IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Special Issue, 5, March 2019 E-ISSN: 2347-2693

Some Cordial Labeling on Human Chain Graph

K. Anitha[1*], B. Selvam[2], K. Thirusangu[3]

[1]Department of Mathematics, Sri Sairam Engineering College, Chennai – 600 044, India
[2,3]Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai – 600 073, India

[*]Corresponding Author: saisrijune@gmail.com, Tel.: +00-12345-54321

DOI: https://doi.org/10.26438/ijcse/v7si5.8184 | Available online at: www.ijcseonline.org

***Abstract*—** A Human chain graph is a simple, finite and undirected geaph. In this paper, we prove that the existence of V-cordial and Homo-cordial labeling for the Human chain graph by using algorithms.

***Keywords*—** Human chain, Cordial, V-cordial, Homo-cordial, Labeling

I. INTRODUCTION

Let G= $HC_{n,m}(p,q)$, n∈N, m≥3 be a human chain graph and it is a simple, finite and undirected graph with p= 2mn+n+1 vertices and q=2mn+2n edges. The concept of Human chain graph was introduced by K.Anitha and B.Selvam[1]. The concept of cordial and total product cordial labeling was introduced by Cahit [2]. The concept of V- cordial labeling was introduced by Nellai Murugan and P.Iyadurai Selvaraj [4]. The concept of Homo-cordial labeling was studied by Nellai Murugan [5,6]. In this paper, we prove that the existence of V-cordial and Homo cordial labeling for the Human chain graph by using algorithms.

II. PRELIMINARIES

Some basic definitions are provided below.

Definition 2.1 Cordial labelling

A function $f : V \rightarrow \{0,1\}$ such that each edge *uv* receives the label $|f(u) - f(v)|$ is said to be cordial labeling if the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

Definition 2.2 V-Cordial labelling

Let G=(V,E) be a graph with p vertices and q edges. A cup (V) cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 0 if f(u)=f(v)=0 or 1 otherwise with the condition that $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. The graph that admits a V-cordial labeling is called V-cordial graph.

Definition 2.3 Total V-Cordial labeling

Let G=(V,E) be a graph with p vertices and q edges. A Total cup (V) cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 0 if f(u)=f(v)=0 or 1 otherwise with the condition that $|ev_f(0)-ev_f(1)|\leq 1$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x(x=0,1). The graph that admits a total V-cordial labeling is called total V-cordial graph.

Definition 2.4 Homo-Cordial labelling

Let G=(V,E) be a graph with p vertices and q edges. A Homo-cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 1 if f(u)=f(v) or 0 if f(u)≠f(v) with the condition that $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. The graph that admits a Homo-cordial labeling is called Homo-cordial graph.

Definition 2.5 Total Homo-Cordial labelling

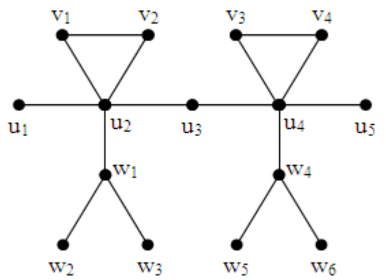
Let G=(V,E) be a graph with p vertices and q edges. A total Homo-cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 1 if f(u)=f(v) or 0 if f(u)≠f(v) with the condition that $|ev_f(0)-ev_f(1)|\leq 1$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x(x=0,1). The graph that admits a total homo-cordial labeling is called total Homo-cordial graph.

Definition 2.6 Human chain graph

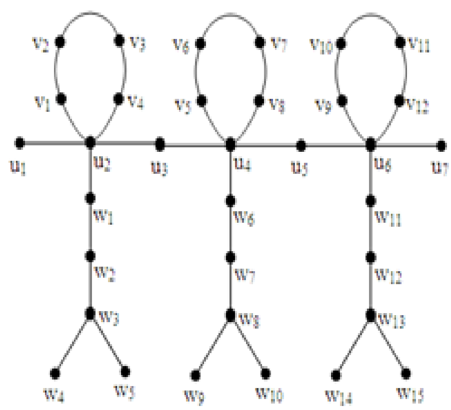
A Human chain graph $HC_{n,m}(p,q)$ is obtained a path $u_1, u_2, \ldots, u_{2n+1}$, n∈N by joining a cycle of length m (C_m) and Y-tree (Y_{m+1}, m≥3) to each u_{2i} for $1\leq i\leq n$. The vertices of C_m and Y-tree Y_{m+1} are $v_1, v_2, \ldots\ldots, v_{(m-1)n}$ and $w_1, w_2, \ldots, w_{mn}$ respectively. The vertex set and the edge set of $HC_{n,m}$ is as follows: $V(HC_{n,m})=\{u_i, v_j, w_k \ / \ 1 \leq i \leq 2n+1, 1 \leq j \leq (m-1)n, \ 1 \leq k \leq mn\}$ and $E(HC_{n,m})=|E|=\{u_iu_{i+1}/ \ 1\leq i\leq 2n\} \cup \{u_{2i} w_{m(i-1)+1};\ u_{2i} v_{(m-1)i};\ u_{2i} v_{(m-1)(i-1)+1};\ w_{mi} w_{mi-2}/1\leq i\leq n\} \cup \{w_{mi+j} w_{mi+j+1};\ v_{(m-1)i+j} v_{(m-i)+j+1}/ \ 0\leq i\leq n-1,\ 1\leq j\leq m-2\}$

