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Fuzzy Decision Trees as a Decision Making Framework in the Private Sector

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Abstract-Systematic approaches to making decisions in the private sector are becoming very common. Most often, these approaches concern expert decision models. The expansion of the idea of the development of e-participation and e-democracy was influenced by the development of technology. The solution presented in this papers concerns fuzzy decision making framework. This framework combines the advantages of the introduction of the decision making problem in a tree structure and the possibilities offered by the flexibility of the fuzzy approach. The possibilities of implementation of the framework in practice are introduced by case studies of investment projects appraisal in a community and assessment of efficiency and effectiveness of private sector.

Keyword - *Decision tree, Appraisal tree, Fuzzy set, Decision making, private sector.*

I. INTRODUCTION

The making decision in the private sector is a common subject of research; however, using systematic approaches is not common when making decisions. The private sector is supposed to act in public interest and consider the interest of all stakeholders. It is obvious that a large number of diverse stakeholders have needs and wishes that must be considered when making decisions, which in the private sector can be clearly stated despite the different views of the definition of the term “The public interest”.

In general, the contribution of the research is the definition of the decision making framework for the private sector, which comprises suitable methods and approaches within the general framework. The core of the solution is decision trees, which represent a common base of qualitative multi-attribute decision models. The use of the fuzzy approach enables the decision makers to appraise the attributes of alternatives more easily and accurately [5]. Within the general definition, a comprehensive definition of the fuzzy appraisal tree is given. The main scientific contribution of the work is the definition of the fuzzy appraisal tree. Decision trees as wells as fuzzy decision trees supporting the appraisal have not been formalized to the stage of classification and comparative trees yet, thus the definition of the fuzzy appraisal tree is an important contribution to the decision trees theory. The solution of enables the use of any types of variable. The aggregation over the appraisal tree combines values of different types of variables without limitations. Furthermore, the solution exceeds the limitation of the number of vertices and their attributes of appraisal trees that use decision rules.

II. DECISION MAKING IN THE PRIVATE SECTOR

In the application of a systematic approach when making decision in the private sector it is important to consider the following points. Any negligence with respect to these points could possible cause difficulties to the systematic approach to making decisions in the private sector [4].

- A complex and less – transparent stakeholder network.
- Many diverse interests,
- Multiple problem perceptions and multiple preferences,
- A large set of appraisal criteria.
- Aggregation of many and often divergent interests of society into such notations as “general welfare”, which only makes the conflict.

The systematic approach to the decision making process is based on systems for decision - making support that include methods, models and tools, and offer help with the quality of decision – making. An approach such as this must suppress the causes for the slow application of this type of solution and must enable:

- The integration of numerous stakeholders and group formation,
- Insight into multiple problem perceptions and multiple preferences and coordination,
- The handling of large sets of appraisal Criteria,
- A simple and understandable introduction to the decision making problem and the decision,
- Analysis of difference in preference and the realization of an opinion reconciliation process and a stakeholder concordance search.

III. FUZZY SETS AND FUZZY LOGIC

Fuzzy logic and approximate reasoning are parts of the framework with the definition of the linguistic variables. The review of fuzzy methods is completed with an introduction to the transformations between crisp and fuzzy and linguistic and fuzzy variables (fuzzyfication, defuzzyfication, linguistic variable to fuzzy number mapping and approximation).

The concept of a characteristic function of a (cantorian or crisp) set was generalized by L.A. Zadeh [11] by replacing, in the co-domain, the two-element set {0, 1} by the unit interval [0, 1]. Logically speaking, this is supposed to work in logic with a continuum of truth values (fuzzy logic) rather than in classical Boolean logic with two values, true and false, only.

Definition: 3.1
Fuzzy set [11]

Given a (crisp) universe of discourse, x, the fuzzy set $\tilde{A}$ (more precisely, the fuzzy subset $\tilde{A}$ of x) is given by its membership function $\mu_{\tilde{A}}(x) : x \to [0, 1]$, and the value $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of x in the fuzzy set $\tilde{A}$. The group of all fuzzy subsets of x is denoted as F(x).

Definition 3.2
Fuzzy number [13]

A fuzzy number $\tilde{A}$ is a convex normalized (Sup px $\mu_{\tilde{A}}(x) = 1$) fuzzy set over the real numbers with a continuous membership function having only one mean value $x_0 \in \mathbf{R} \mu_{\tilde{A}}(x_0) = 1$.

