

Analysis of Finite Source Queueing System with Catastrophe

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DOI: <https://doi.org/10.26438/ijcse/v7si5.59> | Available online at: www.ijcseonline.org

Abstract— In this paper, we are analyzing the finite source queueing system with catastrophe. This model is completely solved by using continued fraction method. We have calculated the compact solutions of steady state probabilities of number of occupants in the system and various system performance measures. Analytical and pictorial studies are also carried out.

Keywords: Finite source queue - Catastrophe - Steady state probabilities - Continued fraction method - performance measures.

I. INTRODUCTION

The study of the crowd in waiting line is called queueing theory. Machine-fixing model is called a finite source queueing model (sometimes called the finite population) Donald Gross [2]. This model is applied to machine fixing, where the calling population is the machines, an arrival corresponds to a machine malfunction, and the repair technicians are the server. The literature on finite source queueing model has grown over the years. During the past few years, a Finite source queues have received considerable attention from researchers who are interested in theoretical and practical applications. We can apply finite source queue in various computer and telecommunication fields. Catastrophe is modeled with computers which affected by viruses. Application of Catastrophe was discussed by various authors in few decades such works include Hanson and Tuck well [3], Artalejo[1], Jain[5]. Muthuganapathi Subramanian.A, Gayathri. S [8] has analyzed a single server queueing system with catastrophe, Krishna Kumar, Krishnamoorthy, PavaiMadheswari and Sadiq Basha[7] analysed single server queues with catastrophe, Hisao Kameda [4] analysed finite source queue with different customers. Kalyanaraman [6] has considered a single server retrial queueing system with two types of arrivals and finite number of repeated customers.

II. MODEL DESCRIPTION

In this model, we consider the calling population is finite of size M . We have assumed that arrival rate λ and the service time μ follows a Poisson and exponential distribution respectively. Further, we assumed that the catastrophe rate γ and δ is the rate of return from inactive state, in which both follows a Poisson process.

We define,

p_{n1} : denotes steady state probability of n occupants

P_{idle} Or P_{00} : Steady state probability that the server is idle.

$P_{inactive}$ Or P_{02} : Steady state probability that the server is inactive.

III. PROBABILITY DISTRIBUTIONS IN STEADY STATE

The balance equations for this model are given by

$$\mu p_{n+1} - ((M-n)\lambda + \gamma + \mu)p_{n1} + (M-(n-1))\lambda p_{n-1} = 0, \text{ for } n = 2, 3, 4, \dots, M-1$$

$$p'_{00}(t) = -(M\lambda + \gamma)p_{00}(t) + \delta p_{02}(t) + \mu p_{11}(t) \quad (1)$$

$$p'_{11}(t) = -((M-1)\lambda + \gamma + \mu)p_{11}(t) + M\lambda p_{00}(t) + \mu p_{21}(t) \quad (2)$$

$$p'_{n1}(t) = -((M-n)\lambda + \gamma + \mu)p_{n1}(t) + (M-(n-1))\lambda p_{n-1}(t) + \mu p_{n+1}(t),$$

$$\text{for } n = 2, 3, \dots, M-1 \quad (3)$$

$$p'_{02}(t) = -(\gamma + \delta)p_{02}(t) + \gamma \quad (4)$$

$$p'_{M1}(t) = -\gamma p_{M1}(t) + \lambda p_{11}(t) \quad (5)$$

When 't' goes to ∞ , then we get the balance equations from (1) to (5) as follows:

$$0 = -(M\lambda + \gamma)p_{00} + \delta p_{02} + \mu p_{11} \quad (6)$$

$$0 = -((M-1)\lambda + \gamma + \mu)p_{11} + M\lambda p_{00} + \mu p_{21} \quad (7)$$

$$0 = -((M - n)\lambda + \gamma + \mu)p_{n1} + (M - (n - 1))\lambda p_{n-11} + \mu p_{n+11},$$

$$\text{for } n = 2, 3, \dots, M-1 \quad (8)$$

$$0 = -(\gamma + \delta)p_{02} + \gamma \quad (9)$$

$$0 = -\gamma p_{M1} + \lambda p_{11} \quad (10)$$

Theorem 1: For the finite source queueing system with catastrophe, if the server is busy and idle then the steady state probability distribution of the occupants are given by

$$p_{n1} = f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}}) p_{\text{idle}} \left[1 - \frac{\gamma}{\gamma + \delta}\right]$$

$$p_{\text{idle}} = \frac{1}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}})}$$

Proof: The steady state probabilities are obtained by using continued fraction method.

