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Proper D - Lucky Labeling on Arbitrary Super Subdivision of New Family of Graphs

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***Abstract*—** In this paper, we prove the existence of proper d – lucky labeling of the arbitrary super subdivision of some new family of graphs ($\boldsymbol{P_m}$: $\boldsymbol{Q_3}$)and [$\boldsymbol{P_m}$: $\boldsymbol{C_n^{(2)}}$]graphs and their proper d- lucky numbers are obtained.

***Keywords*—** *Proper d-lucky labeling, proper d-lucky number, arbitrary super subdivision*

I. Introduction

In recent years, graph labeling is one of the most popular active research area in Graph Theory. It is an assignment of labels to the vertices or edges or both, subject to certain constraints. The concept of d- lucky labeling of graphs was introduced by Mirka Miller et al[7] . It is defined as a function l: V(G) ⟶N, the vertices of G are assigned by positive integers. Define c(u) =$\sum_{v\in N(u)} l(v)$ + d(u) where d(u) denotes the degree of the vertex u and N(u) denotes the open neighborhood of u. The labeling is said to be d- lucky if c(u) ≠ c(v) for every adjacent vertices u and v. The d-lucky numbers for certain graphs are obtained in [1,2,7].The proper lucky number of mesh and it's derived architectures were studied by KinsYenoke et.al[6]. Esakkiammalet.al[3] introduced the concept of proper d-lucky labeling and is defined as follows:A d-lucky labeling is called proper if l (u) ≠ l (v) for every adjacent vertices u and v. The proper d-lucky number of a graph is the least positive integer k such that the graph G has a proper d- lucky labeling with {1, 2,…,k} as the set of labels and is denoted by η_{pdl}(G) . The proper d- lucky number for the arbitrary super subdivision of (P_m : S_n)and (P_m : C_n) graphs were obtained[3].In this paper the proper d-lucky numbers for the arbitrary super subdivision of (P_m : Q_3) and [P_m : $C_n^{(2)}$] graphsare obtained. The following definitions are prerequisites for the present investigation.

A graph is said to be an arbitrary super subdivision of a graph G if it is obtained from G by replacing each edge e_i by a complete bipartite graph K_{2,M_i} (where M_i is any positive integer and may vary for each edge arbitrarily) in such a way that the ends of each edge $e_i$are merged with the two vertices of 2-vertices part of K_{2,M_i} after removing the edge from G and it is denoted by ASS(G). The cubical graph Q_3 is a 3-regular graph with 8 vertices.Also the friendship graph $C_n^{(2)}$ is obtained from two copies of C_n with the common vertex $u_{i1} = v_{i1}$, where the vertices of first copy are denoted by $u_{i1}, u_{i2,\dots,} u_{in}$ and the vertices of second copy are denoted by $v_{i1}, v_{i2,\dots,} v_{in}$. For all terminologies and notions one may refer [4,5].

II. Main Results

A. *Structure of arbitrary super subdivision of (P_m : Q_3) graph*

Let (P_m : Q_3) be a graph obtained from the path graph P_m and m+1 copies of cubical graph Q_3 by joining the vertex u_i in P_m with the vertex v_{i1} in the ith copy of Q_3 by an edge for 1≤ i ≤ m+1, where vertices of P_m are $u_1, u_2,\dots, u_{m+1}$ and the vertices of ith copy Q_3 are $v_{i1}, v_{i2}, \dots, v_{i8}$. The vertex set and the edge set of (P_m : Q_3) are as follows.

V ((P_m : Q_3)) = {u_i , v_{ij} /1≤ i ≤ m+1; 1≤ j ≤ 8};

E ((P_m : Q_3)) = {$e_i = u_i\, u_{(i+1)}$ / 1≤ i ≤ m} ∪{$e_{m+i} = u_i\, v_{i1}$ / 1≤ i ≤ m+1} ∪ {$e_{2m+12i-11+j} = v_{ij}\, v_{i(j+1)}$/1 ≤ i ≤ m + 1; 1 ≤ j ≤ 4 ,for j=4 the subscript j+1 reduced to 1} ∪ {$e_{2m+12i-7+j} = v_{i(j+4)}\, v_{i(j+5)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4,for j=4 the subscript j+5 reduced to 5} ∪ { $e_{2m+12i-3+j} = v_{ij}\, v_{i(j+4)}$ } /1≤ i ≤ m+1; 1≤ j ≤ 4}.This (P_m : Q_3) graph has 9(m+1) vertices and 14m+13 edges.

