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Odd Harmonious Labeling of Subdivided Shell Graphs

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Abstract — A graph $G(p, q)$ is said to be odd harmonious if there exists an injection $f: V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ such that the induced function $f^*: E(G) \to \{1,3,\ldots,2q-1\}$ defined by $f^*(uv) = f(u) + f(v)$ is a bijection. In this paper we prove that the subdivided shell graph, disjoint union of two subdivided shell graph, subdivided shell flower graph and subdivided uniform shell bow graph are odd harmonious.

Keywords — disjoint union of graph, harmonious labeling, subdivided shell graph, odd harmonious labeling.

I. INTRODUCTION

Throughout this paper, by a graph we mean a finite, simple and undirected one. For standard terminology and notation we follow Harary [5]. Graham and Sloane [4] introduced harmonious labeling during their study of modular versions of additive bases problems stemming from error correcting codes. A graph G is said to be harmonious if there exist an injection $f: V(G) \to Z_q$ such that the induced function $f^*: E(G) \to Z_q$ defined by $f^*(uv) = (f(u) + f(v))(\text{mod } q)$ is a bijection and f is called harmonious labeling of G. The concept of odd harmonious labeling was due to Liang and Bai [6]. A graph G is said to be odd harmonious if there exists an injection $f: V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ such that the induced function $f^*: E(G) \to \{1, 3, \ldots, 2q-1\}$ defined by $f^*(uv) = f(u) + f(v)$ is a bijection. If $f(V(G)) = \{0, 1, 2, \ldots, q\}$, then f is called as strongly odd harmonious labeling and G is called as strongly odd harmonious graph. More results about odd harmonious labeling can be found in [6] - [13]. Deb and Limaye [1] have defined a shell graph as a cycle C_n with (n-3) chords sharing a common end point called the apex. Shell graphs are denoted as $C(n, n-3)$. A subdivided shell graph is a shell graph in which the edges in the path of the shell are subdivided. Jeba Jesintha et al. [3] defined disjoint union of two subdivided shell graphs. The same authors defined [2], a bow graph as a double shell in which each shell has any order. In the shell graph, when each edge in the path alone is subdivided, then it is a subdivided shell graph. A subdivided shell flower graph is one vertex union of three subdivided shells and each subdivided shell in this graph called as a petal..

II. MAIN RESULTS

In this section, we prove that the subdivided shell graph, disjoint union of two subdivided shell graph, subdivided shell flower graph and subdivided uniform shell bow graph are odd harmonious.

Theorem 2.1: The subdivided shell graph is odd harmonious.

Proof. Let G be a subdivided shell graph of any order. The apex vertex of G is denoted as u_0 and the remaining vertices in G from bottom to top are denoted as $u_1, u_2, \ldots, u_m$. Let $e_1, e_2, \ldots, e_{\frac{m+1}{2}}$ be the edges $u_0u_1, u_0u_3, u_0u_5, \ldots, u_0u_m$ respectively. Let $e_{\frac{m+3}{2}}, e_{\frac{m+5}{2}}, \ldots, e_{\frac{3m-1}{2}}$ be the edges $u_1u_2, u_2u_3, \ldots, u_{m-1}u_m$ respectively.

Then G has m+1 vertices and $\frac{3m-1}{2}$ edges.

We define labeling

$f: V(G) \to \left\{0,1,2,\ldots\ldots,2\left(\frac{3m-1}{2}\right)-1\right\}$ as follows:

$$f(u_0) = 0.$$
$$f(u_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$$
$$f(u_i) = 3m - 3i + 1, \quad 2 \le i \le m-1 \text{ and } i \text{ is even}.$$

The induced edge labels are

$$f^*(u_0u_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$$
$$f^*(u_iu_{i+1}) = 3m - 2i - 2, \quad 1 \le i \le m-2 \text{ and } i \text{ is odd}.$$
$$f^*(u_iu_{i-1}) = 3m - 2i + 4, \quad 3 \le i \le m \text{ and } i \text{ is odd}.$$

That is, the induced edge labels are {1,3,5,…,m}∪{3m-4,3m-8,…,m+6,m+2}∪{3m-2,3m-6,…,m+8,m+4}= {1,3,..,m,m+2, m+4,…,3m-8, 3m-6, 3m-4, 3m-2}
={1,3,…,2q-1}
and the edge labels are also distinct. Therefore the subdivided shell graph is odd harmonious.

An odd harmonious labeling of a subdivided shell graph with m=11 is shown in Figure 1.