If the mean value covers a subinterval $[a, b] \subseteq [0, 1]$ then we are talking about a fuzzy interval. If the membership function of a fuzzy number of intervals is constructed of linear functions, they are triangular fuzzy numbers and the later are trapezoidal fuzzy numbers.

Definition: 3.3
Trapezoidal fuzzy number

A trapezoidal fuzzy number is expressed as $\tilde{A} = (a, b, \alpha, \beta)$ and defined by the linear membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a-x}{\alpha} & \text{if } a - \alpha \le x \le a \\ 1 & \text{if } a \le x \le b \\ 1 - \dfrac{x-b}{\beta} & \text{if } b \le x \le b + \beta \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

A triangular fuzzy number is a degenerated trapezoidal fuzzy number (a = b). For this reason, from this point the term fuzzy number will be used for fuzzy interval (trapezoidal fuzzy number), as well as for fuzzy number (triangular fuzzy number). As a short break, have a look at a graph of a fuzzy number (more precisely, a fuzzy interval or trapezoidal fuzzy number)

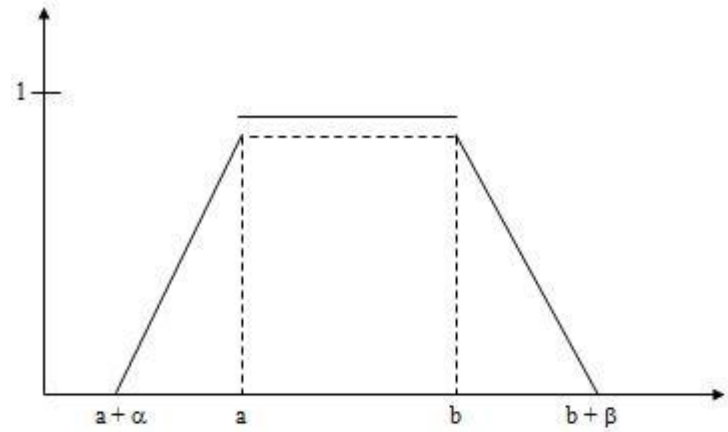

Figure 3.1 Graph of a fuzzy interval

For fuzzy numbers, the computation necessary for algebraic operations are considerably simplified. The calculations within the decision – making framework are only done with positive fuzzy number ($\mu_{\tilde{A}}$ (x) = 0, ∀ x < 0), and therefore only the arithmetic for positive fuzzy numbers is introduced. (The definitions comprise the fuzzy numbers $\tilde{A}$ = (a, b, α, β) and $\tilde{B}$ = (c, d, γ, δ))

TABLE 3.1 Arithmetic operations for trapezoidal fuzzy numbers [2].

Operations	Result	
$\frac{1}{\tilde{A}}$	$\left(\frac{1}{b},\frac{1}{a},\frac{\beta}{b(b+\beta)},\frac{\alpha}{a(a-\alpha)}\right)$	3.2
$\tilde{A}+\tilde{B}$	(a +c, b +d, α + y, β +δ)	3.3
$\tilde{A}-\tilde{B}$	(a – d, b – c, α +δ, β + y)	3.4
$\tilde{A}.\tilde{B}$	(ac, bd, aγ + ca - αγ, bδ + dB + Bδ)	3.5
$\frac{\tilde{A}}{\tilde{B}}$	$\left(\frac{a}{b},\frac{b}{c},\frac{a\delta+d\alpha}{d(d+\delta)},\frac{b\gamma+c\beta)}{c(c-\gamma)}\right)$	3.6

Zadeh introduced mapping between linguistic variables and fuzzy sets by the definition of a linguistic variables.

Definition: 3.4
Linguistic variable [12].
A linguistic variable is defined by a quintuple (K, T(k), U, G, $\tilde{M}$) in which k is the name of the variable, T(K) (or simply T) is the term set of k, that is, the set of names for linguistic values k, with each value being a fuzzy variable denoted generically be x and ranging over a universe of discourse U which is associated with the base variable u ; G is a syntactic rule (which usually has the form of grammar) for generating names x of values of k; and M is a semantic rule for associating each x with its meaning $\tilde{M}$ (x), which is a fuzzy subset of U. A particular X, that is a name generated by G is called a term. A term consisting of a word or words which function as a unit (i.e., always occur together) is called an atomic term. A concatenation of components of a composite term is a sub-term.