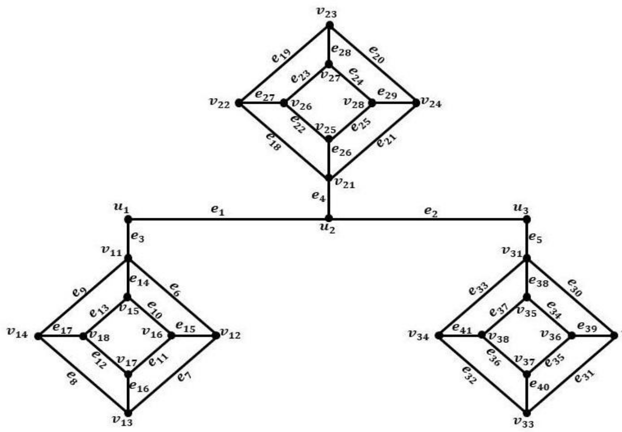

Figure 1. Structure of (P_2 : Q_3) graph

The arbitrary super subdivision of (P_m : Q_3) graph is obtained from (P_m : Q_3) graph in such a way that each edge e_i of (P_m : Q_3) graph is replaced by K_{2,M_i} , where $M_i \epsilon$ N, 1≤ i ≤14m+13. This graph is denoted by ASS ((P_m : Q_3)). The vertex set and edge set are given as follows:

V (ASS((P_m : Q_3))) = $\{u_i, v_{ij}$ /1≤ i ≤ m+1; 1≤ j ≤ 8$\} \cup \{u_{i(i+1)}^{(k)}$ /1≤ i ≤ m; 1≤ k ≤ $M_i\} \cup \{ (uv)_i^{(k)}$ /1≤ i ≤ m+1; 1≤ k ≤ M_{m+i} $\} \cup \{ w_{ij}^{(k)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4 ;1≤ k ≤ $M_{2m+12i+j-3}\} \cup \{ v_{ij}^{(k)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4 ;1≤ k ≤ $M_{2m+12i+j-11}\} \cup \{ v_{i(j+4)}^{(k)}$/ 1≤ i ≤ m+1; 1≤ j ≤ 4;1≤ k ≤ $M_{2m+12i+j-7}\}$.

E(ASS ((P_m : Q_3))) = $\{u_i\, u_{i(i+1)}^{(k)}, u_{i(i+1)}^{(k)} u_{i+1}$ / 1≤ i ≤ m; 1≤ k ≤ M_i $\} \cup \{ u_i\,(uv)_i^{(k)}$, $(uv)_i^{(k)} v_{i1}$ / 1≤ i ≤ m+1; 1≤ k ≤ $M_{m+i}\} \cup \{v_{ij}\, v_{ij}^{(k)}, v_{ij}^{(k)} v_{i(j+1)}$/1≤i≤m+1; 1≤ j ≤ 3 ; 1≤ k ≤ $M_{2m+12i+j-11}$ $\} \cup \{ v_{i4}\, v_{i4}^{(k)}, v_{i4}^{(k)} v_{i1}$ /1≤ i ≤ m+1; 1≤ k ≤ $M_{2m+12i-7}\} \cup \{v_{i(j+4)}\, v_{i(j+4)}^{(k)}$, $v_{i(j+4)}^{(k)} v_{i(j+5)}$ /1≤ i ≤ m+1; 1≤ k ≤ $M_{2m+12i+j-7}$, 1≤ j ≤ 3$\} \cup \{v_{i8}\, v_{i8}^{(k)}, v_{i8}^{(k)} v_{i5}$ /1≤ i ≤ m+1; 1≤ k ≤ $M_{2m+12i-3}\} \cup \{v_{ij}\, w_{ij}^{(k)}, w_{ij}^{(k)} v_{i(j+4)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4 ;1≤ k ≤ $M_{2m+12i+j-3}\}$.

This graph has 9(m+1) + $\sum_{i=1}^{14m+13} M_i$ vertices and $2(\sum_{i=1}^{14m+13} M_i)$ edges.

