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Self- Similar Behaviour Highway Traffic Analysis –Using Queuing Systems

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Abstract: Traffic congestion is a situation of increased disturbance of the motion of traffic. India, accompanied by much growing vehicles on the road, so that congestion of the traffic is quickly increasing. Traffic is still cannot thoroughly forecast under which case Traffic Jam may abruptly occur. This study proposes self-similarity structure; it plays a crucial role in queuing system in the field of congestion traffic. The proposal summarizes that whether vehicle arrival pattern on Highways is self-similar in nature or not? Also depict the results in terms of Length of the Queue, Waiting Time Distribution, Traffic Intensity etc., using Queuing models. For this we provided the data from V.R Technique Consultant Pvt. Ltd, India, as of Toll Plaza reports from Delhi Gurgaon section of National Highway 8(NH8) in India. Few techniques to test the self-similarity have been used and obtained values of Hurst parameter H are reasonably close to each other. Using M/M/1 queuing model and an empirical with Hurst index terms mean queue length has been computed against traffic intensity. Results of the study reveal that mean queue length increases as ρ and H increase. This kind of research is to forecasting the performance analysis and chronic improvement of toll plazas.

***Keywords*:** Queuing Model, System Design, Self-similarity, Hurst Index, Queue Length, Waiting Distribution, Traffic Intensity.

I. INTRODUCTION

One of the major issues in the analysis of any traffic system is the analysis of delay. The analysis of delay normally focuses on delay that results when demand exceeds its capacity; such delay is known as queuing delay, and may be studied by means of queuing theory. This theory involves the analysis of what is known as a queuing system, which is composed of a server; a stream of customers, who demand service; and a queue, or line of customers waiting to be served. Queuing is the synonymous with waiting and waiting lines, waiting is an inevitable part of modern life, from waiting to get served at grocery stores, banks or post offices, to waiting on hold for an operator to pick up telephone calls. Waiting causes not only inconvenience, but also frustration to people’s daily lives. The queuing theory as promulgated by (Agner Krarup Erlang, 1909) is applicable in situations where the customers arrive to service station for service; wait for service, leaving the system after receiving the same. However, it has been shown solution only part of the problem, the efficiency of the process, while the application of these results to real-world service operational settings is restricted because it does not take human factors into consideration.

Since waiting involves people, things, time, and environment, it is essential to incorporate issues related to both the social and psychological perspectives in order to reduce the negative impact of waiting on customer satisfaction and perceived quality. Previously, there have already been numerous researches of queues conducted either in terms of Operations Research, or in terms of the consumer behavior, Even though there has not been any distinct researches conducted in the combination of the two. While this chapter intends to fill the void in this research area of waiting, which has been dominated by mathematical models that is lacking of the consideration of human factors. Essentially, the goal for this paper is to develop a framework that aids the design of a queuing system, which will put together crucial aspects from both psychological and social perspectives into the waiting issue. And it will preserve a universal nature that allows application to all real world operations scenarios. The structure of the queuing system is defined as input or arrival distribution, service distribution, service channels, maximum number of customers in the system, population size or calling source, service discipline.

The premise of this paper is, we observed the characteristics of real time traffic data is self-similar (Qiang Meng and Hooi Ling Khoo, 2009), which is an augmentation to investigate the performance metrics as average waiting time distribution and queue length against the traffic intensity. This research attempts to strike a balance of Mean Queue Length and waiting time distribution for M/M/1 Queuing systems.

II. DATA COLLECTION AND AREA OF STUDY

As discussed in the introduction, we are primarily interested in vehicle arrival pattern on busy highways. For this, we have investigated real time data provided by V R TECHNICHE Consultants Pvt. Ltd, India. For ready reference this data is given in Appendix. This data was collected from toll traffic reports at one of the three operating toll plazas on Delhi – Gurgaon Section of National Highway 8 (NH8) in India. Delhi – Gurgaon Section of NH8 is a 6/8-lane BOT toll road with a 20 year concession period. The data was for seven days in August 2018. Gurgaon is one of the fastest growing cities in the national capital region of India, and Delhi – Gurgaon Section of NH8 is one of the busiest highway sections in India. Delhi – Gurgaon Section has three toll plazas.