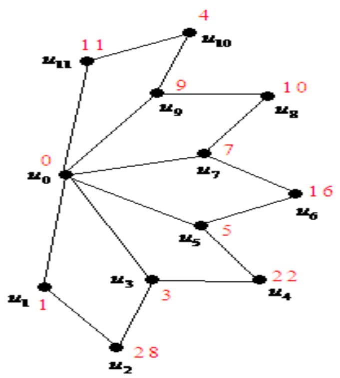

Figure 1: An odd harmonious labeling of a subdivided shell graph with m=11

Theorem 2.2: The disjoint union of two subdivided shell graphs is odd harmonious.

Proof. Let G_1 and G_2 be two subdivided shell graphs of any order. Let G be the disjoint union of G_1 and G_2. The apex of G_1 is denoted as u_0 and the remaining vertices in G_1 from bottom to top are denoted as $u_1, u_2, ..., u_m$. The apex of G_2 is denoted as v_0 and the other vertices from bottom to top are denoted as $v_1, v_2, ..., v_l$. Let $e_1, e_2, ..., e_{\frac{m+1}{2}}$ be the edges $u_0u_1, u_0u_3, u_0u_5, ..., u_0u_m$, $e_{\frac{m+3}{2}}, e_{\frac{m+5}{2}}, ..., e_{\frac{3m-1}{2}}$ be the edges $u_1u_2, u_2u_3, ..., u_{m-1}u_m$, $e_{\frac{3m+1}{2}}, e_{\frac{3m+3}{2}}, ..., e_{\frac{3m+l}{2}}$ be the edges $v_0v_1, v_0v_3, v_0v_5, ..., v_0v_l$ and $e_{\frac{l+3m+2}{2}}, e_{\frac{l+3m+4}{2}}, ..., e_{\frac{3l+3m-2}{2}}$ be the edges $v_1v_2, v_2v_3, ..., v_{l-1}v_l$ respectively.

Then G has $p = m + l + 2$ vertices and $q = \dfrac{3m+3l-2}{2}$ edges.

We define labeling $f : V(G) \to \left\{0,1,2,\ldots\ldots\ldots,2\left(\dfrac{3m+3l-2}{2}\right)-1\right\}$ as follows:

$f(u_0) = 0.$

$f(u_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f(u_i) = 3m - 3i + 1, \quad 2 \le i \le m-1 \text{ and } i \text{ is even}.$

$f(v_0) = 2.$

$f(v_i) = 3(m-1) + i, \quad 1 \le i \le l \text{ and } i \text{ is odd}.$

$f(v_i) = 3l - 3i + 3, \quad 2 \le i \le l-1 \text{ and } i \text{ is even}.$

The induced edge labels are

$f^*(u_0u_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(u_iu_{i+1}) = 3m - 2i - 2, \quad 1 \le i \le m-2 \text{ and } i \text{ is odd}.$

$f^*(u_iu_{i-1}) = 3m - 2i + 4, \quad 3 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(v_0v_i) = 3m + i - 1, \quad 1 \le i \le l \text{ and } i \text{ is odd}.$

$f^*(v_iv_{i+1}) = 3m + 3l - 2i - 3, \quad 1 \le i \le l-2 \text{ and } i \text{ is odd}.$

$f^*(v_iv_{i-1}) = 3m + 3l - 2i + 3, \quad 3 \le i \le l \text{ and } i \text{ is odd}.$

That is, the induced edge labels are {1,3,…,m} ∪{3m-4,3m-8,…,m+6,m+2} ∪{3m-2,3m-6,…,m+8,m+4} ∪ {3m, 3m+2,…,3m+*l*-3,3m+*l*-1} ∪ {3m+3*l*-5, 3m+3*l*-9,…, 3m+*l*+5,3m+*l*+1} ∪{3m+3*l*-3,3m+3*l*-7,…, 3m+*l*+7, 3m+*l*+3}
= {1,3,…,m,m+2,m+4,…,3m-6,3m-4,3m-2, 3m, 3m+2, …, 3m+*l*-1, 3m+*l*+1, 3m+*l*+3,…,3m+3*l*-7, 3m+3*l*-5, 3m+3*l*-3}
={1,3,…,2q-1}and hence the edge labels are distinct.
Therefore, the disjoint union of two subdivided shell graphs is odd harmonious.

An odd harmonious labeling of the disjoint union of two subdivided shell graphs with m=11 and l=7 are shown in Figure 2.

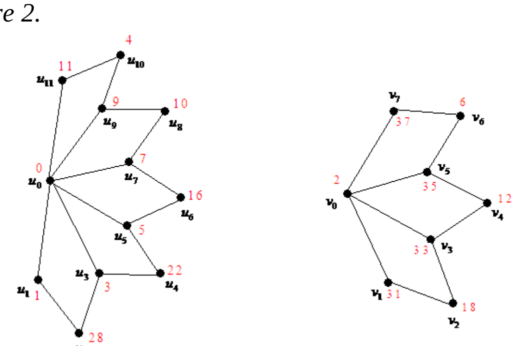

Figure 2: An odd harmonious labeling of disjoint union of two subdivided shell graphs with m=11 and l=7

Theorem 2.3: The subdivided uniform shell bow graph is odd harmonious.