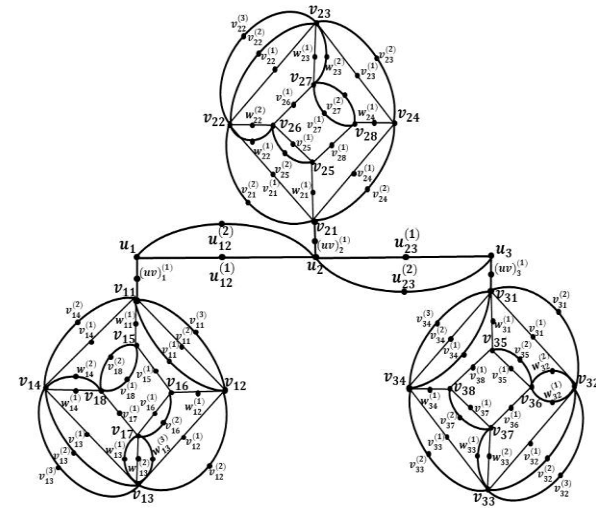

Figure 2. Structure of ASS((P_2 : Q_3)) graph

Here m = 2 ,M_1= 2, M_2= 2, M_3= 1, M_4= 1,M_5= 1, M_6= 3, M_7= 2, M_8= 3,M_9= 2, M_{10}= 1, M_{11}= 2, M_{12}= 1,M_{13}= 2, M_{14}= 1, M_{15}= 1, M_{16}= 3,M_{17}= 2, M_{18}= 2, M_{19}= 3,M_{20}= 2,M_{21}= 2, M_{22}= 2, M_{23}= 1,M_{24}= 2, M_{25}= 1, M_{26}= 1, M_{27}= 2,M_{28}= 2, M_{29}= 1, M_{30}= 2, M_{31}= 3,M_{32}= 2, M_{33}= 3, M_{34}= 2, M_{35}= 1,M_{36}= 2, M_{37}= 1, M_{38}= 1, M_{39}= 2,M_{40}= 2,M_{41}= 1.

Algorithm 2.1.Proper d-lucky labeling of ASS ((P_m : Q_3)) graph

Procedure. Vertex labeling of ASS ((P_m : Q_3)) graph

Input. ASS ((P_m : Q_3)) graph

V←$\{u_i, v_{ij}$ /1≤ i ≤ m+1; 1≤ j ≤ 8$\} \cup \{u_{i(i+1)}^{(k)}$ /1≤ i ≤ m; 1≤ k ≤ $M_i\} \cup \{(uv)_i^{(k)}$ /1≤ i ≤ m+1; 1≤ k ≤ $M_{m+i}\} \cup \{v_{ij}^{(k)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4 ;1≤ k ≤ $M_{2m+12i+j-11}\} \cup \{v_{i(j+4)}^{(k)}$/ 1≤ i ≤ m+1; 1≤ j ≤ 4 ; 1≤ k ≤ $M_{2m+12i+j-7}\} \cup \{ w_{ij}^{(k)}$ /1≤ i ≤ m+1; 1≤ j ≤ 4 ;1≤ k ≤ $M_{2m+12i+j-3}\}$

fori = 1 to m do

for k = 1 to M_i do

$u_{i(i+1)}^{(k)}$ ← k+1;

end for

end for

fori = 1 to m+1 do

u_i ← 1;

for j = 1 to 8 do

v_{ij} ← 1;

end for

for k = 1 to M_{m+i} do

$(uv)_i^{(k)}$ ← k+1;

end for

for j = 1 to 4 do

for k = 1 to $M_{2m+12i+j-11}$ do

$v_{ij}^{(k)}$ ← k+1;

end for

for k = 1 to $M_{2m+12i+j-7}$ do

$v_{i(j+4)}^{(k)}$ ← k+1;

end for

for k = 1 to $M_{2m+12i+j-3}$ do

$w_{ij}^{(k)}$ ← k+1;

end for

end for

end for

end procedure

Output: The vertex labeled ASS ((P_m : Q_3)) graph.

Theorem 2.1. The arbitrary super subdivision of (P_m : Q_3) graph admits proper d- lucky labeling and the proper d-lucky number is

η_{pdl}(ASS((P_m:Q_3))) = max{M_i/ $1 \leq$ i ≤ 14m $+ 13\} + 1$.