Out of these, the toll plaza at km 24 (between Delhi and Gurgaon) is busiest and has 32 toll lanes as shown in the Fig.1., It may be noted that the km 24 toll plaza with 32 toll lanes is the largest toll plaza in India and is one of the largest in the world.

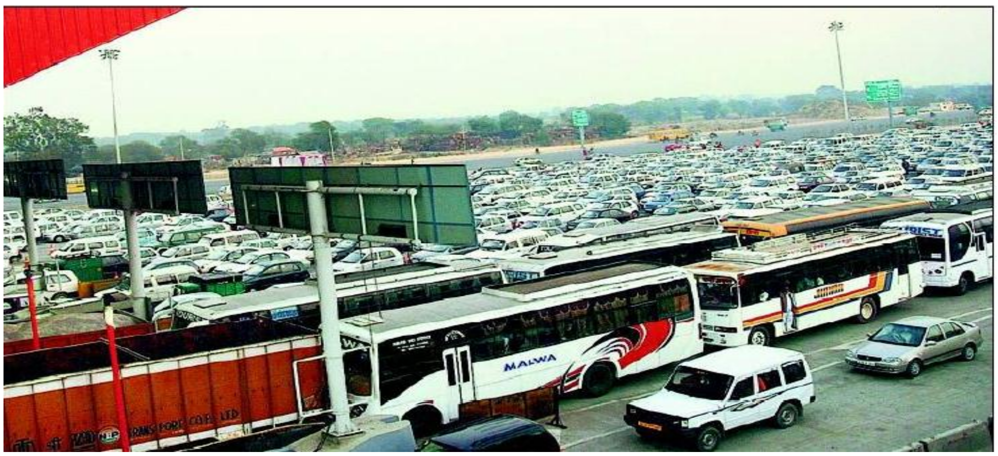

Figure .1: km 24 Toll Plaza on Delhi – Gurgaon Section of NH8

III. SELF-SIMILAR PROCESS- HURST INDEX

Self-Similar Process: Self-similarity is a property in which the arrangement of the intact is enclosed in its parts. The word self-similar was invented by Mandelbrot. He and his co-workers obtained self-similar processes to the awareness of statisticians, mostly as functions in such regions as geophysics and hydrology (Mandelbrot, 1968)

As mentioned above, a procedure is considered to be precisely self similar if the aggregated processes have first and second-order statistical properties that are impossible to differentiate from those of the process itself. On the contrary, an asymptotically self-similar process is a process where the autocorrelation function of the aggregated process approaches that of the process itself for a large degree of aggregation.

When a self-similar process X (t) has the property of stationary increments (that is when the finite dimensional distributions of X $(t+\tau)$ - X (t) do not depend upon t), then the process may serve as an underlying process yielding a fractal process. That means, one can construct a (discrete time) stationary increment process.

$$X_n=X[nT_i]-X[(n-1)T_i] \quad (3.1)$$

With long-range dependence, slowly decaying variance and 1/f - noise properties, those are specific for fractal processes. The most widely-known example of self-similar process is the fractional Brownian motion (fBm) process (with infinitely long-run correlations), which is a generalization of the Brownian motion with uncorrelated and independent intervals (Beran, 1995).

Hurst Index: The intensity of self-similarity is given by Hurst parameter H . The parameter H was named after the hydrologist (Hurst, 1951) who spent many years to investigate the problem of water storage and also to determine the level patterns of the Nile River. Hurst parameter is perfectly well defined mathematically, measuring if it is a problematic one. The data must be measured at high lags or low frequencies where fewer readings are available. The parameter H has range $0.5 \le H \le 1$. Estimation of H is a difficult task. Several methods are available to estimate degree of self-similarity in a time-series (Roughness 2003, Jerzy Wawszczak, 2005).

