

## **Manpower Levels for Business with Various Recruitment Rates in the Ten Point State Space System through Stochastic Models**

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**Abstract**— Aim of the present study is to find the steady rate of crisis and steady state of probabilities with different situations which may be manpower, in irregular situations of complete availability, moderate availability and zero availability inside the case of manpower, business and recruitment. The various states have been discussed under the different assumptions that the transition from one state to another both business and manpower arise in exponential time with different parameters.

**Keywords**- Steady state, Crisis rate, Markov chain.

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### **I. INTRODUCTION**

At the present time we establish that labor has become a buyers' market as well as sellers' market. Any business which usually runs on profitable base wishes to keep only the optimum level of all resources needed to meet company's responsibility, at any time during the course of the business and so manpower is not an exemption. This is spelt in the sense that a company may not want to keep manpower more than what is needed. Thus, retrenchment and recruitment are general and recurrent in most of the companies now. Recruitment is done when the business is busy and lean-to manpower. Equally true with the labor, has the choice to switch over to other jobs because of improved working condition, better emolument, nearness to their living place or other reasons. Under such conditions the company may face crisis because business may be there but skilled manpower may not be available. If skilled laborers and technically qualified persons leave the business the seriousness is most awfully felt and the company has to appoint paying deep price or pay overtime to employees.

Manpower problems have been dealt in many different ways as early as 1947 by Vajda [11] and others. Markov model are designed for promotion and wastages in manpower system by Vassilou [12] and Subramaniam [10]. Manpower planning models have been dealt in depth in Grinold & Marshal [3], Barthlomew [1] and Vajda[11]. The methods to calculate wastages (Dismissal, resignation and death) and promotion intensities which make the proportions corresponding to some selected planning proposals have been dealt by Lesson [4]. For n unit standby system may refer to Ramanarayanan and Usha [8]. A two unit stand by system has been studied

by Chandrasekar and Natrajan [2] with confidence limits under steady state. For an application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [9]. The study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5]. Stochastic Analysis of a Business with Varying Levels in Manpower and Business has been studied by C. Mohan and P.Selvaraju [7]. For three characteristics system involving machine, manpower and money one may refer to C. Mohan and R. Ramanarayanan [6]. Stochastic analysis of manpower levels affecting business with varying recruitment rates by K. Harikumar et.al [13]

### **II. MARKOV CHAIN MODEL WITH VARIOUS STATES**

In this paper we consider three characteristics that is manpower, business and recruitment. The situations may be that the manpower may be hardly available fully available or business may fluctuate between complete availability to nil availability and the recruitment is full from the nil level. It goes off when the manpower becomes nil. This is so because the experts may take the business along with them or those who have brought good will to the concern may bring the client's off the concern. The business depends on steady state probabilities and the availability of manpower. The steady state probabilities of the continuous Markov chain connecting the transitions probabilities with various states are considered for presenting the cost analysis. Numerical examples are also provided.

III. ASSUMPTIONS

1. There are two levels of manpower namely manpower is nil and manpower is full.
2. There are two levels of business namely,
  - (i) Business is full and exponentially distributed with the parameter ‘μ’
  - (ii) Nil level of business with the parameter ‘λ’
3. When the manpower is full and business is full it is denoted by λ<sub>111</sub>. It changes to λ<sub>121</sub>. The parameter of the distribution is λ<sub>201</sub> when the manpower is full and business is nil and the same parameter λ<sub>201</sub> changes to λ<sub>212</sub> with exponential time β<sub>201</sub>.
4. The recruitment distribution with parameter is μ<sub>001</sub> when the manpower is zero the business is vanished. It changes to μ<sub>002</sub> with exponential time and the parameter α<sub>002</sub>.
5. Although manpower becomes zero, the business is vanished and becomes nil.

IV. SYSTEM ANALYSIS

The stochastic process X(t) describing the system is a continuous time Markov chain with ten points state space as given below in the order of manpower, business and recruitment

$$S = \{(0 0 1), (0 0 2), (1 0 0), (1 0 1), (1 0 2), (1 1 0), (1 1 1), (1 2 1), (2 0 1), (2 1 2)\} \text{ ---- (1)}$$

Where, the first co-ordinate refers to shortage / non availability of manpower. Second co-ordinate refers to the business and the third co-ordinate indicates recruitment level. The continuous time markov chain of the state space is given below which is a matrix of order ten.