Proof. Consider ASS ((P_m : Q_3)) , the arbitrary super subdivision of (P_m : Q_3) graph whose vertices and edges are given as in structure 2.1. The vertices of ASS ((P_m : Q_3)) graph are labeled by defining a function l:V(ASS((P_m : Q_3)))→ N as given in the algorithm 2.1. Clearly all the adjacent vertices have distinct labels. Hence the graph is properly labeled. The degrees of the vertices are:

d (u_1) = M_1+ M_{m+1};d (u_{m+1}) = M_m+ M_{2m+1};

For 2≤ i ≤ m, d (u_i) = M_{i-1}+M_i+M_{m+i};

For 1≤ i ≤ m, 1≤ k ≤ M_i , d ($u_{i(i+1)}^{(k)}$) = 2;

For 1≤ i ≤ m+1, 1≤ k ≤ M_{m+i} , d (($(uv)_i^{(k)}$) = 2;

For 1≤ i ≤ m+1, d (v_{i1}) = M_{m+i} + $M_{2m+12i-10}$ + $M_{2m+12i-7}$+ $M_{2m+12i-2}$;

For 1≤ i ≤ m+1, 2≤ j ≤ 4, d (v_{ij}) = $M_{2m+12i+j-12}$+ $M_{2m+12i+j-11}$+ + $M_{2m+12i+j-3}$;

For 1≤ i ≤ m+1, 6≤ j ≤ 8, d (v_{ij}) = $M_{2m+12i+j-12}$ + $M_{2m+12i+j-11}$+$M_{2m+12i+j-7}$;

For 1≤ i ≤ m+1, d (v_{i5}) = $M_{2m+12i-6}$ + $M_{2m+12i-3}$+$M_{2m+12i-2}$;

For 1≤ i ≤ m+1, 1≤ j ≤ 8, 1≤ k ≤ $M_{2m+12i+j-11}$; d ($v_{ij}^{(k)}$) = 2 ;

For 1≤ i ≤ m+1, 1≤ j ≤ 4, 1≤ k ≤ $M_{2m+12i+j-3}$; d ($w_{ij}^{(k)}$) = 2

Now c (u) ′s are calculated as follows:

c (u_1) = $\frac{1}{2}$ (M_1^2 + M_{m+1}^2 + 5(M_1 + M_{m+1})) ;

c (u_{m+1}) = $\frac{1}{2}$ (M_m^2 + M_{2m+1}^2 + 5(M_m + M_{2m+1})) ;

For 2≤ i ≤ m, c (u_i) = $\frac{1}{2}$ (M_{i-1}^2 + M_i^2 + M_{m+i}^2 + 5(M_{i-1} + M_i +M_{m+i})) ;

For 1≤ i ≤ m, $1 \leq$ k $\leq M_i$, c ($u_{i(i+1)}^{(k)}$) = 4;

For 1≤ i ≤ m+1, 1≤ k ≤ M_{m+i} ,c (($(uv)_i^{(k)}$) = 4 ;

For 1≤ i ≤ m+1,

c (v_{i1}) = $\frac{1}{2}$ (M_{m+i}^2+ $M_{2m+12i-10}^2$ +$M_{2m+12i-7}^2$+ $M_{2m+12i-2}^2$+

5(M_{m+i}+ $M_{2m+12i-10}$ +$M_{2m+12i-7+}$$M_{2m+12i-2}$));

For 1≤ i ≤ m+1, 2≤ j ≤ 4,

c (v_{ij}) = $\frac{1}{2}$ ($M_{2m+12i+j-12}^2$ + $M_{2m+12i+j-11}^2$ + $M_{2m+12i+j-3}^2$+

5($M_{2m+12i+j-12}$ + $M_{2m+12i+j-11}$+ $M_{2m+12i+j-3}$));

For 1≤ i ≤ m+1, 6≤ j ≤ 8,

c (v_{ij}) = $\frac{1}{2}$ ($M_{2m+12i+j-12}^2$ + $M_{2m+12i+j-11}^2$ + $M_{2m+12i+j-7}^2$ +

5($M_{2m+12i+j-12}$ +$M_{2m+12i+j-11}$ +$M_{2m+12i+j-7}$));

For 1≤ i ≤ m+1,

c (v_{i5}) = $\frac{1}{2}$ ($M_{2m+12i-6}^2$ + $M_{2m+12i-3}^2$ + $M_{2m+12i-2}^2$ +