IV. QUEUING ANALYSIS OF TRAFFIC DATA- QUEUE LENGTH DISTRIBUTION

A. M/M/1 QUEUING SYSTEM

Queues or waiting lines are the most extensive phenomenon in our everyday life. Queuing system is one of the main segments of an Operations Research. It is a scientific and systematic approach to analyze and solve the complicated problems also for making better decisions. The Researchers have given unique importance to the development and the use of techniques like queuing theory. Queuing theory is used to solve problems concerned with traffic congestion in bank counters, ration shop, railway reservation counters, toll plazas, doctor's clinic, and automobile service etc.., its main reason is to predict the congestion situations of a precise urban transportation network and suggest improvements in the traffic Areas. The ultimate idea is to offer a better optimization of the traffic communications. Those optimizations are supposed to conclude into a decrease of pollution, travelling times and fuel consumption. In this paper we introduce markov processes that play a central role in the analysis of all the basic queuing systems. Queuing theory is an intricate and yet highly practical field of mathematical study that has vast applications in performance evaluation (Bhat U.N, 2015). The basic concept and results are instrumental to the understanding of queuing theory that is outlined in this section.

B. INTERPRETATION OF M/M/1 MODEL

Queuing models enlighten the researchers and engineers to ensure an optimal flow with a minimum number of traffic jams. It main purpose is to predict congestion states of specific network traffic. In this we specifies M/M/1 queuing model which is the simplest model and is commonly used. This model is based extensively on two theoretical distributions, the Poisson distribution for arrivals and the negative exponential distribution for service times. In this model, it assumed that single waiting line has no restriction on length of queue and the Poisson distribution of arrivals.

The objective of queuing analysis is to offer a reasonably satisfactory service to waiting customers. The model M/M/1 represents the queue length in a system having a single server where arrivals are determined by a Poisson process and service times have an exponential distribution. The model name is written in Kendall's notation. Kendall's notation is used to describe and classify a queuing node. (Kendall,1953) proposed to describing queuing models using three factors written A/S/C where A denotes time between arrivals to the queue, S denotes the size of jobs and C the number of servers at the node. It has since been extended to A/S/C/K/N/D, where K is the system capacity to hold customers, N denotes population size which can be finite or infinite, and D is the queue discipline.

In this model the rate of arrival and the service depend on the length of the line. This model is also called the birth and death model. Both the arrivals and service rates are independent of the number of customers in the waiting line. The arrivals are completely at random according to Poisson distribution. There is only one queue and one service facility, arrivals are handled on FCFS (first come first service) basis and service is provided to the customers according to FCFS rules. Arrivals form a single queue, there is a single server in the service facility. When arrivals do not get influenced by the length of the queue then leave the system only after receiving the service. The Poisson and the exponential distributions are related to each other, both of them are denoted by the same letter "M" is used due to markovian property of exponential process.

The exponential distribution is used to describe the inter arrival time in the pure birth model means arrivals only allowed and the inter departure time in the pure death model means departures only can takes place is to show the close relationship between the exponential and the Poisson distributions. The mean service rate is higher than the mean arrival rate (i.e. $\mu>\lambda$). When $\mu>\lambda$, no queue will be formed and the arriving customers will not have to wait. when $\mu=\lambda$, in this case, if the initial queue length was zero then new arrival will not have to wait, and in case the initial queue length was not zero, then every person arriving in the system will have to join the queue i.e. the length of the queue would remain constant. When $\mu<\lambda$, in this case, the length will increase indefinitely and this will not be a steady system. The ratio λ/μ is known as the utilization factor.

C. MEAN PERFORMANCE METRICS

- The average number of customers in the system.