Defining

$$A_n = (M - n)\lambda, \text{ for } n = 0, 1, 2, \dots, M-1 \quad (11)$$

$$B_n = (M - n)\lambda + \gamma + \mu, \text{ for } n = 1, 2, 3, \dots, M-1 \quad (12)$$

$$\phi(1) = \frac{p_{11}}{p_{00}} \quad (13)$$

$$\phi(n) = \frac{p_{n1}}{p_{n-11}}, \text{ for } n = 2, 3, \dots, M \quad (14)$$

From (6), we get

$$\phi(1) = \frac{p_{11}}{p_{00}} = \frac{1}{\mu} [(M\lambda + \gamma) - \frac{\delta p_{02}}{p_{00}}] = f_1(p_{00}) \quad (15)$$

From (7), we get

$$\phi(2) = \frac{p_{21}}{p_{11}} = \frac{1}{\mu} [B_1 - \frac{A_0}{\phi(1)}] = f_2(p_{00}) \quad (16)$$

Consider the difference equation
(17)

Replace n by (n-1) in equation (17), we get

$$\mu p_{n1} = B_{n-1} p_{n-11} - A_{n-2} p_{n-21} \quad (18)$$

From the equation (18), we get

$$\frac{p_{n1}}{p_{n-11}} = \frac{1}{\mu} \left(B_{n-1} - \frac{A_{n-2}}{\left(\frac{p_{n-11}}{p_{n-21}}\right)} \right), \text{ for } n = 3, 4, 5, \dots, M \quad (19)$$

$$\phi(n) = \frac{p_{n1}}{p_{n-11}} = \frac{1}{\mu} \left[B_{n-1} - \frac{A_{n-2}}{\phi(n-1)} \right], \text{ for } n = 3, 4, 5, \dots, M \quad (20)$$

From equations (15), (16) and (20), we get

$$p_{11} = f_1(p_{\text{idle}}) p_{\text{idle}} \quad (21)$$

$$p_{21} = f_1(p_{\text{idle}})f_2(p_{\text{idle}}) p_{\text{idle}} \quad (22)$$

In general,

$$p_{n1} = f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}}) p_{\text{idle}} \quad (23)$$

Using the normalized condition

$$p_{00} + p_{02} + p_{11} + p_{21} \dots + p_{M1} = 1 \quad (24)$$

$$\text{From the equation (9), } p_{02} = \frac{\gamma}{\gamma + \delta} \quad (25)$$

$$\text{From the equation (10), } p_{M1} = \frac{\lambda}{\gamma} p_{11} \quad (26)$$

Using equations (23) & (25) in the equation (26)

$$p_{\text{idle}} = \frac{\left[1 - \frac{\gamma}{\gamma + \delta}\right]}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}})} \quad (27)$$

p_{idle} is obtained by solving the equation (27), it represents the steady state probability of the server is idle in the system and using this, we get

$$p_{n1} = f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}}) p_{\text{idle}} \quad (28)$$

Therefore, the equation (28) represents the probability of n occupants when the server is busy in the steady state.

Theorem 2: Expected number of occupants in the system in steady state is given by

$$L_s = \frac{[\sum_{n=1}^M n (f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}}))] \left[1 - \frac{\gamma}{\gamma + \delta}\right]}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_k(p_{\text{idle}})}$$

Proof:

Expected number of occupants in the system is given by

$$L_s = \sum_{n=0}^M n p_{n1} \quad (29)$$

Using the equation (27) in (29), we get

$$L_s = \frac{[\sum_{n=1}^M n (f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_n(p_{\text{idle}}))] \left[1 - \frac{\gamma}{\gamma + \delta}\right]}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{\text{idle}})f_2(p_{\text{idle}}) \dots f_k(p_{\text{idle}})} \quad (30)$$

This is the Expected number of occupants in the system for finite source queueing system with Catastrophe. The remaining system measures L_q , W_s and W_q can be obtained by using Little's formula.