An example of a term set is:

T = {Reject, lowest, very low, Low, Middle, High, Very high, Highest, Must Be} (3.7)

The modelling of linguistic variables with trapezoidal fuzzy numbers was proposed by Bonissone and Decker [2]. A choice of the cardinality of the term set depends on the characteristics of the problem in this case, and the same is true for the membership functions of the corresponding fuzzy numbers any kind of term set can be considered without any major changes, and in that respect the framework is flexible.

A metric of the fuzzy sets is required as a definition of all the mappings between crisp values (real numbers), fuzzy numbers and linguistic values. The Tran – Dickstein distance takes into account the fuzziness of the fuzzy sets and is confirmed in practice in an environmental-vulnerability assessment [9]. We have, therefore, decided to choose it for our framework. For trapezoidal fuzzy numbers the general definition is simplified as:

Definition: 3.5
Tran – Dickstein distance for trapezoidal fuzzy numbers (f (α) = α) [9].

$$D_T^2(\tilde{A},\tilde{B},\alpha)=\left(\frac{a+b}{2}-\frac{c+d}{2}\right)^2+\frac{1}{3}\left(\frac{a+b}{2}-\frac{c+d}{2}\right)$$

[β - α - δ, λ] +

$$\frac{2}{3}\left(\frac{b-a}{2}\right)^2+\frac{1}{9}\left(\frac{b-a}{2}\right)$$

$$[\beta + \alpha] + \frac{2}{3}\left(\frac{d-c}{2}\right)^2 + \frac{1}{9}\left(\frac{d-c}{2}\right)$$

(3.8)

$$[\delta + \gamma] + \frac{1}{18}[\beta^2 + \alpha^2 + \delta^2 + \gamma^2] - \frac{1}{18}$$

$$[\alpha\beta + \gamma\delta] + \frac{1}{12}[\beta\gamma + \alpha\delta + \beta\delta + \alpha\gamma]$$

The proposed framework introduces parallel use of three types of variables, the real number (crisp value), the fuzzy number and the linguistic variable.

Definition: 3.6

Real number ← → fuzzy number ← → linguistic variable transformations

Fuzzyfication $V_F : l \rightarrow \tilde{L}$

Fuzzyfication makes the transformation from normalized real numbers $l \in \mathbf{R}$ to fuzzy sets $L \in F(x)$ (in our case, fuzzy numbers) using membership functions. It is carried out in two steps:

Mapping $T_M : L \rightarrow \tilde{L}$ of the real number $l \in \mathbf{R}$ to the fuzzy set $\tilde{L} \in F(x)$, where in the case of multiple corresponding fuzzy sets the weighted average operator is used.

$$\tilde{L}_F = \frac{1}{\sum_K \tilde{\mu}_k(l)} \sum_K \tilde{\mu}_k . \tilde{L}_F ; K = 1, N;$$

N is number of fuzzy sets tuched by l, $\tilde{L}_K$; are the fuzzy sets tuched by l and $\tilde{\mu}_k(x)$ are the membership functions of the fuzzy sets $\tilde{L}_F$

Translation T_T ; $\tilde{L}_F \rightarrow \tilde{L}_l$ of the fuzzy set $\tilde{L} \in F(x)$ so that the result of defuzzyfication of fuzzy set $\tilde{L}_l$, T_{DF} : $\tilde{L}_l \rightarrow x$ is equal to the input real number $l \in \mathbf{R}$.

Defuzzyfication T_{DF} : $\tilde{L} \rightarrow l$.

Defuzzyfication makes the transformation from fuzzy sets $\tilde{L} \in F(x)$ to real numbers $l \in \mathbf{R}$. A "centre of gravity" method was choosen for all the possible transformations of fuzzy sets into crisp values. The method is the most trivial weighted average and has a distinct geometrical meaning

$$x_{COG} = \frac{\int_x x.\mu(x)dx}{\int_x \mu(x)dx}$$

(3.9)

A simple calculation for a fuzzy number $\tilde{A}$ (a, b, α, β) gives the simple formula

$$x_{COG} = \frac{-a^2 + b^2 + a\alpha + b\beta - \frac{\alpha^2}{3} + \frac{\beta^2}{3}}{-2a + 2b + \alpha + \beta}$$

(3 .10)

linguistic variable $L \in T(k)$ to fuzzy variable $\tilde{L} \in F(x)$ mapping $T_M : L \rightarrow \tilde{L}$.

