

# An Improvement in Maximum Difference Method to Find Initial Basic Feasible Solution for Transportation Problem

**Lakhveer kaur<sup>1\*</sup>, Madhuchanda Rakshit<sup>2</sup>, Sandeep Singh<sup>3</sup>**

<sup>1,2</sup>Department of Applied Science, Guru Kashi University, Talwandi Sabo, Bathinda, INDIA

<sup>3</sup>Department of Mathematics, Akal University, Talwandi Sabo, Bathinda, INDIA

Available online at: [www.ijcseonline.org](http://www.ijcseonline.org)

Accepted: 26/Sept/2018, Published: 30/Sept/2018

**Abstract-** It is very important to find initial solution of transportation problems to reach optimal solution. In this paper Maximum Difference Method (MDM) is improved to get best initial solution of transportation problems. Our improved method overcomes the limitations of MDM given by Smita Sood and Keerti Jain. This modified approach most of times give better solution than MDM specially in case of tie and very close to the optimal solution. Also sometimes gives optimal solution.

**AMS Subject Classification:** 90C08, 90C05

**Keywords:** Transportation Problem, Optimal Solution, Initial Basic Feasible Solution, MDM

## I. INTRODUCTION

The transportation problem (TP) is a special case of Linear Programming Problems in which commodities are transported from several sources to different destinations in such a way that transportation cost should be minimum while satisfying both the supply unit and the demand requirement. These transportation problems are usually solved by modified distribution method (MODI) [3] or Stepping Stone Method [2]. To solve TP by using these methods, it is essential to proceed with initial basic feasible solution (IBFS). Existing method to find IBFS are, NWCM, LCM and VAM [17], Improved versions of VAM [1, 4, 15, 19, 20], Total opportunity cost method [8], TOCM-VAM [12], MDM [21], JHM Method [5], ICVM [13] and more methods to find IBFS are given by [6, 7, 9–11, 14, 16, 18]. In all these existing algorithms to find IBFS of transportation problems one of the algorithm is developed by Smita Sood and Kirti Jain [21] which is named as Maximum Difference Method (MDM). This method not always gives better solution because some of its limitations which are explained below: In MDM, Penalties are calculated by the difference of maximum cost and next to maximum cost of transportation table in rows and columns. If maximum and next maximum cost are same then penalties are taken to be zero which seems to be inappropriate because in many cases row or column having two maximum costs are equal but there is more difference between maximum and next to the maximum (not same) cost. But allocation is given to another cell which may have comparatively more cost. So in this case we do not get best feasible solution. Secondly, In MDM, If same maximum penalty in two or more rows or columns is occur then top most row and extreme left column is selected for the allocation. But in the top most row and extreme left corner if minimum cost is comparatively greater than the cost in row or column which is not on the top most row and extreme left column then greater cost is selected for allocation and then transportation cost is not minimized in that case.

So main reason of all these limitations is no proper rule in MDM for presence of tie, which can be occur in selecting cost cells for calculating penalties or for making allocations or in both cases. For removing all these limitations we have given improvement in MDM. We have applied our modified algorithm over number of examples (specially those examples in which tie occurs at different steps of solution processor).

In next sections modified algorithm is presented. Then numerical examples are provided and important observations are summarized in last section. Optimal solution of Numerical examples is obtained by modified distribution method [3].

## II. PROPOSED HEURISTIC

To modify MDM, Given steps are followed:

*Step 1:* Balance the transportation problem if it is unbalanced by adding dummy column with zero cost if total demand is less than total supply and add dummy row if total supply is less than total demand.

Step 2: Identify the cells which have the maximum cost and next to the maximum (not same) cost of each row and calculate the difference (penalty) between them and write it along the side of transportation table against corresponding row.

Step 3: Identify the cell which have the maximum cost and next to the maximum (not same) cost of each column and calculate the difference (penalty) between them and write it below the transportation table against corresponding column.