© 2019, IJCSE All Rights Reserved

Example: 1 ($HC_{2,3}$)



Example: 2 ($HC_{3,5}$)



Structural properties of $HC_{n,m}$

1. The vertex set of $HC_{n,m}$ $=\{u_i, v_j, w_k / 1\leq i\leq 2n+1, 1\leq j\leq (m-1)n, 1\leq k\leq mn\}$.
2. The total number of vertices of $HC_{n,m}= |V| = 2mn+n+1$.
3. The edge of set of $HC_{n,m}= \{u_i\, u_{i+1} / 1\leq i\leq 2n\} \cup \{u_{2i}\, w_{m(i-1)+1};\ u_{2i}\, v_{(m-1)i};\ u_{2i}\, v_{(m-1)(i-1)+1};\ w_{mi}\, w_{mi-2} / 1\leq i\leq n\} \cup \{ w_{mi+j}\, w_{mi+j+1};\ v_{(m-1)i+j}\, v_{(m-i)+j+1} / 0\leq i\leq n-1, 1\leq j\leq m-2\}$.
4. The total number of edges of $HC_{n,m}=|E|= 2mn+2n$.
5. The maximum degree of $HC_{n,m}= \Delta=5$.
6. The minimum degree of $HC_{n,m}= \delta=1$.

III. METHODOLOGY

In this section, we present algorithms and prove that the existence of V-cordial, total V-cordial,Homo cordial and total Homo-cordial labeling for the Human chain graph.

Algorithm 3.1

We use the following algorithm to prove that the existence of V-cordial labeling for the human chain graph.

Procedure (V- Cordial labeling of $HC_{n,m}$)

$V\leftarrow\{u_1,u_2,\ldots,u_{2n+1}, v_1,v_2,\ldots,v_{(m-1)n}, w_1,w_2,\ldots,w_{mn}\}$

$E\leftarrow\{ e_1,e_2,\ldots,e_{2mn+2n}\}$

if m≥3

    for i= 1 to mn do
        $w_i\leftarrow 1$
    end for
    for i= 1 to (m-1)n do
        $v_i\leftarrow 0$
    end for
    for i= 1 to n do
        $u_{2i}\leftarrow 0$
    end for
    for i= 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do
        $u_{4i-1}\leftarrow 0$
    end for
    for i= 1 to $\left\lfloor\frac{n+2}{2}\right\rfloor$
        $u_{4i-3}\leftarrow 1$
    end for

end if

end procedure

Theorem 3.1 For m≥3 and n≥1, the Human chain graph admits V-cordial labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with $p= 2mn+n+1$ vertices and $q= 2mn+2n$ edges. Using algorithm 3.1, the $2mn+n+1$ vertices are labeled with {0,1}. Here if n is odd, the number of vertices labeled with '0' and '1' is $mn+(n+1)/2$ respectively and if n is even, the number of vertices labeled with '0' is $mn+(n/2)+1$ and the number of vertices labeled with '1' is $mn+(n/2)$. Thus $2mn+n+1$ vertices are labeled with 0 and 1 differ by at most one. The resulting edge labels are given in the following way. If n≥1, for 1≤i≤n, $f(u_{2i}\ v_{(m-1)i})=0$, $f(u_{2i}\ v_{(m-1)(i-1)+1})=0$, $f(u_{2i}\ w_{m(i-1)+1})=1$, $f(w_{mi}\ w_{mi-2})=1$, for 0≤i≤n-1, 1≤j≤m-2, $f(w_{mi+j}\ w_{mi+j+1})=1$, $f(v_{(m-1)i+j}\ v_{(m-1)i+j+1})=0$, for $1\leq i\leq\left\lfloor\frac{n+1}{2}\right\rfloor$, $f(u_{4i-3}\ u_{4i-2})=1$, $f(u_{4i-2}\ u_{4i-1})=0$ and if n>1, for $1\leq i\leq\left\lfloor\frac{n}{2}\right\rfloor$, $f(u_{4i-1}\ u_{4i})=0$, $f(u_{4i}\ u_{4i+1})=1$. Thus mn+n edges are labeled with '0' and mn+n edges are labeled with '1' . Thus 2mn+2n edges are labeled with '0' and '1' differ by at most one. Hence the human chain graph $HC_{n,m}$ admits V-cordial labeling.