IV. SYSTEM PERFORMANCE MEASURES

The system performance measures are most important for every queueing system. We are formulated various formulae to find such measures by using the steady state probabilities which are described by the equations (25), (27), (28) and (30)

1. The Probability that the server is idle

$$P_{idle} = \frac{\left[1 - \frac{\gamma}{\gamma + \delta}\right]}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{idle})f_2(p_{idle}) \dots f_n(p_{idle})}$$

2. The Probability that the server is inactive

$$P_{inactive} = \frac{\gamma}{\gamma + \delta}$$

3. The Probability that the server is busy is given by,

$$\sum_{n=1}^M p_{11} = \sum_{n=1}^M \prod_{k=1}^n f_1(p_{idle})f_2(p_{idle}) \dots f_k(p_{idle}) P_{idle}$$

4. Expected number of occupants in the system is given by

$$L_s = \frac{\left[\sum_{n=1}^M n (f_1(p_{idle})f_2(p_{idle}) \dots f_n(p_{idle}))\right] \left[1 - \frac{\gamma}{\gamma + \delta}\right]}{1 + \sum_{k=1}^M \prod_{n=1}^k f_1(p_{idle})f_2(p_{idle}) \dots f_n(p_{idle})}$$

V. NUMERICAL ANALYSIS

We discussed that the different values of λ , μ , δ and γ are chosen to satisfy the stability condition. **Table 1**, **Table 2** and **Table 3** show Steady State Probabilities for server idle and server busy for numerous values of the system parameters, we infer that

- The probability that the server is idle gets decreases and that the server busy gets increases if arrival rate λ increases.
- For a fixed value of λ and μ , $\{P_n\}$ tends to zero.

Table 4, **Table 5** and **Table 6** show System Performance Measures for server idle and busy for numerous values of the system parameters, we infer that the probability that the server idle gets decreases and that the server busy gets increases and expected no. of occupants increases, if arrival rate λ increases.

Table:1 Steady State Probabilities for different λ when $\mu = 10$, $\delta = 5$ and $\gamma = 2$ for finite population size $M = 3$

λ	P_{idle}	$P_{inactive}$	P_{11}	P_{21}	P_{31}
1.0000	0.5554	0.2857	0.1349	0.0222	0.0018
1.5000	0.4901	0.2857	0.1757	0.0430	0.0054
2.0000	0.4333	0.2857	0.2038	0.0661	0.0110
2.5000	0.3841	0.2857	0.2221	0.0894	0.0186
3.0000	0.3416	0.2857	0.2329	0.1118	0.0280
3.5000	0.3049	0.2857	0.2382	0.1325	0.0387
4.0000	0.2731	0.2857	0.2395	0.1513	0.0504
4.5000	0.2456	0.2857	0.2378	0.1679	0.0630

5.0000	0.2217	0.2857	0.2341	0.1824	0.0760
5.5000	0.2010	0.2857	0.2290	0.1950	0.0894
6.0000	0.1829	0.2857	0.2229	0.2057	0.1029
6.5000	0.1670	0.2857	0.2162	0.2148	0.1163
7.0000	0.1530	0.2857	0.2091	0.2224	0.1297
7.5000	0.1407	0.2857	0.2020	0.2287	0.1429
8.0000	0.1299	0.2857	0.1948	0.2338	0.1558
8.5000	0.1202	0.2857	0.1877	0.2379	0.1685

Table: 2 Steady State Probabilities for different λ when $\mu = 10$, $\delta = 5$ and $\gamma = 2$ for finite population size $M = 7$