Proof. Let G be a subdivided uniform shell bow graph with p vertices and q edges. Denote the apex of G as v_0. Let m be the number of vertices in each path. Denote the vertices in the path of the right shell of G from bottom to top as

$v_1, v_2, \dots, v_m$. The vertices in the path of the left shell are denoted from top to bottom as $v_{m+1}, v_{m+2}, \dots, v_{2m-1}, v_{2m}$. Then G has $p = 2m+1$ vertices and $q = 3m-1$ edges.

We define labeling

$f : V(G) \rightarrow \{0,1,2,\dots\dots\dots,2(3m-1)-1\}$ as follows:

$f(v_0) = 0.$

$f(v_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f(v_i) = i+1 \quad , m+1 \le i \le 2m \text{ and } i \text{ is even}.$

$f(v_i) = 6m - 3i \quad , \quad 2 \le i \le m-1 \text{ and } i \text{ is even}.$

$f(v_i) = 6m - 3i + 1 \quad , m+2 \le i \le 2m-1 \text{ and } i \text{ is odd}.$

The induced edge labels are

$f^*(v_0 v_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(v_0 v_i) = i+1, \quad m+1 \le i \le 2m \text{ and } i \text{ is even}.$

$f^*(v_i v_{i+1}) = 6m - 2i - 3, \quad ,1 \le i \le m-2 \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i-1}) = 6m - 2i + 3, \quad 3 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i+1}) = 6m - 2i - 1, \quad ,m+1 \le i \le 2m-2 \text{ and } i \text{ is even}.$

$f^*(v_i v_{i-1}) = 6m - 2i + 5, \quad m+3 \le i \le 2m \text{ and } i \text{ is even}.$

That is, the induced edge labels are {1,3,…,m-2,m} ∪{ m+2, m+4,…,2m-1,2m+1} ∪ {6m-5, 6m-9,…,4m+5, 4m+1}∪{6m-3,6m-7,…,4m+7,4m+3}∪{4m-3,4m-7,…, 2m+7,2m+3}∪{4m-1,4m-5,…,2m+9,2m+5}
={1,3,…,m,m+2,…,2m+1, 2m+3,…,4m-3, 4m-1, 4m+1, 4m+3,…,6m-5, 6m-3}
={1,3,…,2q-1} and also the edge labels are distinct.
Therefore, the subdivided uniform shell bow graph is odd harmonious.

An odd harmonious labeling of subdivided uniform shell bow graph with m=9 is shown in Figure 3.

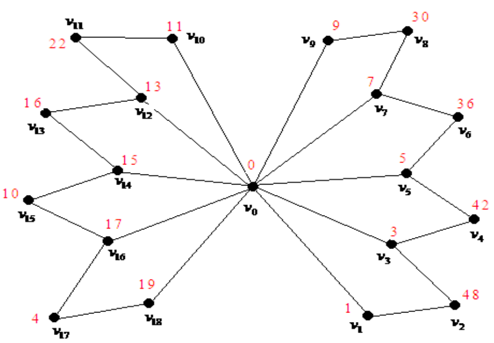

Figure 3: An odd harmonious labeling of subdivided uniform shell bow graph with m=9

Theorem 2.4: All subdivided shell flower graphs are odd harmonious.

Proof. Let G be a subdivided shell flower graph with three petals. We describe G as follows: G is one vertex union of three subdivided shells of same order. The apex is denoted as v_0. Denote the vertices in the first petal of G as $v_1, v_2, \dots, v_m$. The vertices in the second petal of G are denoted as $v_{m+1}, v_{m+2}, \dots, v_{2m}$. The vertices in the third petal of G are denoted as $v_{2m+1}, v_{2m+2}, \dots, v_{3m}$. Let

$e_1, e_2, \dots, e_{\frac{m+1}{2}}$ be the edges $v_0v_1, v_0v_3, v_0v_5, \dots, v_0v_m$,

$e_{\frac{m+3}{2}}, e_{\frac{m+5}{2}}, \dots, e_{m+1}$ be the edges $v_0v_{m+1}, v_0v_{m+3}, \dots, v_0v_{2m}$,

$e_{m+2}, e_{m+3}, \dots, e_{\frac{3m+3}{2}}$ be the edges $v_0v_{2m+1}, v_0v_{2m+3}, v_0v_{2m+5}, \dots, v_0v_{3m}$,