$$Q = \begin{matrix} M/B/R & (001) & (002) & (100) & (101) & (102) & (110) & (111) & (121) & (201) & (212) \\ \begin{matrix} (001) \\ (002) \\ (100) \\ (101) \\ (102) \\ (110) \\ (111) \\ (121) \\ (201) \\ (212) \end{matrix} & \begin{matrix} \eta_1 & \alpha & \beta_{001} & 0 & 0 & b & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_2 & \beta_{002} & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{100} & 0 & \eta_3 & \beta_{101} & b & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{101} & \mu & 0 & \eta_4 & 0 & \beta_{110} & 0 & 0 & 0 & 0 & 0 \\ \alpha_{102} & 0 & a & 0 & \eta_5 & \beta_{110} & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{110} & 0 & \eta_6 & 0 & b & 0 & 0 & 0 \\ \alpha_{111} & 0 & 0 & 0 & a & 0 & \eta_7 & \beta_{121} & 0 & 0 & 0 \\ \alpha_{121} & 0 & 0 & 0 & 0 & a & 0 & \eta_8 & \beta_{201} & 0 & 0 \\ \alpha_{201} & 0 & 0 & 0 & \mu & 0 & 0 & 0 & \eta_9 & \lambda & 0 \\ \alpha_{202} & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & \eta_{10} \end{matrix} \end{matrix} \text{ ----(2)}$$

Where,

$$\left. \begin{matrix} \eta_1 = -(\beta_{001} + \alpha + b) ; \eta_2 = -(\beta_{002} + \lambda) \\ \eta_3 = -(\alpha_{100} + \beta_{101} + b) ; \eta_4 = -(\alpha_{101} + \beta_{110} + \mu) \\ \eta_5 = -(\alpha_{102} + a + b + \beta_{110}) ; \eta_6 = -(\alpha_{110} + b) \\ \eta_7 = -(\alpha_{111} + a + \beta_{121}) ; \eta_8 = -(\alpha_{121} + a + \beta_{201}) \\ \eta_9 = -(\alpha_{201} + \mu + \lambda) ; \eta_{10} = -(\alpha_{202} + a) \end{matrix} \right\}^3$$

The events of transition in manpower, business and recruitment are independent, after that the individual infinitesimal generator of them are given by

Let  $\pi = [\pi_{001} \pi_{002} \pi_{100} \pi_{101} \pi_{102} \pi_{110} \pi_{111} \pi_{121} \pi_{201} \pi_{212}]$  be the steady state probability vector of the Q matrix, then

$$\pi Q = 0, \quad \pi e = 1 \quad \text{----- (4)}$$

From (4), we get the steady state probabilities:

$$\begin{matrix} \pi_{001} = \frac{d_0 \lambda}{z \sum_{i=0}^2 d_i} ; & \pi_{002} = \frac{d_1 \lambda}{z \sum_{i=0}^2 d_i} ; \\ \pi_{100} = \frac{d_2 \mu}{z \sum_{i=0}^2 d_i} & \pi_{101} = \frac{d_0 \mu}{z \sum_{i=0}^2 d_i} \\ \pi_{102} = \frac{d_1 \beta_{102}}{z \sum_{i=0}^2 d_i} ; & \pi_{110} = \frac{d_2 \beta_{110}}{z \sum_{i=0}^2 d_i} \\ \pi_{111} = \frac{d_0 \beta_{201}}{z \sum_{i=0}^2 d_i} ; & \pi_{121} = \frac{d_1 \beta_{201}}{z \sum_{i=0}^2 d_i} \quad \text{----- (5)} \\ \pi_{201} = \frac{d_2 \beta_{201}}{z \sum_{i=0}^2 d_i} ; & \pi_{212} = \frac{d_2 \beta_{002}}{z \sum_{i=0}^2 d_i} \end{matrix}$$

Where,

$$\begin{matrix} d_0 = \alpha_{100} \alpha_{201} + \alpha_{101} \alpha_{212} + \alpha_{201} \beta_{101} + \alpha_{111} \beta_{201} ; \\ d_1 = \alpha_{110} \alpha_{201} + \alpha_{101} \alpha_{212} + \alpha_{201} \beta_{002} ; \\ d_2 = \alpha_{100} \alpha_{212} + \alpha_{001} \alpha_{201} + \alpha_{201} \beta_{101} ; \\ Z = [a + b] \text{ and } \sum_{i=0}^2 d_i = [d_0 + d_1 + d_2] \end{matrix}$$

While manpower is available business is full or nil. Manpower is inadequate or nil will lead to crisis state.

In this system the crisis states are {(1 1 1), (1 2 1), (2 0 1), (2 1 2)} and the crisis arise when there is nil business or moderate business and the manpower is moderate or full also the recruitment is moderate or full. Now the rate of crisis in the steady state is given by

$$C_{\infty} = \alpha_{111}\pi_{111} + \alpha_{121}\pi_{121} + \alpha_{201}\pi_{201} + \alpha_{212}\pi_{212} \text{ ---- (6)}$$

Using steady state probabilities, we obtain the rate of crisis

$$C_{\infty} = \frac{\mu\lambda}{Z \sum_{i=0}^2 d_i} \left[ (\alpha_{111}d_0\beta_{201} + \alpha_{201}d_2\beta_{002} + d_1\alpha_{121}\beta_{101} + d_1\alpha_{212}\beta_{002}) \right] \text{ ---- (7)}$$