  This is the number of customers in the queue plus the number of Customers being served and is denoted by

$$L_S = \frac{\lambda}{\mu-\lambda} \quad (4.1)$$

- The average number of customers in queue (i.e queue length)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \quad (4.2)$$

- Average waiting time in the queue system.
  It is the time that a customer spends waiting in queue plus the time it takes for servicing the customer.

$$W_s = \frac{1}{\mu-\lambda} \quad (4.3)$$

- The average waiting time in the queue.

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} \quad (4.4)$$

- Traffic intensity ρ= (mean arrival rate) λ /(mean service rate)μ. (4.5)

In this section, we present some numerical results of mean queue length ($\bar{L}$) against traffic intensity. For this, we use the formula (Gunther, 2000) given under:

$$\bar{L} = \frac{\rho^{0.5/(1-H)}}{(1-\rho)^{H/(1-H)}} . \quad (4.6)$$

In the equation (4.6), ρ is traffic intensity. H is Hurst parameter is an index of Self-similarity

V. NUMERICAL RESULTS

Using percentile method the H value is computed for the data given. The obtained value of H in this case is 0.767. From the Residuals of Regression Method, the obtained value of H is 0.754 and by the periodogram method obtained value of H is 0.746 (Malla Reddy Perati et al., 2012).

In the Equation (4.6) ρ is the traffic intensity, Results are illustrated in Figure 2(a)-8(b). From these figures, we bring to a notice that as ρ increases average queue length increases which is predictable. Additionally, as H increases, average length of the queue increases. This inclination concurs with our perception.