$e_{\frac{3m+5}{2}}, e_{\frac{3m+7}{2}}, \dots, e_{\frac{5m+1}{2}}$ be the edges $v_1v_2, v_2v_3, \dots, v_{m-1}v_m$,

$e_{\frac{5m+3}{2}}, e_{\frac{5m+5}{2}}, \dots, e_{\frac{7m-1}{2}}$ be the edges $v_{m+1}v_{m+2}, v_{m+2}v_{m+3}, \dots, v_{2m-1}v_{2m}$ and

$e_{\frac{7m+1}{2}}, e_{\frac{7m+3}{2}}, \dots, e_{\frac{9m-3}{2}}$ be the edges $v_{2m+1}v_{2m+2}, v_{2m+2}v_{2m+3}, \dots, v_{3m-1}v_{3m}$ respectively.

Then G has $p = 3m+1$ vertices and $q = \frac{9m-3}{2}$ edges.

We define labeling

$f : V(G) \rightarrow \left\{0,1,2,\dots\dots\dots,2\left(\frac{9m-3}{2}\right)-1\right\}$ as follows:

$f(v_0) = 0.$

$f(v_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f(v_i) = i+1 \quad , m+1 \le i \le 2m \text{ and } i \text{ is even}.$

$f(v_i) = i+2 \quad , \quad 2m+1 \le i \le 3m \text{ and } i \text{ is odd}.$

$f(v_i) = 9m - 3i - 1, \quad 2 \le i \le m-1 \text{ and } i \text{ is even}.$

$f(v_i) = 9m - 3i \quad , m+2 \le i \le 2m-1 \text{ and } i \text{ is odd}.$

$f(v_i) = 9m - 3i + 1, \quad 2m+2 \le i \le 3m-1 \text{ and } i \text{ is even}.$

The induced edge labels are

$f^*(v_0 v_i) = i, \quad 1 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(v_0 v_i) = i+1, \quad m+1 \le i \le 2m \text{ and } i \text{ is even}.$

$f^*(v_0 v_i) = i+2, \quad 2m+1 \le i \le 3m \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i+1}) = 9m - 2i - 4, \quad ,1 \le i \le m-2 \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i-1}) = 9m - 2i + 2, \quad 3 \le i \le m \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i+1}) = 9m - 2i - 2, \quad ,m+1 \le i \le 2m-2 \text{ and } i \text{ is even}.$

$f^*(v_i v_{i-1}) = 9m - 2i + 4, \quad m+3 \le i \le 2m \text{ and } i \text{ is even}.$

$f^*(v_i v_{i+1}) = 9m - 2i, \quad ,2m+1 \le i \le 3m-2 \text{ and } i \text{ is odd}.$

$f^*(v_i v_{i-1}) = 9m - 2i + 6, \quad 2m+3 \le i \le 3m \text{ and } i \text{ is odd}$

That is, the induced edge labels are {1,3,…,m}∪{m+2,m+4,…,2m+1}∪ {2m+3,…,3m,3m+2} ∪ {9m-6, 9m-10,…,7m+4,7m} ∪{ 9m-4,…,7m+2} ∪ {7m-4,7m-8,…,5m+2} ∪ {7m-2,…, 5m+4} ∪{ 5m-2,5m-6,…, 3m+4} ∪ {5m, 5m-4,…,3m+6}
={1,3,…,m,m+2,…,2m+1,2m+3,…,3m+2,3m+4,…, 5m-4,5m-2,5m,5m+2,…, 7m-4, 7m-2, 7m, 7m+2,…,9m-6, 9m-4} ={1,3,…,9m-4}
= {1,3,…,2q-1}and also the edge labels are distinct.

Therefore, the subdivided shell flower graphs are odd harmonious.

An odd harmonious labeling of the subdivided shell flower graph with m=7 is shown in Figure 4.

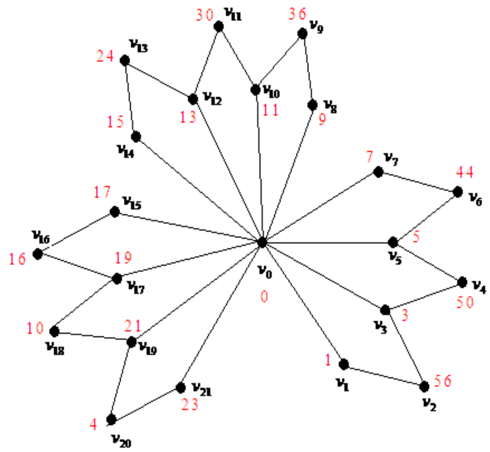

Figure 4: An odd harmonious labeling of subdivided shell flower graph with m=7

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