λ	P_{idle}	$P_{inactive}$	P_{11}	P_{21}	P_{31}	P_{41}	P_{51}	P_{61}	P_{71}
1.0000	0.3776	0.2857	0.1969	0.0902	0.0352	0.0112	0.0027	0.0004	0.0000
1.5000	0.2712	0.2857	0.1962	0.1272	0.0714	0.0332	0.0119	0.0029	0.0004
2.0000	0.1983	0.2857	0.1744	0.1409	0.1008	0.0606	0.0285	0.0092	0.0015
2.5000	0.1492	0.2857	0.1482	0.1389	0.1180	0.0860	0.0497	0.0200	0.0042
3.0000	0.1162	0.2857	0.1243	0.1290	0.1245	0.1054	0.0718	0.0345	0.0086
3.5000	0.0934	0.2857	0.1046	0.1164	0.1237	0.1179	0.0921	0.0513	0.0149
4.0000	0.0773	0.2857	0.0889	0.1037	0.1186	0.1246	0.1092	0.0690	0.0230
4.5000	0.0655	0.2857	0.0765	0.0922	0.1114	0.1268	0.1228	0.0867	0.0325
5.0000	0.0567	0.2857	0.0668	0.0821	0.1035	0.1258	0.1329	0.1035	0.0431
5.5000	0.0498	0.2857	0.0589	0.0734	0.0955	0.1228	0.1399	0.1192	0.0546
6.0000	0.0444	0.2857	0.0526	0.0661	0.0880	0.1186	0.1445	0.1334	0.0667
6.5000	0.0401	0.2857	0.0475	0.0598	0.0811	0.1136	0.1470	0.1461	0.0791
7.0000	0.0365	0.2857	0.0432	0.0545	0.0748	0.1083	0.1480	0.1573	0.0918
7.5000	0.0335	0.2857	0.0396	0.0500	0.0691	0.1029	0.1476	0.1671	0.1045
8.0000	0.0309	0.2857	0.0365	0.0460	0.0640	0.0976	0.1464	0.1757	0.1171
8.5000	0.0287	0.2857	0.0339	0.0427	0.0595	0.0925	0.1444	0.1830	0.1296

Table:3 Steady State Probabilities for different λ when $\mu = 10$, $\delta = 5$ and $\gamma = 2$ for finite population size $M = 9$

λ	P_{idle}	$P_{inactive}$	P_{11}	P_{21}	P_{31}	P_{41}	P_{51}	P_{61}	P_{71}	P_{81}	P_{91}
1.0000	0.3065	0.2857	0.1943	0.1128	0.0588	0.0269	0.0105	0.0033	0.0008	0.0001	0.0000
1.5000	0.2002	0.2857	0.1675	0.1316	0.0952	0.0617	0.0347	0.0161	0.0058	0.0014	0.0002
2.0000	0.1377	0.2857	0.1325	0.1233	0.1084	0.0876	0.0626	0.0377	0.0177	0.0057	0.0010
2.5000	0.1008	0.2857	0.1041	0.1064	0.1057	0.0990	0.0842	0.0613	0.0355	0.0143	0.0030
3.0000	0.0781	0.2857	0.0835	0.0899	0.0962	0.0998	0.0964	0.0815	0.0556	0.0267	0.0067
3.5000	0.0632	0.2857	0.0688	0.0761	0.0853	0.0949	0.1008	0.0961	0.0751	0.0418	0.0122
4.0000	0.0529	0.2857	0.0581	0.0652	0.0751	0.0876	0.1002	0.1052	0.0923	0.0583	0.0194
4.5000	0.0454	0.2857	0.0501	0.0567	0.0663	0.0799	0.0965	0.1098	0.1063	0.0751	0.0281
5.0000	0.0398	0.2857	0.0440	0.0500	0.0589	0.0725	0.0913	0.1111	0.1173	0.0914	0.0381
5.5000	0.0354	0.2857	0.0392	0.0446	0.0528	0.0658	0.0855	0.1100	0.1253	0.1067	0.0489
6.0000	0.0318	0.2857	0.0353	0.0403	0.0477	0.0599	0.0797	0.1074	0.1309	0.1208	0.0604
6.5000	0.0289	0.2857	0.0322	0.0366	0.0434	0.0547	0.0741	0.1039	0.1345	0.1336	0.0724
7.0000	0.0265	0.2857	0.0295	0.0336	0.0398	0.0502	0.0689	0.0998	0.1364	0.1450	0.0846
7.5000	0.0245	0.2857	0.0273	0.0311	0.0367	0.0463	0.0641	0.0954	0.1369	0.1550	0.0969
8.0000	0.0227	0.2857	0.0253	0.0289	0.0341	0.0430	0.0597	0.0910	0.1365	0.1638	0.1092
8.5000	0.0212	0.2857	0.0237	0.0269	0.0318	0.0400	0.0558	0.0867	0.1353	0.1715	0.1215