The mapping of linguistic values into fuzzy numbers is part of linguistic variable definition where suitable parameters are defined

- The name of the linguistic variable,
- The cardinality of the term set and the terms, the elements of the term set.
- For each term the corresponding fuzzy number (mapping functions).

The linguistic variable "Appraisal", with, nine values and names was used for this study:

TABLE 3.2
LINGUISTIC VARIABLES "APPRAISAL" MAPPING FUNCTION

Reject	Lowest	Very Low	Low	Medium	High	Very High	Highest	Must Be
0	.01	.10	.22	.41	.63	.78	.98	1
0	.02	.18	.36	.58	.80	.92	.99	1
0	.01	.06	.05	.09	.05	.06	.05	0
0	.05	.05	.06	.07	.06	.05	.01	0

Fuzzy set $\tilde{L} \in F(x)$ to linguistic value $L \in T(k)$ approximation T_A : $\tilde{L} \rightarrow L$.

The fuzzy number $\tilde{A}$ is approximated to a linguistic value $\tilde{L}_{approx}$ so that the closet fuzzy number $\tilde{L}$, representative of the nearest linguistic value, is found:

$L_{approx} = L : D_T(\tilde{A}, \tilde{L}, \alpha) = \min D_T(\tilde{A}, \tilde{L}_i, \alpha)$; i =1, …., n (3.11)

For higher granularity of the ends results we introduced the approximation deviation. This is defined as the relative number of the difference in distance of the approximated fuzzy number and the fuzzy number image of the linguistic approximation and the difference between two adjacent linguistic values [1]:

$\tilde{A}$

The approximation with the deviation is then labelled as

$\leftarrow L_{approx}$, if Dev % L – 25 %
L_{approx}, if - 25% $\leq$ Dev % $\leq$ 25 %
$L_{approx} \rightarrow$, if Dev %<25%
(3.12)

At this point, we are well equipped with all that is needed to define the proposed model. We know that in order to perform the appraisal, an appraisal tree should be constructed and that in the private sector it is very suitable to perform an appraisal with the help of fuzzy variables and fuzzy aggregation. Therefore, in a comprehensive definition of the fuzzy appraisal framework and within it, the definition of the fuzzy appraisal tree is presented.

IV. FUZZY APPRAISAL FRAMEWORK

The suggested appraisal framework resulted from the problem when solving the group multi-attribute decision making in the private sector. An investigation of the problem and the development of the solution lead to a general appraisal framework combining the advantages of the introduction of a tree structure and the use of a fuzzy approach for the appraisal of attributes or indicator as well as a comparison of the criteria and perspectives.

The entire fuzzy appraisal framework includes the definition of the fuzzy appraisal tree, averaging operators for the calculation of the average value of forests (groups of trees, with respect to groups of evaluators, group of alternatives, organization units of the same kind etc.) method for tree comparison, and tree classification (regarding the root, regarding the individual nodes, regarding the structure, etc.) methods for the analysis of tree variability (regarding the root, regarding the individual nodes, regarding the structure, etc.) and methods for tree optimization (efficiency, information, entropy, etc.)

Definition: 4.1
Fuzzy appraisal framework
Fuzzy appraisal frame constitute a forest with fuzzy appraisal tees over which the following is defined:

- ❖ Fuzzy appraisal trees
- ❖ Averaging operators O_{AVg} : ($(\tilde{T}_1,....,\tilde{T}_n) \rightarrow \tilde{T}_{Avg}$

For the calculation of average tree values in chosen sub – forests.

- ❖ Methods for fuzzy tree comparison and fuzzy tree classification,
- ❖ Variability measures, and
- ❖ Methods for fuzzy tree optimization.

With a given framework, it is possible to use three types of variables – real, fuzzy and linguistic, the values of which represent an equivalent appraisal of an attribute, criteria, indicator of perspective represented by the nodes.

Ingoing values (in the leaves) and calculated values (in the nodes) are recalculated from the ingoing type into the other two –real number, fuzzy number, fuzzy number, linguistic variable. All the necessary transformations are defined in each node and proceed during the recalculations. The values in the inner nodes are filled from the aggregation functions over fuzzy numbers. The aggregation functions over linguistic values are not considered (simplicity, distinction from existing systems based on system rules). For special cases, the aggregation function is defined over real variables. Ingoing variables for aggregation operators are defined by the connections from the successors. Like the nodes, the connections, which also represent the weights are evaluated with all three types of variables and equipped by transformations to transform one into the other.