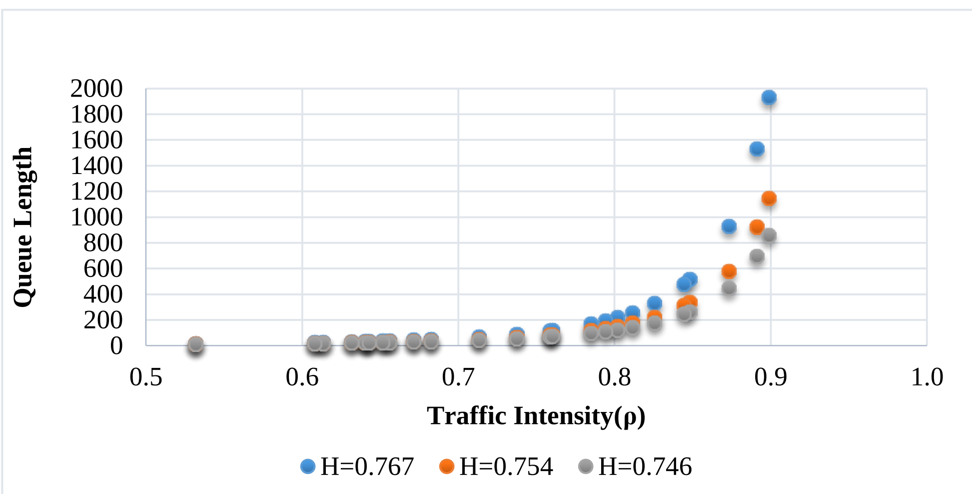

Figure 2(a): Queue length V/s Traffic intensity of traffic data on day 1

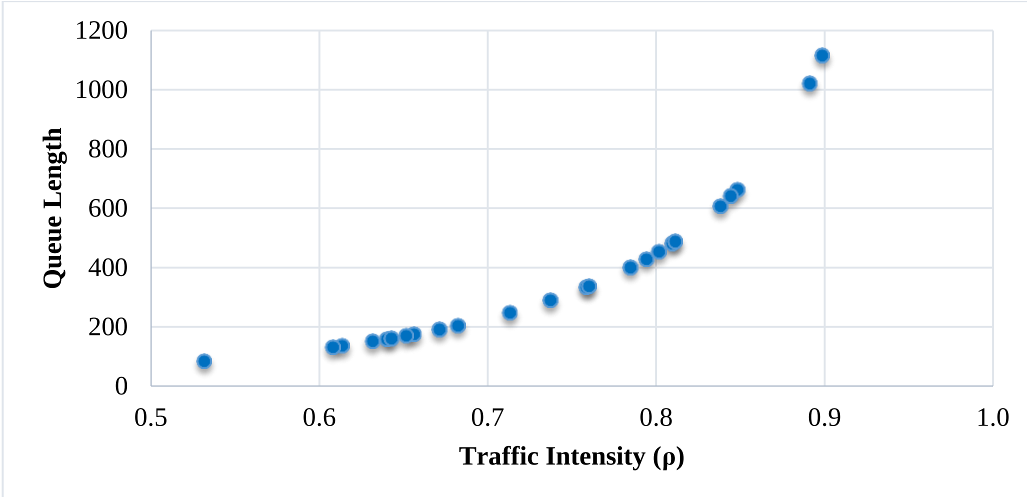

Figure 2(b): Queue length V/s Traffic intensity of traffic data on day 1 (M/M/I)