Step 4: Identify maximum penalty, if it is below the table then make allocation in cell having minimum cost in the column corresponding to the maximum penalty and if it is along the side of table then make allocation in the cell having minimum cost in the row corresponding to maximum penalty.

Step 5: If maximum penalty in two or more rows or columns is equal, then make allocation in row or column having minimum cost. If minimum cost is also same then choose the row or column in which maximum allocation can be given to the cell having minimum cost.

Step 6: Adjust the supply and demand and cross out the satisfied row or column. If both row and column are satisfied simultaneously then cross out only one of them and assign zero supply (or demand) in the remaining column (or row). Further calculation is not required for the row or column which is crossed out.

Step 7: Calculate the penalties again for the remaining rows and columns by repeating the step 2 and step 3 and make allocations by repeating step 4 to step 6 until all the demand requirements and supply units are not satisfied.

Step 8: Calculate transportation cost by taking sum of product of the allocations and corresponding cells.

### III. NUMERICAL EXAMPLES

Numerical examples are considered for the application of proposed modified method. So that modified method can be clearly specified. Input data and results obtained by applying proposed approach and MDM are given in table 1.

Table 1: Input Data for numerical examples and IBFS using Proposed Modified Approach and MDM

Ex.	Input Data	Obtained Allocations Using proposed Approach	Obtained Cost	Obtained Allocations Using MDM	Obtained Cost
1	$[c_{ij}]_{3 \times 5} = [2 \ 7 \ 8 \ 8 \ 3; 5 \ 6 \ 5 \ 5 \ 6; 5 \ 7 \ 8 \ 8 \ 3]; [a_i]_{3 \times 1} = [2, 3, 5]; [b_j]_{1 \times 5} = [3, 1, 1, 2, 3]$	$x_{11} = 2, x_{23} = 1, x_{24} = 2, x_{31} = 1, x_{32} = 1, x_{35} = 3$	40	$x_{12} = 1, x_{13} = 1, x_{21} = 3, x_{34} = 2, x_{35} = 3$	55
2	$[c_{ij}]_{4 \times 6} = [9 \ 12 \ 9 \ 6 \ 9 \ 10; 7 \ 3 \ 7 \ 5 \ 5 \ 6; 6 \ 5 \ 9 \ 11 \ 3 \ 11; 6 \ 8 \ 11 \ 2 \ 2 \ 10]; [a_i]_{4 \times 1} = [5, 6, 2, 9]; [b_j]_{1 \times 6} = [4, 4, 6, 2, 4, 2]$	$x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 1, x_{33} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4,$	112	$x_{13} = 5, x_{22} = 4, x_{26} = 2, x_{31} = 2, x_{41} = 2, x_{43} = 1, x_{44} = 2, x_{45} = 4$	114
3	$[c_{ij}]_{4 \times 6} = [2 \ 6 \ 4 \ 2 \ 5 \ 9; 3 \ 4 \ 3 \ 3 \ 2 \ 1; 2 \ 5 \ 1 \ 2 \ 1 \ 4; 6 \ 4 \ 3 \ 1 \ 7 \ 3]; [a_i]_{4 \times 1} = [75, 50, 30, 20]; [b_j]_{1 \times 6} = [25, 50, 20, 40, 30, 10]$	$x_{11} = 25, x_{13} = 10, x_{14} = 40, x_{22} = 40, x_{26} = 10, x_{35} = 30, x_{42} = 10, x_{43} = 10$	430	$x_{11} = 25, x_{12} = 30, x_{14} = 20, x_{22} = 20, x_{25} = 20, x_{26} = 10, x_{33} = 20, x_{35} = 10, x_{44} = 20$	450