5($M_{2m+12i-6}$ + $M_{2m+12i-3}$+$M_{2m+12i-2}$));

For 1≤ i ≤ m+1, 1≤ j ≤ 8,1≤ k ≤ $M_{2m+12i+j-11}$, c ($v_{ij}^{(k)}$) = 4 ;

For 1≤ i ≤ m+1, 1≤ j ≤ 4,1≤ k ≤ $M_{2m+12i+j-3}$, c ($w_{ij}^{(k)}$) = 4 ;

Thus c (u) ≠ c (v) for every uv ∈ E, Therefore ASS ((P_m : Q_3)) is a proper d- lucky labeled graph and η_{pdl}(ASS ((P_m : Q_3))) = max{M_i/$1 \leq$ i ≤ 14m $+ 13\} + 1$.

Example

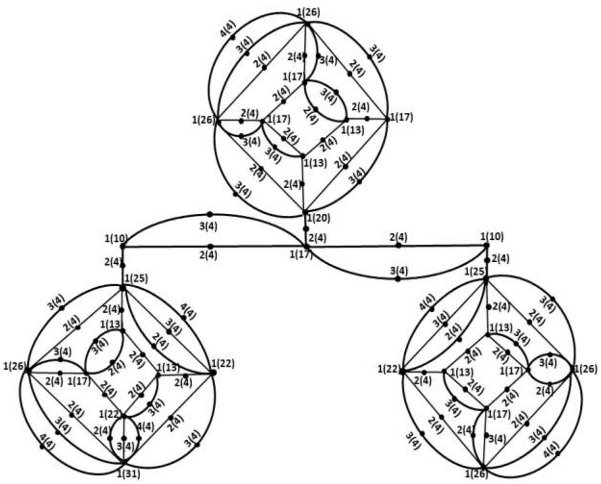

Figure 3. The proper d-lucky labeled ASS ((P_2 : Q_3))

B. Structure of arbitrary super subdivision of [P_m : $C_n^{(2)}$]

Let [P_m : $C_n^{(2)}$] be a graph obtained the path graph P_m and m+1 copies of friendship graph $C_n^{(2)}$, where the vertex u_i in P_m are merged with the vertex $v_{i1} = u_{i1}$ in the i[th] copy of $C_n^{(2)}$for 1≤ i ≤ m+1,where the vertices of the path graph P_m are u_1 ,u_2 ,…,u_{m+1} and the vertices of i[th] copy $C_n^{(2)}$ are v_{i1}=u_{i1} , v_{ij} , u_{ij} , $1 \leq i \leq m+1;\ 2 \leq j \leq n$ .The vertex set and the edge set of [P_m : $C_n^{(2)}$] are as follows.

V ([P_m : $C_n^{(2)}$]) = { $u_i = v_{i1} = u_{i1}, v_{ij}$, u_{ij} /1≤ i ≤ m+1; 2≤ j ≤ n}.

E ([P_m : $C_n^{(2)}$]) = { $e_i = u_i u_{(i+1)}$ /1≤ i ≤ m} ∪ {$e_{m+2n(i-1)+j} = u_{ij} u_{i(j+1)}$ / 1≤ i ≤ m+1; 1 ≤ j ≤ n} ∪ {$e_{m+n(2i-1)+j} = v_{ij} v_{i(j+1)}$ / 1≤ i ≤ m+1; 1 ≤ j ≤ n},where the subscript j=n+1 reduces to modulo n. This [P_m : $C_n^{(2)}$] graph has (m+1) (2n-1) vertices and 2n (m+1)+ m edges.