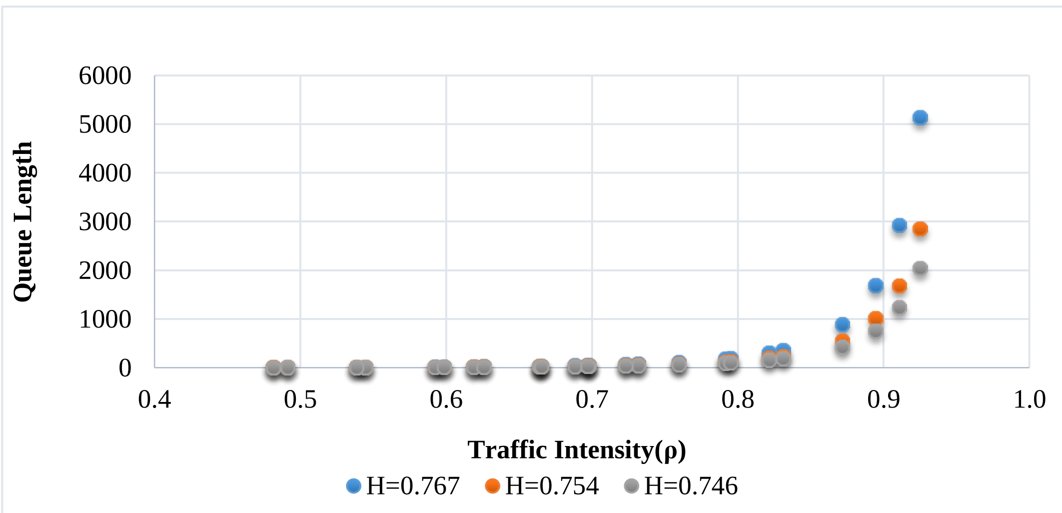

Figure 3(a): Queue length V/s Traffic intensity of traffic data on day 2

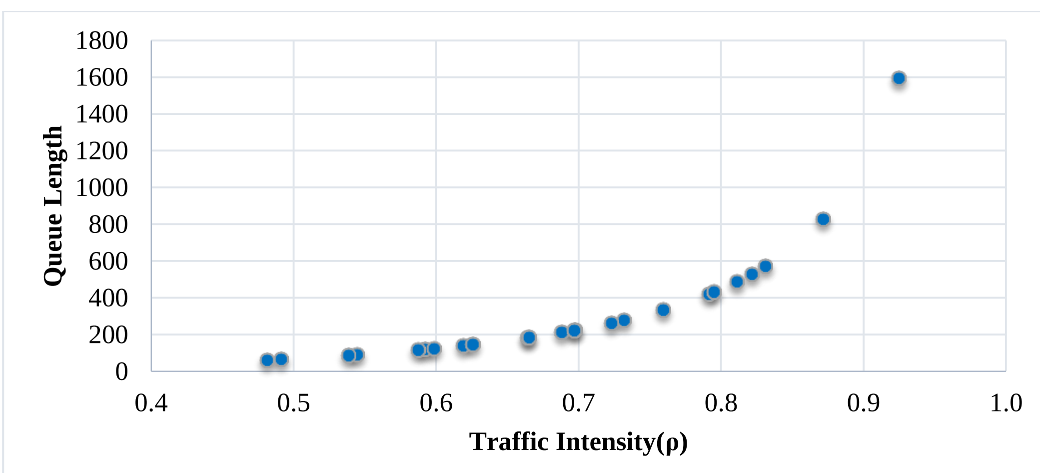

Figure 3(b): Queue length V/s Traffic intensity of traffic data on day 2 (M/M/I)

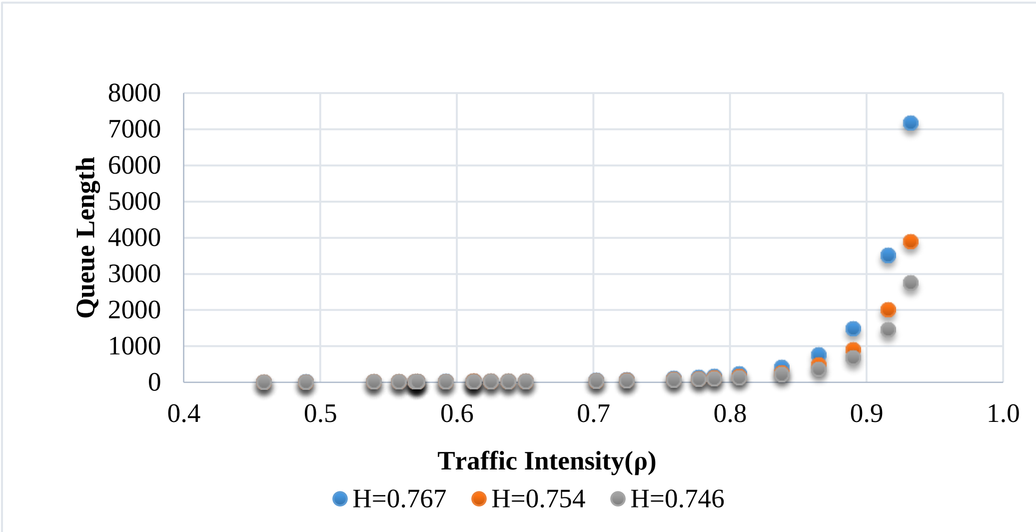

Figure 4(a): Queue length V/s Traffic intensity of traffic data on day 3

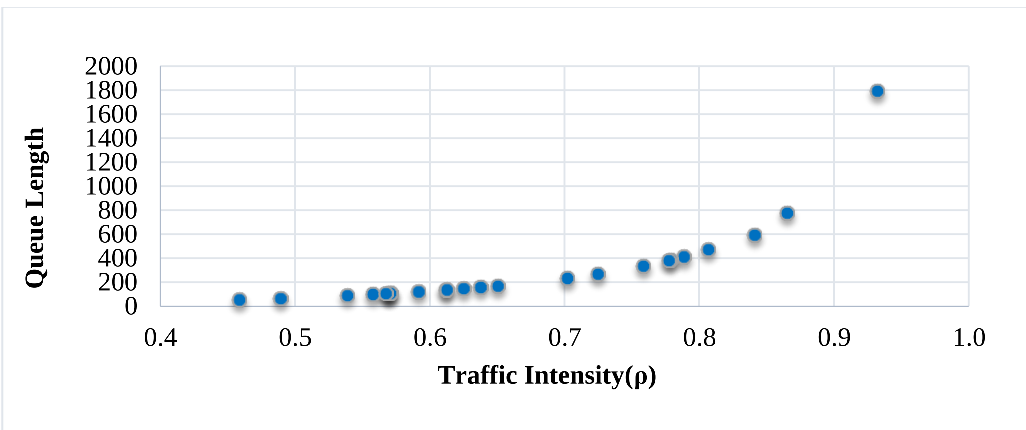

Figure 4(b): Queue length V/s Traffic intensity of traffic data on day 3 (M/M/I)