4	$[c_{ij}]_{3 \times 4} = [7 \ 2 \ 3 \ 7; 1 \ 5 \ 4 \ 5; 7 \ 2 \ 4 \ 5]; [a_i]_{3 \times 1} = [2, 1, 3]; [b_j]_{1 \times 4} = [2, 1, 1, 2]$	$x_{12} = 1, x_{13} = 1, x_{21} = 1, x_{31} = 1, x_{34} = 2$	23	$x_{11} = 1, x_{12} = 1, x_{21} = 1, x_{33} = 1, x_{34} = 2$	24
5	$[c_{ij}]_{5 \times 6} = [3 \ 4 \ 5 \ 3 \ 6 \ 6; 2 \ 3 \ 4 \ 5 \ 2 \ 5; 7 \ 5 \ 4 \ 7 \ 6 \ 5; 4 \ 5 \ 2 \ 3 \ 3 \ 2; 7 \ 3 \ 5 \ 7 \ 5 \ 6]; [a_i]_{5 \times 1} = [8, 7, 3, 2, 6]; [b_j]_{1 \times 6} = [3, 5, 7, 2, 1, 8]$	$x_{13} = 1, x_{14} = 2, x_{16} = 5, x_{21} = 3, x_{23} = 3, x_{25} = 1, x_{33} = 3, x_{46} = 2, x_{52} = 5, x_{56} = 1$	98	$x_{11} = 3, x_{14} = 2, x_{16} = 3, x_{22} = 5, x_{23} = 1, x_{25} = 1, x_{33} = 3, x_{43} = 2, x_{53} = 1, x_{56} = 5$	105
6	$[c_{ij}]_{3 \times 4} = [21 \ 16 \ 25 \ 13; 17 \ 18 \ 14 \ 23; 32 \ 27 \ 18 \ 41]; [a_i]_{3 \times 1} = [11, 13, 19]; [b_j]_{1 \times 4} = [6, 10, 12, 15]$	$x_{14} = 11, x_{21} = 6, x_{22} = 3, x_{24} = 4, x_{32} = 7, x_{33} = 12$	796	$x_{14} = 11, x_{21} = 6, x_{22} = 3, x_{24} = 4, x_{32} = 7, x_{33} = 12$	796
7	$[c_{ij}]_{3 \times 5} = [6 \ 4 \ 4 \ 7 \ 5; 5 \ 6 \ 7 \ 4 \ 8; 3 \ 4 \ 6 \ 3 \ 4]; [a_i]_{3 \times 1} = [100, 125, 175]; [b_j]_{1 \times 5} = [60, 80, 85, 105, 70]$	$x_{12} = 15, x_{13} = 85, x_{21} = 60, x_{22} = 65, x_{34} = 105, x_{35} = 70$	1685	$x_{12} = 15, x_{13} = 85, x_{21} = 60, x_{22} = 65, x_{34} = 105, x_{35} = 70$	1685

On solving above TPs by MDM, it is observed that there is no rule for selecting minimum cost cell (in case of tie) for making allocation. For example, in example 2, there is a tie in selecting cell c31 and c41 for making allocation. If we first select cell c31 for giving allocation according to MDM rule, i.e topmost row, then solution obtained as 114, which is not best solution. But according to our proposed approach, we first select cell c41, to which maximum allocation is possible and obtained solution is same as optimal solution.

#### IV. RESULT ANALYSIS

In this section, all the results obtained by solving TPs of Section 3 are interpreted and comparison is made between proposed modified approach, MDM and VAM. Optimal solution is also obtained by MODI method. Input data is given in table 2.

Examples → Methods ↓	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7
Proposed Approach	40	112	430	23	98	796	1685
MDM	46	114	450	24	105	796	1685
VAM	40	112	450	23	97	796	1690
Optimal Solution	40	112	430	23	97	796	1580

#### V. Conclusion

In this paper, we have find out the limitations of MDM and improved it by removing its limitations so that best initial solution for minimizing the cost of transportation problem should be obtained. This proposed modified approach gives better solution than MDM when tie occurs in selecting cost cells and making allocations. Also most of times obtained solution is same as optimal solution or very close to the optimal solution.

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