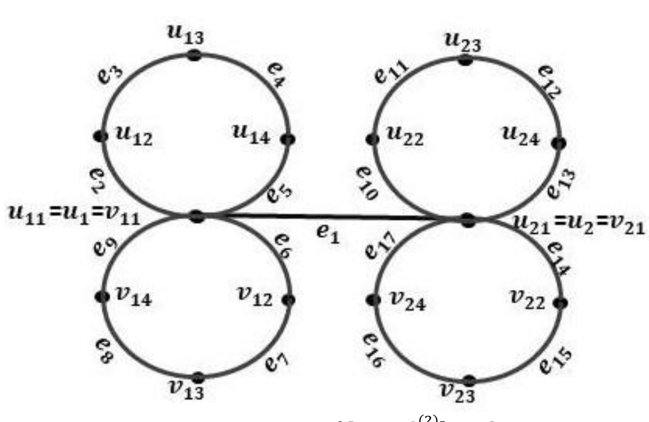

Figure 4. Structure of [P_1 : $C_4^{(2)}$] graph

The arbitrary super subdivision of [P_m : $C_n^{(2)}$] graph is obtained from [P_m : $C_n^{(2)}$] graph in such a way that each edge e_i of [P_m : $C_n^{(2)}$] graph is replaced by K_{2,M_i} , where M_iϵ N, 1≤ i ≤ 2n(m+1)+m. The vertex set and edge set are given as follows:

V (ASS([P_m : $C_n^{(2)}$])) = {$w_{i(i+1)}^{(k)}$ /1≤ i ≤ m; 1≤ k ≤M_i }∪{u_i =v_{i1} =u_{i1} , v_{ij} ,u_{ij} / 1≤ i ≤ m+1; 2 ≤ j ≤ n}∪{$u_{ij}^{(k)}$ /1≤ i ≤ m+1;1≤ j ≤ n; 1≤ k ≤ $M_{m+2n(i-1)+j}$} ∪ {$v_{ij}^{(k)}$ / 1≤ i ≤ m+1;1≤ j ≤ n; 1≤ k ≤ $M_{m+n(2i-1)+j}$}.

E(ASS ([P_m : $C_n^{(2)}$]))= {$u_i\, w_{i(i+1)}^{(k)}$, $w_{i(i+1)}^{(k)} u_{(i+1)}$ / 1≤ i ≤ m; 1≤ k ≤ M_i }∪ {$u_{ij}\, u_{ij}^{(k)}$, $u_{ij}^{(k)} u_{i(j+1)}$ / 1≤ i ≤ m+1;1≤ j ≤ n; 1≤ k ≤ $M_{m+2n(i-1)+j}$ } ∪ {$v_{ij} v_{ij}^{(k)}$, $v_{ij}^{(k)} v_{i(j+1)}$/1≤ i ≤m+1;1≤ j≤ n;1≤k≤ $M_{m+n(2i-1)+j}$ }, where the subscript j=n+1 reduces to modulo n . This graph is denoted by ASS ([P_m : $C_n^{(2)}$]) and has (m+1) (2n-1)+$\sum_{i=1}^{2n(m+1)+m} M_i$ vertices and 2($\sum_{i=1}^{2n(m+1)+m} M_i$) edges.

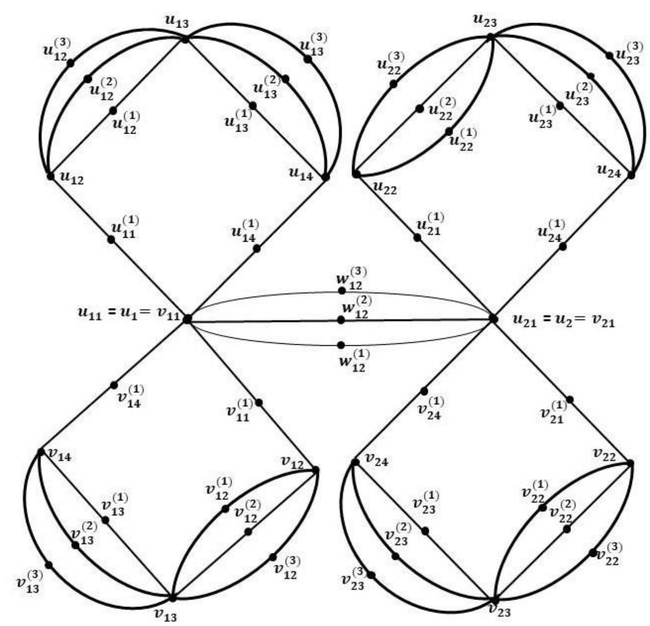

Figure 5. Structure of ASS([P_1 : $C_4^{(2)}$]) graph

Here m = 1, n =4, ,M_1= 3, M_2= 1, M_3= 3, M_4= 3,M_5= 1, M_6= 1, M_7= 3, M_8= 3, M_9= 1, M_{10}= 1, M_{11}= 3, M_{12}= 3, M_{13}= 1, M_{14}= 1, M_{15}= 3, M_{16}= 3, M_{17}= 1.

