IJCSE International Journal of Computer Sciences and Engineering **Open Access**

Research Paper **Vol.-7, Special Issue, 5, March 2019** **E-ISSN: 2347-2693**

The Nonsplit Bondage Number of Graphs

R. Jemimal Chrislight[1*], Y. Therese Sunitha Mary [2]

[1] Dept. of Mathematics, St. Xavier's College (Autonomous), Palayamkottai - 627 002, Tamil Nadu, INDIA
[2]Dept. of Mathematics, St. Xavier's College (Autonomous), Palayamkottai - 627 002, Tamil Nadu, INDIA
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627 012, Tamil Nadu, INDIA

Corresponding Author: jemimachrislight@gmail.com, Tel.: +91-99946-35396

DOI: https://doi.org/10.26438/ijcse/v7si5.7476 | Available online at: www.ijcseonline.org

Abstract— A set D of vertices in a graph $G = (V, E)$ is a nonsplit dominating set if the induced subgraph $< V - D >$ is connected. The minimum cardinality of a nonsplit dominating set is called the nonsplit domination number of G and denoted $\gamma_{ns}(G)$. In this paper, we define the nonsplit bondage number $b_{ns}(G)$ of a graph G to be the minimum cardinality of a set E of edges for which $\gamma_{ns}(G - E) > \gamma_{ns}(G)$. We obtain sharp bounds for $b_{ns}(G)$ and obtain the exact values for some standard graphs.

Keywords— *Nonsplit dominating set, Nonsplit domination number, Bondage number, Nonsplit bondage number.*

I. INTRODUCTION

In this paper, the graphs $G = (V, E)$ considered here are finite and undirected without loop or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Harary [2] and for domination we refer Haynes et al. [3].

For any vertex $p \in V$, the open neighbourhood of p, denoted by $N(p)$, is the set of vertices adjacent to p and the closed neighbourhood of p is $N[p] = N(p) \cup p$. A set D subset of V is a dominating set of G if every vertex $p \in V$ is either an element of D or is adjacent to an element of D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. Kulli V. R. et al. [4] introduced the concept of nonsplit domination in graphs. A dominating set D of a graph G is a nonsplit dominating set if the induced graph $\langle V - D \rangle$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ is the minimum cardinality of a nonsplit dominating set.

In 1990, J. F. Fink et al. [1] introduced the notion of bondage number of a graph. The bondage number $b(G)$ of a nonempty graph G is the minimum cardinality among all sets of edges E for which $\gamma(G - E) > \gamma(G)$.
The purpose of this paper is to introduce the concept of nonsplit bondage number $b_{ns}(G)$ of a graph G. The nonsplit bondage number $b_{ns}(G)$ of a graph G be the minimum cardinality of a set E of edges for which $\gamma_{ns}(G - E) > \gamma_{ns}(G)$. In this paper, we obtain the exact values of the nonsplit bondage number for some standard graphs.

We need the following theorem in [4].

Theorem 1.1 For any complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$, $\gamma_{ns}(K_{m,n}) = 2$.

2. Main Results

Theorem 2.1. For any complete graph with $p \geq 2$,

$$b_{ns}(G) = \begin{cases} \left\lfloor \frac{p}{2} \right\rfloor & if\ p \leq 3 \\ \left\lceil \frac{p}{2} \right\rceil & otherwise \end{cases}.$$

Proof. If G is K_2, Clearly $b_{ns}(G) = \left\lfloor \frac{p}{2} \right\rfloor = 1$.
Let H is a spanning subgraph of K_p. If $p = 3$, then H is obtained by removing $\left\lfloor \frac{p}{2} \right\rfloor$ edges of K_3 which increase the nonsplit domination number. Thus, $b_{ns}(K_3) = \left\lfloor \frac{p}{2} \right\rfloor$.
If $p \geq 4$, then H is obtained by removing less than $\left\lceil \frac{p}{2} \right\rceil$ edges from K_p and so H contains a vertex of degree $p - 1$, whence the nonsplit dominating set of H is not increasing. Thus $b_{ns}(K_p) \geq \left\lceil \frac{p}{2} \right\rceil$.
Suppose p is even, the removing $\frac{p}{2}$ independent edges from K_p decrease the degree of each vertex to $p - 2$ and therefore gives a connected graph H with the nonsplit domination number $\gamma_{ns}(H) = 2$.
Let v be a vertex of K_p and suppose p is odd, then removing $\frac{p-1}{2}$ independent edges from K_p leaves a graph having exactly one vertex of degree $p - 1$ say v by eliminating one

edge incident with v. So H is a connected graph with $\gamma_{ns}(H) = 2$.