### V. NUMERICAL ILLUSTRATION AND STEADY STATE COST CALCULATION

#### Ten point state space

##### Case (i)

The steady state probabilities and the rate of crises are measured by using the formulas (6) and (7) respectively. Taking,  $a = 10, b = 9, \lambda = 3, \mu = 5, \beta_{201} = 8, \alpha_{001} = 12, \alpha_{110} = 8, \alpha_{100} = 5, \beta_{002} = 4, \beta_{101} = 12, \alpha_{101} = 11, \alpha_{111} = 3, \alpha_{201} = 9, \alpha_{121} = 2, \alpha_{212} = 6, \beta_{102} = 4, \beta_{110} = 8.$

We get  $\pi_{001} = 0.05912, \pi_{002} = 0.0298,$

$$\pi_{100} = 0.09432, \pi_{101} = 0.09853, \pi_{102} = 0.05575,$$

$$\pi_{110} = 0.151886. \pi_{111} = 0.157651,$$

$$\pi_{121} = 0.111511, \pi_{201} = 0.151886,$$

$$\pi_{212} = 0.075943$$

The crisis rate is 21.67206

##### Case (ii)

If we assume that the value of a and b,  $a = 8, b = 5, \lambda = 6, \mu = 3, \beta_{101} = 4, \beta_{102} = 7, \beta_{201} = 10, \beta_{202} = 11, \alpha_{001} = 7, \alpha_{112} = 5, \alpha_{201} = 9, \alpha_{202} = 4.$

Table: 1 Relationship among a, b and  $C_{\infty}$

<b>a</b>	5	7	14	20	22	28
<b>b</b>	6	8	17	22	26	35
$C_{\infty}$	47.297	34.684	16.783	12.387	10.839	8.2583
	63	93	03	48	04	17

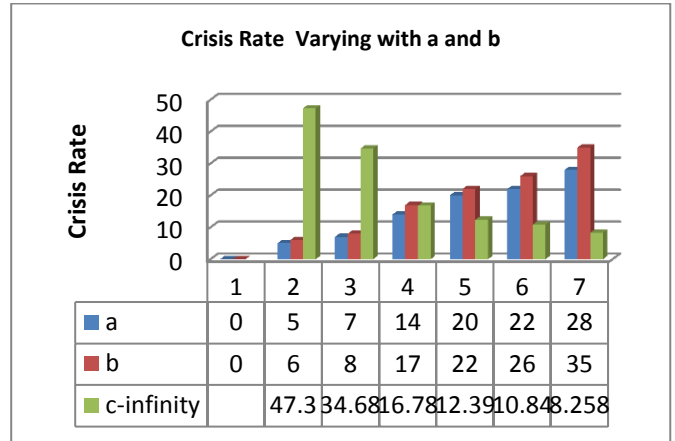


Figure: 1 Relationship among a, b and  $C_{\infty}$

When the values of a and b increases and the corresponding crisis rate decreased.

##### Steady state cost

The costs of steady state are determined by using the formula

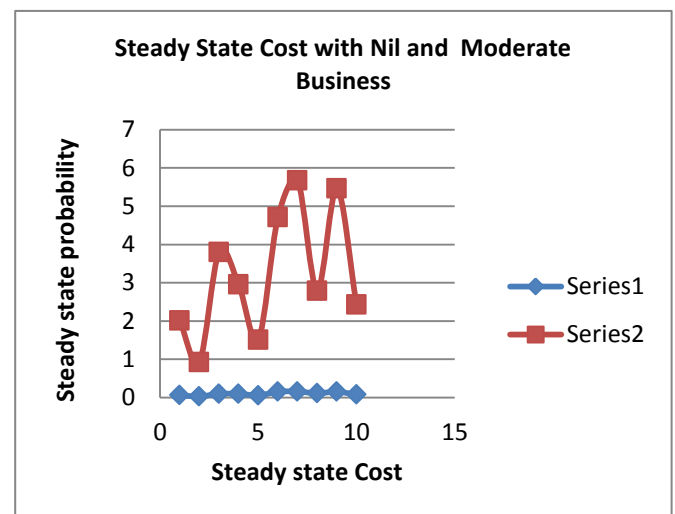
$$C_{ijk} = (c_{MP}^i + c_B^j + c_R^k)\pi_{ijk}$$

Where,

$c_{MP}^i$  stands for cost of manpower at the states  $i = 0$  or  $i = 1,$   
 $c_B^j$  stands for business cost at the states  $j = 0$  or  $j = 1, c_R^k$   
stands for the departure or recruitment cost at the states  $k = 1$   
or  $k = 2.$  We assume that the costs

$$c_{MP}^0 = 12, c_{MP}^1 = 8, c_{MP}^2 = 7, c_B^0 = 12, c_B^1 = 18, c_B^2 = 7, c_R^0 = 5, c_R^1 = 10 \text{ and } c_R^2 = 7$$

Table: 2 Relationship between steady state probability and steady state cost



## V. CONCLUSION

From the above concept we found that the steady state cost increases, while there is full business also departure/recruitment rate increases. When there is moderate/no business, the steady state cost increases and the corresponding recruitment rate increases. Also it is observed that if there is full business and recruitment rate increases but the steady state cost decreases.

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