Algorithm 2.2. Proper d- lucky labeling of ASS ([P_m : $C_n^{(2)}$]) graph

Procedure. Vertex labeling of ASS ([P_m : $C_n^{(2)}$]) graph

Input. ASS ($[P_m : C_n^{(2)}]$) graph

V←{$u_i = v_{i1} = u_{i1}$, v_{ij} ,u_{ij} /1≤ i ≤ m+1; 2 ≤ j ≤ n}∪{$u_{ij}^{(k)}$ /1≤ i ≤ m+1; 1 ≤ j ≤ n; 1≤ k ≤ $M_{m+2n(i-1)+j}$}∪{$v_{ij}^{(k)}$/1≤i≤ m+1;1 ≤j≤ n;1≤k≤ $M_{m+n(2i-1)+j}$}∪ {$w_{i(I+1)}^{(k)}$ /1≤ i ≤ m; 1≤ k ≤ M_i}.

for i = 1 to m+1 do

u_i ← 1;

for j = 2 to n do

u_{ij} ← 1;v_{ij} ← 1

end for

for j = 1 to n do

for k = 1 to $M_{m+2n(i-1)+j}$ do

$u_{ij}^{(k)}$ ← k+1;

end for

for k = 1 to $M_{m+n(2i-1)+j}$ do

$v_{ij}^{(k)}$ ← k+1;

end for

end for

end for

fori = 1 to m

for k = 1 to M_i do

$w_{ij}^{(k)}$ ← k+1;

end for

end for

end procedure.

Output: The vertex labeled ASS ($[P_m : C_n^{(2)}]$) graph.

Theorem 2.2. The arbitrary super subdivision of $[P_m : C_n^{(2)}]$ graph admits proper d- lucky labeling and the proper d- lucky number number is

η_{pdl}(ASS($[P_m : C_n^{(2)}]$)) is $\max\{M_i/1 \le i \le 2n(m+1) + m\} + 1$.

Proof.Consider ASS ($[P_m : C_n^{(2)}]$) , the arbitrary super subdivision of $[P_m : C_n^{(2)}]$ graph, where each edge e_i of $[P_m : C_n^{(2)}]$ graph is subdivided by K_{2,M_i}, $M_i \epsilon$ N, 1 ≤ i ≤ 2n(m+1)+m. The vertices of ASS($[P_m : C_n^{(2)}]$) graph are labeled by defining a function l:V(ASS($[P_m : C_n^{(2)}]$) → N as given in the algorithm 2.2.Since all the adjacent vertices have distinct labels, the vertices are properly labeled. The degrees of the vertices are:

d($u_1 = u_{11} = v_{11}$) = $M_1 + M_{m+1} + M_{m+n} + M_{m+n+1} + M_{m+2n}$;

d($u_{(m+1)} = v_{(m+1)1} = u_{(m+1)1}$) = $M_m + M_{m(2n+1)+1} + M_{n(2m+1)+m} + M_{n(2m+1)+m+1} +$

$M_{2n(m+1)+m}$;

For 2 ≤ i ≤ m,

d ($u_i = v_{i1} = u_{i1}$)= $M_{i-1} + M_i + M_{m+2n(i-1)+1} + M_{m+2ni-n} + M_{m+n(2i-1)+1} + M_{m+2ni}$;

For 1≤ i ≤ m+1,2≤ j ≤ n , d (u_{ij}) = $M_{m+2n(i-1)+j-1} + M_{m+2n(i-1)+j}$;

d (v_{ij}) = $M_{m+n(2i-1)+j-1} + M_{m+n(2i-1)+j}$;

For 1≤ i ≤ m , 1≤ k ≤M_i , d ($w_{i(i+1)}^{(k)}$) = 2 ;

For 1≤ i ≤ m+1, 1≤ j ≤ n ;1≤ k ≤ $M_{m+2n(i-1)+j}$, d ($u_{ij}^{(k)}$) = 2 ;

For 1≤ i ≤ m+1, 1≤ j ≤ n, 1≤ k ≤ $M_{m+n(2i-1)+j}$, d ($v_{ij}^{(k)}$) = 2.

