applications, Vol. 7, No. 2 ID: 1550020, 2015.

[14] B. S. Panda, S. Paul, and D. Pradhan, "*Hardness Results, Approximation and Exact Algorithms for Liars Domination Problem in Graphs*", Theoretical Computer Science Vol. 573, pp. 26-42, 2015. □

[15] Ponraj, Raja and Narayanan, S. Sathish, "*Difference cordiality of some snake graphs*", Journal of applied mathematics and informatics, Vol. 32, pp. 377-387, 2014.

[16] I. Rajasingh, B. Rajan, A. S. Shanthi, and Albert Muthumalai, "*Induced matching partition of sierpinski and honeycomb networks*", in International Conference on Informatics Engineering and Information Science, Vol. 253, pp. 390-399, 2011.

[17] M. L. Roden and P. J. Slater, "*Liar's domination in graphs*", Discrete Math., Vol. 309, pp. 5884-5890, 2008.

[18] P. J. Slater, "*Liar's Domination*", Networks, Vol. 54, pp. 70-74, 2009.

[19] A. M. Teguia, A. P. Godbole, "*Sierpinski Gasket Graphs and Some of their Properties*", Australasian Journal of Combinatorics, Vol. 35, pp. 181-192, 2006.

[20] J. Vernold Vivin and M. Venkatachalam, "*On b-chromatic number of Sun let graph and wheel graph families*", Journal of the Egyptian Mathematical Society, Vol. 23, pp. 215-218, 2014.