Definition: 4.2
Fuzzy appraisal tree
A fuzzy tree $\tilde{T} = (\tilde{V}, \tilde{E})$ consists of a finite, nonempty set of fuzzy nodes (or vertices) $\tilde{V}$ and a set of fuzzy edges $\tilde{E}$. A fuzzy vertex $\tilde{V}$ consists of: Three variables $l \in \mathbf{R}$, $\tilde{L} \in$ M (x), L$\in$L(k); (crisp variable *l*, fuzzy number $\tilde{L}$ and linguistic variable L), four transformations between them,

fuzzyfication $T_F : l \rightarrow \tilde{L}$, defuzzyfication $T_{DF} : \tilde{L} \rightarrow l$, approximation T_A: $\tilde{L} \rightarrow$l, and mapping T_M : L$\rightarrow \tilde{L}$ A fuzzy aggregation operator over the fuzzy variables of children (for internal nodes)

f : ($\tilde{L}_{i+1,j1}$, …. $\tilde{L}_{i+1,j,kij}$) $\rightarrow \tilde{L}_{ij}$ where i is the level of the node, j is the position of the node at the level i, and K_{ij} is the number of children of the node.

A fuzzy edge $\tilde{E}_{i,j}$ = ($\tilde{V}_{i,j}$, $\tilde{V}_{i+1,j,k}$) consists of a path from the parent to a child and of the weight $\tilde{W}_{i,j,k}$ which consists of three variables and four transformations between them

$\tilde{V}_{i,j}$,				
$l \in \mathbf{R}$	T_F: l$\leftarrow$	$\tilde{L}$ $\in$F(x)	T_A: $\tilde{L}$ $\leftarrow$	L$\in$L(K)
O_{Agg}: $\tilde{L}_{i+1, j,Kij} \rightarrow \tilde{L}_{ij}$				

Figure – 6.1 the structure of the fuzzy vertex $\tilde{V}$

For a function appraisal framework to work, averaging operators to drive aggregation functions and to calculate averages of fuzzy forests are needed. Because of the simplicity principle, we have opted, among the many averaging operators [13], for generalized operators of the weighted mean of fuzzy numbers expressed by the formula in Definition 4.3.

Definition: 4.3
[13]Generalised operators of the weighted mean of fuzzy numbers are:

$$h_a^w(a_1,....,a_n) = (\sum_{i=1}^{n} w_i a_i^a)^{\frac{1}{a}} a \in [0,1], i \in N_n, a \in \mathbf{R} \ (\alpha \neq 0) \quad (4.1)$$

Where for the vector $\tilde{W}$ = (W_1, …, W_n) it holds $\sum_{i=1}^{n}$ W_i=1, $W_i \geq 0 \forall_i \in \mathbf{N}_n$. Then vector $\tilde{W}$ is termed the weighted vector, and its components W_1 the weights. In the simplest version (equal weights $W_1 = \frac{1}{n}$ and α =1), it is simply the arithmetic mean.

Comparison and classification is based on the comparison of calculated average values approximated into linguistic values.

The proximity measure and consensus measure are chosen for the analysis of the variability in a forest of appraisal results.

V. APPLICATION EXAMPLES

The specific definition of a fuzzy appraisal framework is in general a choice of system elements according to the needs and possibilities of a specific problem. In this chapter the implementation of the fuzzy appraisal framework for two cases is presented. The first one, the optimal selection of community investment projects, was the environment where the idea of the fuzzy appraisal framework was born. The second one is the project balanced scorecard as an assessment and benchmarking tool is private sector running at Faculty of administration where the first implementation after definition of framework is going on.

5.1 Selection of investment project in a municipality for private sector

The case is focused on the question of the optimum choice of investment projects in a local community burned by various circumstances that could results in the municipality is inopportune investment orientation decision making in municipalities takes place successively with two groups of participants. Professional services asses the investments projects and merge them into investment options according to professional Criteria.
The proposals are then revised and approved by the mayor and forwarded to the municipal council, which then decides independently and autonomously. The decision makers are confronted with various difficulties resulting from un systematic approach political decision makers are reluctant to take professional arguments into consideration, while professional tend to disregard the political circumstances: however, an optimum decision is achieved only if all opinions and comments are dealt with in the decision making process.