Table:4 System Performance Measures for different λ when $\mu=10, \gamma=2$ and $\delta=5$ for finite population size $M = 3$

λ	P_{idle}	$P_{inactive}$	P_{busy}	L_s	L_q	W_s	W_q
1.0000	0.5554	0.2857	0.1589	0.1847	0.0259	0.1847	0.0259
1.5000	0.4901	0.2857	0.2241	0.2779	0.0538	0.1853	0.0359
2.0000	0.4333	0.2857	0.2809	0.3691	0.0881	0.1845	0.0441
2.5000	0.3841	0.2857	0.3301	0.4568	0.1267	0.1827	0.0507
3.0000	0.3416	0.2857	0.3727	0.5404	0.1677	0.1801	0.0559
3.5000	0.3049	0.2857	0.4094	0.6192	0.2098	0.1769	0.0600
4.0000	0.2731	0.2857	0.4412	0.6933	0.2521	0.1733	0.0630
4.5000	0.2456	0.2857	0.4687	0.7625	0.2938	0.1694	0.0653
5.0000	0.2217	0.2857	0.4925	0.8270	0.3344	0.1654	0.0669
5.5000	0.2010	0.2857	0.5133	0.8870	0.3737	0.1613	0.0679
6.0000	0.1829	0.2857	0.5314	0.9429	0.4114	0.1571	0.0686
6.5000	0.1670	0.2857	0.5473	0.9948	0.4475	0.1530	0.0688
7.0000	0.1530	0.2857	0.5612	1.0431	0.4818	0.1490	0.0688
7.5000	0.1407	0.2857	0.5735	1.0880	0.5145	0.1451	0.0686
8.0000	0.1299	0.2857	0.5844	1.1299	0.5455	0.1412	0.0682
8.5000	0.1202	0.2857	0.5941	1.1689	0.5748	0.1375	0.0676

Table:5 System Performance Measures for various different λ when $\mu=10, \gamma=2$ and $\delta=5$ for finite population size $M = 7$

λ	P_{idle}	$P_{inactive}$	P_{busy}	L_s	L_q	W_s	W_q
1.0000	0.3776	0.2857	0.3367	0.5442	0.2075	0.5442	0.2075
1.5000	0.2712	0.2857	0.4431	0.8769	0.4339	0.5846	0.2892
2.0000	0.1983	0.2857	0.5160	1.2100	0.6939	0.6050	0.3470
2.5000	0.1492	0.2857	0.5650	1.5221	0.9571	0.6089	0.3828
3.0000	0.1162	0.2857	0.5981	1.8037	1.2056	0.6012	0.4019
3.5000	0.0934	0.2857	0.6209	2.0529	1.4320	0.5865	0.4091
4.0000	0.0773	0.2857	0.6370	2.2716	1.6346	0.5679	0.4086
4.5000	0.0655	0.2857	0.6488	2.4634	1.8146	0.5474	0.4032
5.0000	0.0567	0.2857	0.6576	2.6320	1.9743	0.5264	0.3949
5.5000	0.0498	0.2857	0.6645	2.7807	2.1163	0.5056	0.3848
6.0000	0.0444	0.2857	0.6699	2.9127	2.2428	0.4854	0.3738
6.5000	0.0401	0.2857	0.6742	3.0303	2.3561	0.4662	0.3625
7.0000	0.0365	0.2857	0.6778	3.1358	2.4580	0.4480	0.3511
7.5000	0.0335	0.2857	0.6808	3.2307	2.5499	0.4308	0.3400
8.0000	0.0309	0.2857	0.6834	3.3166	2.6333	0.4146	0.3292
8.5000	0.0287	0.2857	0.6855	3.3947	2.7092	0.3994	0.3187
9.0000	0.0268	0.2857	0.6874	3.4660	2.7785	0.3851	0.3087
9.5000	0.0252	0.2857	0.6891	3.5312	2.8421	0.3717	0.2992

Table:6 System Performance Measures for different λ when $\mu=10, \gamma=2$ and $\delta=5$ for finite population size $M = 9$