Now c (u)′s are calculated as follows:

c ($u_1 = u_{11} = v_{11}$) = $\frac{1}{2}$ ($M_1^2 + M_{m+1}^2 + M_{m+n}^2 + M_{m+n+1}^2 + M_{m+2n}^2 +$

5($M_1 + M_{m+1} + M_{m+n} + M_{m+n+1} + M_{m+2n}$));

c ($u_{m+1} = u_{(m+1)1} = v_{(m+1)1}$) = $\frac{1}{2}$ ($M_m^2 + M_{m(2n+1)+1}^2 + M_{n(2m+1)+m}^2 + M_{n(2m+1)+m+1}^2 +$

$M_{2n(m+1)+m}^2$ +5($M_m + M_{m(2n+1)+1} + M_{n(2m+1)+m} + M_{n(2m+1)+m+1} + M_{2n(m+1)+m}$));

For 2 ≤ i ≤ m,

c ($u_i = u_{i1} = v_{i1}$) = $\frac{1}{2}$ ($M_{i-1}^2 + M_i^2 + M_{m+2n(i-1)+1}^2 + M_{m+2ni-n}^2 + + M_{m+2ni}^2$ + 5($M_{i-1} + M_i + M_{m+2n(i-1)+1} + M_{m+2ni-n} + M_{m+n(2i-1)+1} + M_{m+2ni}$));

For 1≤ i ≤ m+1, 2≤ j ≤ n,

c (u_{ij}) = $\frac{1}{2}$ ($M_{m+2n(i-1)+j-1}^2 + M_{m+2n(i-1)+j}^2$ + 5($M_{m+2n(i-1)+j-1} + M_{m+2n(i-1)+j}$)) ;

c (v_{ij}) = $\frac{1}{2}$ ($M_{m+n(2i-1)+j-1}^2 + M_{m+n(2i-1)+j}^2$ + 5($M_{m+n(2i-1)+j-1} + M_{m+n(2i-1)+j}$)) ;

For 1≤ i ≤ m,1≤ k ≤ M_i , c ($w_{i(i+1)}^{(k)}$) = 4 ;

For 1≤ i ≤ m+1, 1≤ j ≤ n, 1≤ k ≤ $M_{m+2n(i-1)+j}$, c ($u_{ij}^{(k)}$) = 4;

For 1≤ i ≤ m+1, 1≤ j ≤ n, 1≤ k ≤ $M_{m+n(2i-1)+j}$, c ($v_{ij}^{(k)}$) = 4;

Thus all the adjacent vertices have distinct c (u)'s. Hence ASS ([$P_m : C_n^{(2)}$]) is a proper d- lucky labeled graph and the proper d – lucky number is

η_{pdl}(ASS ([$P_m : C_n^{(2)}$])) $= \max\{M_i / 1 \leq \text{ i } \leq 2n\,(m+1) + m\} + 1$.

Example.

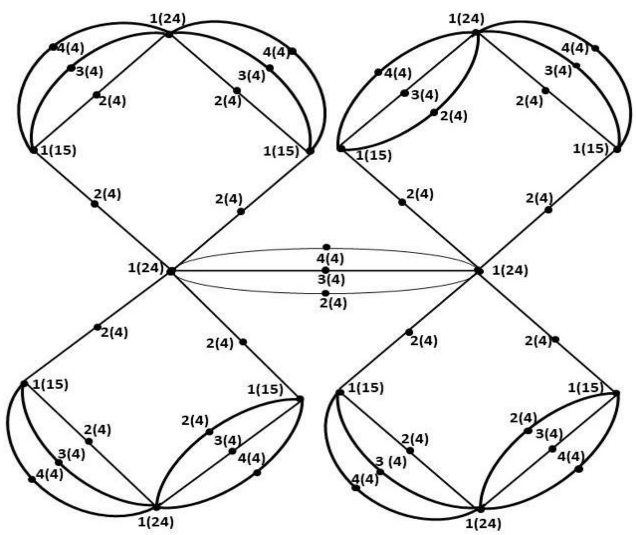

Figure 6. The proper d-lucky labeled ASS([P_1 : $C_4^{(2)}$])

III. CONCLUSION

In this paper, we have proved the existence of proper d- lucky labeling of arbitrary super subdivisions of $(\boldsymbol{P_m} : \boldsymbol{Q_3})$and $[\boldsymbol{P_m} : \boldsymbol{C_n^{(2)}}]$ graphs and obtained their proper d- lucky numbers .

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