We have therefore been seeking a solution to the issue of making optimal decisions on investment in local government, in the phase of preparing the investments as well as in the phase of initiating then realization and financing. The solution would have to establish a process that allow confrontation and coordination of diverse opinions and interests on the professional and political levels, in professional political as well as in professional – professional and political – political relations.

Based on the previous discussions, the fuzzy appraisal framework presented in section 4 represents an appropriate approach to the solution of the given problem. The decision tree contains knowledge of the structure of the values that determine to what extent an individual alternative is suitable for inclusion in the budget. We have determined the structure of the appraisal tree, taking in to account framework of deciding on capital investments in the private sector [3], legally prescribed definitions and the analysis of the method of decision making in local communities in Slovenia.

The appraisal model for investment projects in local communities was defined according to the needs and possibilities, based on the general definition of the fuzzy appraisal framework (Definition 4.2) with adaptations as follows.

Definition: 5.2
Fuzzy appraisal model for selection of investment projects in a municipality.

(1) Fuzzy appraisal model for selection of investment projects in a municipality is a fuzzy appraisal framework.
(2) The input values are linguistic variables (among the transformations in definition 4.1 point 1 fuzzyfication $T_F: l \rightarrow \tilde{L}$ is not needed).
(3) The fuzzy aggregation operators over the fuzzy variables of children (for internal nodes) O_{Agg} : ($\tilde{L}_{i+1,j,1}$,…. $\tilde{L}_{i+1,j,kij}$) $\rightarrow \tilde{L}_{ij}$, is derived from (Def 4.3), where α =1 and equal weights $w_i = \frac{1}{n}$ for all edges are chosen:

$$\tilde{A}_{ij} = \frac{1}{K_{ij}} \sum_{K} \tilde{A}_{i+1,j,k}; i = I-1,...i; j = 1,....J \quad K = 1, ...K_{ij} \quad (5.1)$$

Where I is the number of levels of the tree, i is the current level of the tree, J_i is the branching of the tree, j is the position of the node at the i-th level, K_{ij} is the number of children of the parent in question at the level i+1, and K is the position of the child of the parent in question.

4) The averaging operator O_{Avg} : ($\tilde{T}_1$,…., $\tilde{T}_n$) → $\tilde{T}_{Avg}$ for the average tree value calculation in chosen sub – forests is derived ( ), where α =1 and equal weights $W_i = \frac{1}{n}$ for all edges are chosen:

$$\tilde{A}_{ij} = \frac{1}{|G|} \sum_{G} \tilde{A}_{i+j}; i = I-1,...I; j = 1,.... j_i \quad (5.2)$$

Where G is the set of appraisers.

5) Variability measures are the proximity and consensus measure over the set of appraisers G (Definition 4.2)

The appraisal tree including three nodes (project contribution, feasibility and risk and cost / benefit appraisal), where the first two nodes each included three leaves and the third node included only two leaves [1]. The model was tested in three Slovenian municipalities. The set of appraisal projects include from seven to nine investment projects. Two types of appraisal were invited, representatives of municipal government and municipal councillors. Due to the reluctance of municipal councillors, the appraisal groups were rather small, comprising from nine to fifteen appraisers. We analysed the results represented with linguistic values and prepared a qualitative representation of results, where we considered the differences between projects and appraisal groups. The proposed solution attracted great interest, since the problem is of everyone's concern. It has been proven that the chosen method of appraisal is suitable for the chosen environments. An interview was performed after each case study concerning the usefulness and suitability of the suggested approach for decision making in a chosen environments. The results proved the approach to be suitable due to the evaluators having no problems during the appraisal. The content of the appraisal was a bigger problem due to the evaluators not being introduced to it and / or the importance of the project was underestimated, also financially. This is a matter of preparation and organization appraisals processes, in which case the fuzzy appraisal framework can contribute to but not solve the problem.

5.3 BALANCED SCORECARD AS AN ASSESSEMENT AND BENCH MAKING TOOL IN THE PRIVATE SECTOR

The fuzzy approach can be also effectively used when solving the problem of how to measure the successfulness of organizations with balanced scorecard. The balanced scorecard joins success indicators into four business perspectives. Customers, finance, process and learning and growth. In the profit sector final result is measured with the financial perspective. It is enabled by the other three perspectives, which indicate success fullness of the organization in the near future [6].