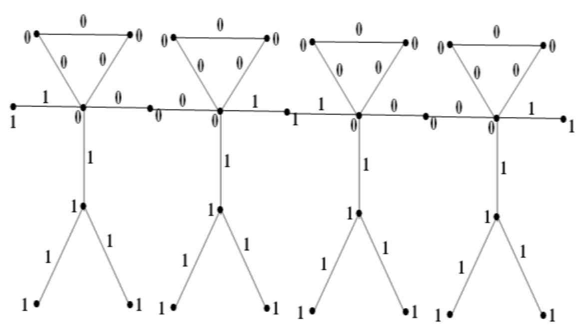
Theorem 3.2 For m≥3 and n≥1, the Human chain graph admits total V-cordial labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges.

Case (i) If n is odd, in theorem 3.1, the number of vertices labeled with '0' and '1' is mn+(n+1)/2 respectively and the number of edges labeled with '0' and '1' is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with '0' and with '1' is mn + (n+1)/2 + mn + n = 2mn + (3n/2) + (1/2), which differ by at most one.

Case (ii) If n is even , in theorem 3.1, the number of vertices labeled with '0' is mn+(n/2)+1 and labeled with '1' is mn+(n/2) and the number of edges labeled with '0' and '1' is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with '0' is mn + (n/2) + 1 + mn + n = 2mn + (3n/2) + 1 and with '1' is mn + (n/2) + mn + n = 2mn + (3n/2) which differ by at most one. Hence the human chain graph $HC_{n,m}$ admits total V-cordial labeling.

Example : 3 V-Cordial labeling of $HC_{3,4}$



Algorithm 3.2

Procedure (Homo-Cordial labeling of $HC_{n,m}$)

$V \leftarrow \{u_1,u_2,\ldots,u_{2n+1}, v_1,v_2,\ldots,v_{(m-1)n}, w_1,w_2,\ldots,w_{mn}\}$

$E \leftarrow \{ e_1,e_2,\ldots,e_{2mn+2n}\}$

if m≥3

for i= 1 to $\left\lfloor \frac{n+2}{2} \right\rfloor$ do

$u_{4i-3} \leftarrow 1$

end for

for i= 1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ do

$u_{4i-1} \leftarrow 0$

end for

for i= 1 to n do

$u_{2i} \leftarrow 0$

$w_{mi} \leftarrow 0$

$w_{mi-1} \leftarrow 1$

end for

for i= 1 to n do

for j= 1 to $\left\lfloor \frac{m-1}{2} \right\rfloor$ do

$w_{mi-m+2j-1} \leftarrow 0$

$w_{mi+2j-m} \leftarrow 1$

end for

end for

for i= 1 to n do

for j= 1 to $\left\lfloor \frac{m+1}{2} \right\rfloor$ do

$v_{(m-1)i-m+j+1} \leftarrow 1$

end for

end if

if m>3

for i= 1 to n do

for j= 1 to $\left\lfloor \frac{m-2}{2} \right\rfloor$ do

$v_{(m-1)i-j+1} \leftarrow 0$

end for

end for

end if

end procedure

Theorem 3.3 For m≥3 and n≥1, the Human chain graph admits Homo-cordial labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges. Using algorithm 3.2, the 2mn+n+1 vertices are labeled with {0,1}. Here if n is odd, the number of vertices labeled with '0' and '1' is mn+(n+1)/2 respectively and if n is even, the number of vertices labeled with '0' is mn+(n/2)+1 and the number of vertices labeled with '1' is mn+(n/2). Thus 2mn+n+1 vertices are labeled with 0 and 1 differ by at most 1.

The resulting edge labels are given in the following way:

If n≥1, for $1 \le i \le \left\lfloor \frac{n+1}{2} \right\rfloor$, $f(u_{4i-2}\, u_{4i-1})=0$, $f(u_{4i-3}\, u_{4i-2})=0$

for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f(u_{4i-1}\, u_{4i})=1$, $f(u_{4i}\, u_{4i+1})=0$

for $1 \le i \le n$, $f(u_{2i}\, v_{(m-1)(i-1)+1})=0$, $f(u_{2i}\, w_{m(i-1)+1})=1$