In both cases, we obtain the graph H after the removal of $\left\lceil\frac{p}{2}\right\rceil$ edges from K_p and so $b_{ns}(K_p) \le \left\lceil\frac{p}{2}\right\rceil$. Thus, $b_{ns}(K_p) = \left\lceil\frac{p}{2}\right\rceil$.

Theorem 2.2. For any complete bipartite graph $K_{m,n}$ with $2 \le m \le n$, then $b_{ns}(G) = m$.

Proof. Let H be the spanning subgraph of $K_{m,n}$. Suppose the graph H is obtained by removing m edges of $K_{m,n}$ and we get two components of H namely $K_{m,n-1}$ and a isolated vertex. Clearly $\gamma_{ns}(K_{m,n-1}) = 2$ by theorem 1.1. So $\gamma_{ns}(H) = 3$. Since, $\gamma_{ns}(G) < \gamma_{ns}(H)$. Thus, $b_{ns}(G) = m$.

Theorem 2.3. For any wheel graph W_p, $b_{ns}(W_p) = 1$.

Proof. Let v be the center vertex of Wheel graph. Let u be any adjacent vertex of v such that $\deg(v) = \Delta(G)$. The removal of any edge uv from W_p which increase the nonsplit domination number of H. Thus, $b_{ns}(G) = 1$.

Theorem 2.4. If T is a tree which is not a star with $p \ge 4$, then $b_{ns}(T) = 1$.

Proof. Since every edge of tree is a bridge and hence $b_{ns}(T) \ge 1$. Suppose that T has any two adjacent vertices, say x and y.

Case 1. If $diam(T)$ is odd and x and y be the center. The graph H is the subgraph of T that is obtained by removing the edge xy from T. Then the graph H has two components and the nonsplit domination number must be increase. Thus, $b_{ns}(T) \le 1$.

Case 2. If $diam(T)$ is even and x is the center. If H is obtained by removing one edge adjacent to a center x which is not a pendent edge namely xy_1 or xy_2. Then the graph divided into two components and so $\gamma_{ns}(H) > \gamma_{ns}(T)$. Thus, $b_{ns}(T) \le 1$. Hence, $b_{ns}(T) = 1$.

Remark 2.5. $b_{ns}(G)$ is not defined if G is isomorphic to galaxy.

Theorem 2.6. For any helm graph H_p, then $b_{ns}(H_p) = 3$.

Proof. Let $X = \{u_1, u_2, u_3, \dots, u_p\}$ be *the* end vertices of H_p and $Y = \{v_1, v_2, v_3, \dots, v_p\}$ be the adjacent vertices or rim vertices of H_p. Let K_1 is a center of H_p. The graph H is the subgraph of H_p that is obtained by removing lessthan three edges of H_p whose nonsplit domination number is not increasing. Thus, $b_{ns}(H_p) \ge 3$.

Let $v_i \in Y$ $(1 \le i \le p)$ be the rim vertices of H_p and degree of every rim vertex of H_p is four. H is the subgraph obtained by removing atmost three edges from any one rim vertex of H_p which is not a pendent edge and it increase the nonsplit domination number of H. Thus, $b_{ns}(H_p) \le 3$. Hence, $b_{ns}(H_p) = 3$.

Theorem 2.7. For any $\overline{C_P}$ with $p \ge 4$, then $b_{ns}(\overline{C_p}) = \begin{cases} 2 & if\ p \le 5 \\ p-4 & otherwise \end{cases}$.

Proof. Let $V = \{v_1, v_2, v_3, \dots, v_p\}$ be the vertices of C_p and H be the subgraph of $\overline{C_p}$. If $p = 4 = 5$, then the nonsplit domination number of $\overline{C_p}$ is 3. The graph H is obtained by removing two edges in $\overline{C_4}$ or $\overline{C_5}$. Then $\gamma(H)$ increase. Thus, $b_{ns}(\overline{C_p}) = 2$.

If $p \ge 6$, then the graph H is obtained by removing at least $p-4$ edges in $\overline{C_p}$. Thus $b_{ns}(\overline{C_p}) \ge p-4$.