An organization is tree structured, where leaves are single employees or small departments and nodes combine subordinate units. The result of a unit is given by indicators defined for the unit, where some of them are calculated from equal indicators of subordinate units, and the others carry the results of the unit in question. The indicators of a unit are leaves of the appraisal tree of the unit. The nodes at the first level of the tree represent four perspectives of successfulness. The indicators of such an appraisal tree are defined over different variables which are hard to aggregate into joint value. The situation is the natural environment of the fuzzy appraisal framework which offers somehow simple solutions for quite difficult problems.

As a result of the research studying the problem of the implementation of the balanced scorecard into the private sector organizations we introduce the structure of the fuzzy appraisal framework for balanced scorecard follows:

- Each organizational unit is the carrier of indicators, which represent the result of the unit in question, and joint indicators of the result of subordinate units,
- The indicators which are measuring the same results are by definition equal indicators,
- The equal indicators of unit at the chosen level are aggregated into the equal indicators of the unit at the upper level of the organisational tree,
- The indicators of an organizational unit are linked into the appraisal tree of the unit, at the top of which four perspective nodes are defined, and the root of the tree represents the general appraisal of a unit,
- The root of the organizational tree is "the organization", which links all the indicators defined for the subunits into joint appraisal tree.

Definition of the appraisal model for the balanced scorecard is based on the general definition of the fuzzy appraisal framework with adaptations as follows.

Definition: 5.4
Fuzzy appraisal model of the balanced scorecard.

1) Fuzzy appraisal model for selection of investment projects in a municipality is a fuzzy appraisal framework.
2) Each node is evaluated with three variables (Crisp, fuzzy and linguistic), where one of them is the input variable.

   The fuzzy aggregation operator over the fuzzy variables of children.

$$O_{Agg} : (\tilde{L}_{i+1,j,1}, \ldots \tilde{L}_{i+1,j,kij}) \rightarrow \tilde{L}_{ij},$$

$$\tilde{A}_{ij} = \sum_{K} \tilde{W}_{i+1,j,k}\ \tilde{A}_{i+1,\ j,k,} \sum_{K} \tilde{W}_{i+1,j,k} = 1;\ i = I - 1, 1;\ j = 1, J_i, K = 1, \ldots, K_{ij} \qquad (5.3)$$

Where I is the number of levels of the tree, i is the current level of the tree, j_i is the branching of the tree, j is the position of the node at the i-th level, K_{ij} is the number of children of the parent in question at the level i+1, and K is the position of the child of the parent in question. The weights regulate the contribution of the children to the aggregation value of the parent.

3) The model of balanced scorecard comprises single organizational tree structure, so the averaging operator over forests of trees is not needed.
4) Variability is not a greater issues in the balanced scorecard model, but in any case the measure of proximity and consensus measure are available.

V. CONCLUSIONS

The structure of a fuzzy appraisal framework in the private sector is presented in this paper. The purpose of the framework is to develop solutions with properties adjusted specially for use in the public people and private sector. The methods and approaches that lead to the satisfactory conclusion were systematically combined in the framework.

The theory of decision trees and the theory of fuzzy sets and fuzzy logic. This led to incorporating the desired properties[8],[10]:

- Clarity and conciseness, context sensitivity, flexibility:
- Allow the representation of cognitive uncertainties in decision making, providing more information to the decision maker, using linguistic terms with soft boundaries to accommodate vagueness and ambiguous in human thinking and perception, into a framework.

The appraisal framework defined general elements of the system and gives guidelines to form concrete solutions. The approach was realized through its use in practice one case is observed.

The assessment of the performance of organizations with indicators balanced scorecard. Case studies proved the framework to be a suitable basis for implementing solutions of different decision making problems in private sector. However the accommodation of the framework to the specific environment of the private sector is not a restriction but a generalization.

It incorporates more flexibility in the appraisal, which makes the solution easier to use. The framework gives practitioners and researchers a change to broaden their research method and tools, designed to make their appraisal application better and more user friendly for all kinds of use, both public and non-public.

However, new research challenges and motivation are perhaps even more important than a contribution to solving the solving the specific problem in practice. The most important research for the future is:

- The definitions of suitable membership function of fuzzy sets and fuzzy numbers.
- The modelling of linguistic variables with fuzzy sets in accordance with the operators' comprehension and understanding.
- The definition of adequate functions and operators over a fuzzy set (aggregating and averaging operators, distance etc.),
- The discussions of data variability defined with fuzzy tree structures
- The methods for fuzzy tree optimisation.

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