$f(w_{mi}\, w_{mi-2})=1$, m is odd

$f(w_{mi-1}\, w_{mi-2})=0$, m is odd

$f(w_{mi}\, w_{mi-2})=0$, m is even

© 2019, IJCSE All Rights Reserved

$f(w_{mi-1}\, w_{mi-2})=1$, m is even

for $0 \le i \le n-1$, $1 \le j \le \left\lfloor \frac{m-1}{2} \right\rfloor$, $f(v_{(m-1)i+j}\, v_{(m-1)i+j+1})=1$

If m=3, for $1 \le i \le n$, $f(u_{2i}\, v_{(m-1)i})=0$

If m>3 , for $0 \le i \le n-1$, $1 \le j \le m-3$, $f(w_{mi+j}\, w_{mi+j+1})=0$

for $1 \le i \le n$, f(u2i v(m-1)i)=1,

$f(v_{(m-1)i-\left\lfloor \frac{m}{2} \right\rfloor+1}\, v_{(m-1)i-\left\lfloor \frac{m}{2} \right\rfloor+2})=0$

If m>5, for $1 \le i \le n$, for $1 \le i \le \left\lfloor \frac{m-4}{2} \right\rfloor$, $f(v_{(m-1)i+j-1}\, v_{(m-1)i+j-2})=1$

Thus mn+n edges are labeled with ‘0’ and mn+n edges are labeled with ‘1’ . Thus 2mn+2n edges are labeled with ‘0’ and ‘1’ differ by at most one. Hence the human chain graph $HC_{n,m}$ admits Homo-cordial labeling.

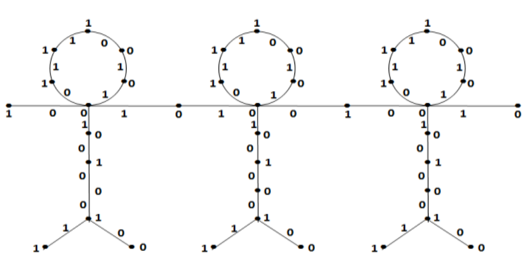
Theorem 3.4 For $m \ge 3$ and $n \ge 1$, the Human chain graph admits total Homo-cordial labeling.

Proof: Let $HC_{n,m}(p,q)$ be a human chain graph with p= 2mn+n+1 vertices and q= 2mn+2n edges.

Case (i): If n is odd, in theorem 3.3, the number of vertices labeled with ‘0’ and ‘1’ is mn+(n+1)/2 respectively and the number of edges labeled with ‘0’ and ‘1’ is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with ‘0’ and with ‘1’ is mn + (n+1)/2 + mn + n = 2mn + (3n/2) + (1/2) which differ by at most one.

Case (ii): If n is even , in theorem 3.2.1, the number of vertices labeled with ‘0’ is mn+(n/2)+1 and labeled with ‘1’ is mn+(n/2) and the number of edges labeled with ‘0’ and ‘1’ is mn+n respectively. From this we conclude that, the number of vertices and edges labeled with ‘0’ is mn + (n/2) + 1 + mn + n = 2mn + (3n/2) + 1 and with ‘1’ is mn + (n/2) + mn + n = 2mn + (3n/2) which differ by at most one. Hence the human chain graph $HC_{n,m}$ admits Homo-cordial labeling.

Example : 4 Homo-Cordial labeling of $HC_{6,3}$



IV. CONCLUSION

In this paper, we have constructed algorithms for labeling the vertices and edges and also proved the existence of V-cordial, total V-cordial, Homo-cordial and total Homo-cordial labeling for human chain graph.

REFERENCES

[1] K. Anitha, B. Selvam, “*Human chain graph*”, International journal of Engineering, Science and Mathematics, Vol. 7, Issue 8, 2018.

[2] Cahit, “*On cordial and 3-equitable labelings of graphs*”, Util. Math., Vol. 37, pp. 189–198, 1990.

[3] J.A. Gallian, “*A Dynamic Survey of graph labeling*”, the Electronic Journal of combinatories, 19, # DS6, 2012.

[4] A. Nellai Murugan, P. Iyadurai Selvaraj, “*Path Related Cup Cordial Graphs*”, Indian Journal of Applied Research. Vol. 4, Issue 8, pp. 433–436, 2011.

[5] A. Nellai Murugan, A. Madhubala, “*Path Related Homo-Cordial Graphs*”, International Journal of Innovative Science Engineering and Technology, Vol. 2, Issue 8, 2015.

[6] A. Nellai Murugan, A. Madhubala, “*Special case of Homo-Cordial Graphs*”, International Journal of Emerging Technologies in Engineering research, Vol. 2, Issue 3, pp. 1–5, 2015.

[7] Selvam, Avadayappan, M. Bhuvaneshwari, M. vasanthi, “*Homo Cordial Graphs*”, International Journal of Scientific Research, Vol. 5, Issue 5, pp. 700–724, 2016.