Let the graph $(\overline{C_p})$ is a $p-3$ regular graph and so the nonsplit domination number of $\overline{C_p}$ is 2. Suppose $b_{ns}(\overline{C_p}) < p-4$, the graph H is obtained by removing atmost $p-3$ edges in $\overline{C_p}$. Since $\gamma(\overline{C_p}) = \gamma(H)$, which is impossible. Now, let v_i be any vertex of $\overline{C_p}$. If the removal of $p-4$ edges of $\overline{C_p}$ in H such that each $p-4$ edges are incident with v_i where $i = 1,2,3,\dots,p$ in $\overline{C_p}$ which increase the nonsplit domination number. Thus $b_{ns}(\overline{C_p}) \le p-4$.

Theorem 2.8. For any $\overline{P_P}$ with $p \ge 3$, then $b_{ns}(\overline{P_p}) = \begin{cases} 1 & if\ p \le 4 \\ p-3 & otherwise \end{cases}$.

Proof. It follows from theorem 2.7.

Theorem 2.9. For any $\overline{K_{m,n}} \ncong \overline{K_{2,2}}$ with $1 \le m \le n$, then

$$b_{ns}(\overline{K_{m,n}}) = \begin{cases} \left\lfloor\frac{n}{2}\right\rfloor & if\ m \le 2\ and\ 2 \le n \le 3 \\ \left\lceil\frac{n}{2}\right\rceil & if\ 3 \le m \le n \end{cases}.$$

Proof. Let $V_1 = \{u_1, u_2, \dots, u_m\}$ and $V_2 = \{v_1, v_2, \dots, v_n\}$ be two partitions of $K_{m,n}$ and $\overline{K_{m,n}}$ has two components, say V_1 and V_2. let H be the subgraph of $\overline{K_{m,n}}$.

If $m \le 2$ and $2 \le n \le 3$, then the removal of one edge of V_2 in H which increase the nonsplit domination number. Thus $b_{ns}(\overline{K_{m,n}}) = \left\lfloor\frac{n}{2}\right\rfloor = 1$ with $m \le 2$ and $2 \le n \le 3$.

Case 1. If $n = m = 3$, then $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2, v_3\}$. The nonsplit dominating set D_1 of $\overline{K_{3,3}}$ is $\{u_1, u_2, u_3, v_1\}$. Now, the graph H is obtained by removing at least two edges of $\overline{K_{3,3}}$. Then the set D_2 of H is $\{u_1, u_2, u_3, v_1, v_2\}$. Since $\gamma_{ns}(H) > \gamma_{ns}(\overline{K_{3,3}})$, $b_{ns}(\overline{K_{3,3}}) = \left\lceil\frac{n}{2}\right\rceil = 2$.

Case 2. If $3 \le m < n$, then the two components V_1 and V_2 are complete in $\overline{K_{m,n}}$ and by theorem 2.1, $b_{ns}(\overline{K_{m,n}}) = \left\lceil\frac{n}{2}\right\rceil$.

Proposition 2.10. For any corona graph $(G.K_1)$ with $p \ge 2$, then $b_{ns}(G.K_1) = 1$.

Proof. We find the bondage number to remove any edge which incident with end vertex.

Proposition 2.11. For any friendship graph F_p, then $b_{ns}(F_p) = 1$.
Proof. We find the bondage number to remove any rim edge of F_p.

Proposition 2.12. For any fan graph f_p, then $b_{ns}(f_p) = 1$.
Proof. We find the bondage number to remove any edge which incident with K_1.

Proposition 2.13 For any Book graph B_p with $p \geq 2$, then $b_{ns}(B_p) = 1$.
Proof. We find the bondage number to remove any edge of B_p, then the nonsplit domination number increase.

REFERENCES

[1] J. F. Fink, M. S. Jacobson, L. F. Kinch and J. Roberts, The Bondage number of a graph, Discrete Math. 86(1990) 47-57.
[2] F. Harary, Graph Theory, Addision-Wesley, Reading Mass. (1969).
[3] L. Hartnell, Douglas F. Rall, Bounds on the bondage number of a graph, Discrete Mathematics 128 (1994) 173-177.
[4] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Disc.Math., Marcel Dekker, Inc., New York, 1998.
[5] V. R. Kulli and B. Janakiram, The Nonsplit Domination Number of a Graph, Indian J.Pure appl. Math., 31(4):441-447, April 2000.