λ	P_{idle}	$P_{inactive}$	P_{busy}	L_s	L_q	W_s	W_q
1.0000	0.3065	0.2857	0.4077	0.7837	0.3760	0.7837	0.3760
1.5000	0.2002	0.2857	0.5141	1.2863	0.7722	0.8575	0.5148
2.0000	0.1377	0.2857	0.5766	1.7728	1.1962	0.8864	0.5981
2.5000	0.1008	0.2857	0.6135	2.2082	1.5947	0.8833	0.6379
3.0000	0.0781	0.2857	0.6362	2.5847	1.9485	0.8616	0.6495
3.5000	0.0632	0.2857	0.6511	2.9071	2.2560	0.8306	0.6446
4.0000	0.0529	0.2857	0.6614	3.1834	2.5220	0.7958	0.6305
4.5000	0.0454	0.2857	0.6689	3.4215	2.7526	0.7603	0.6117
5.0000	0.0398	0.2857	0.6745	3.6282	2.9537	0.7256	0.5907
5.5000	0.0354	0.2857	0.6789	3.8090	3.1301	0.6926	0.5691
6.0000	0.0318	0.2857	0.6825	3.9683	3.2859	0.6614	0.5476
6.5000	0.0289	0.2857	0.6854	4.1097	3.4243	0.6323	0.5268
7.0000	0.0265	0.2857	0.6878	4.2358	3.5480	0.6051	0.5069
7.5000	0.0245	0.2857	0.6898	4.3491	3.6593	0.5799	0.4879

VI. GRAPHICAL STUDY

Figure 1, 2 and 3 show the Expected number of occupants in the system over the arrival rate λ . The following graphs show that the Expected no. of occupant's increases when arrival rate λ increases.

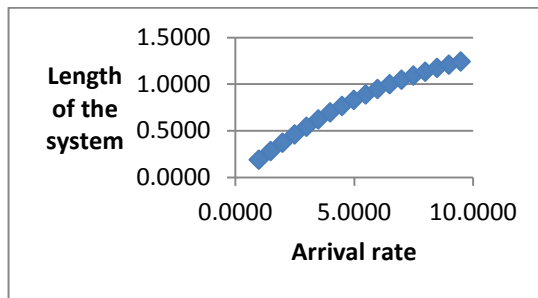


Figure 1: Expected no. of occupants for finite population size = 3, $\mu = 10, \gamma = 2, \delta = 5$ and numerous values of λ

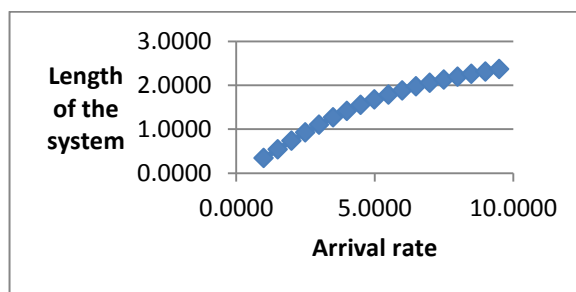


Figure 2: Expected no. of occupants for finite population size = 7, $\mu = 10, \gamma = 2, \delta = 5$ and numerous values of λ

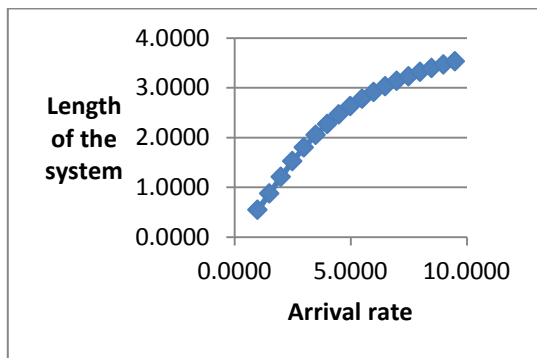


Figure 3: Expected no. of occupants for finite population size = 9, $\mu = 10$, $\gamma = 2$, $\delta = 5$ and numerous values of λ

VII. CONCLUSION

In this research work, we have analyzed the catastrophe effect in finite source queueing system. If the catastrophe happens then it removes all the occupants in the system, it becomes null and then the server goes to the state of inactive. This model goes to basic finite source queues when the catastrophe rate $\gamma \rightarrow 0$.

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