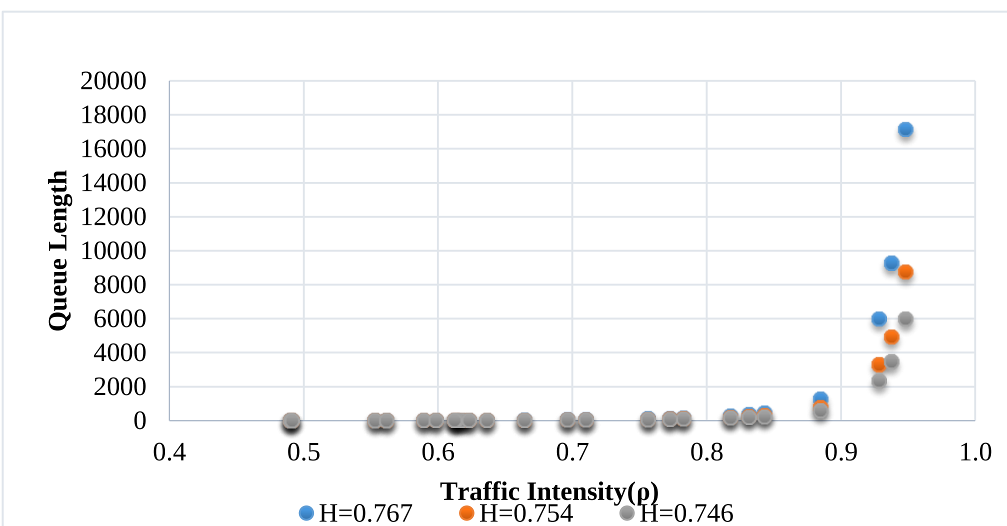

Figure 5(a): Queue length V/s Traffic intensity of traffic data on day 4

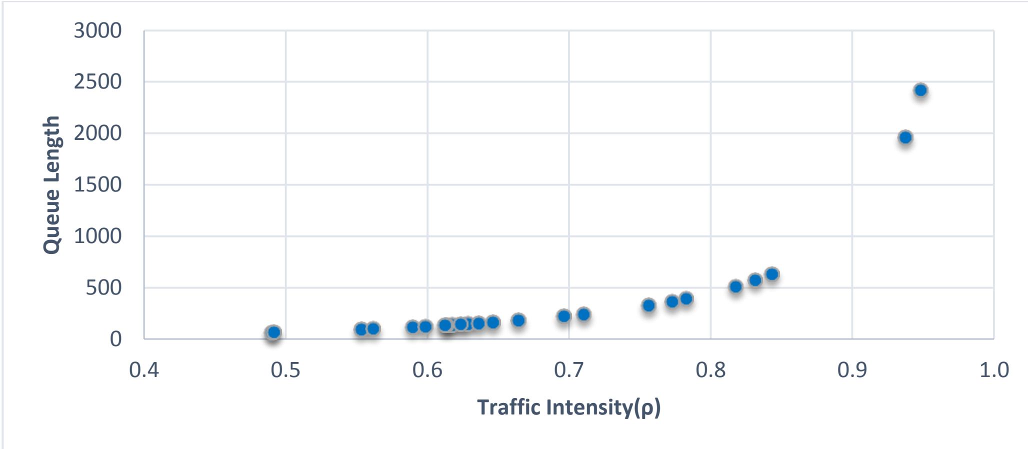

Figure 5(b): Queue length V/s Traffic intensity of traffic data on day 4 (M/M/I)

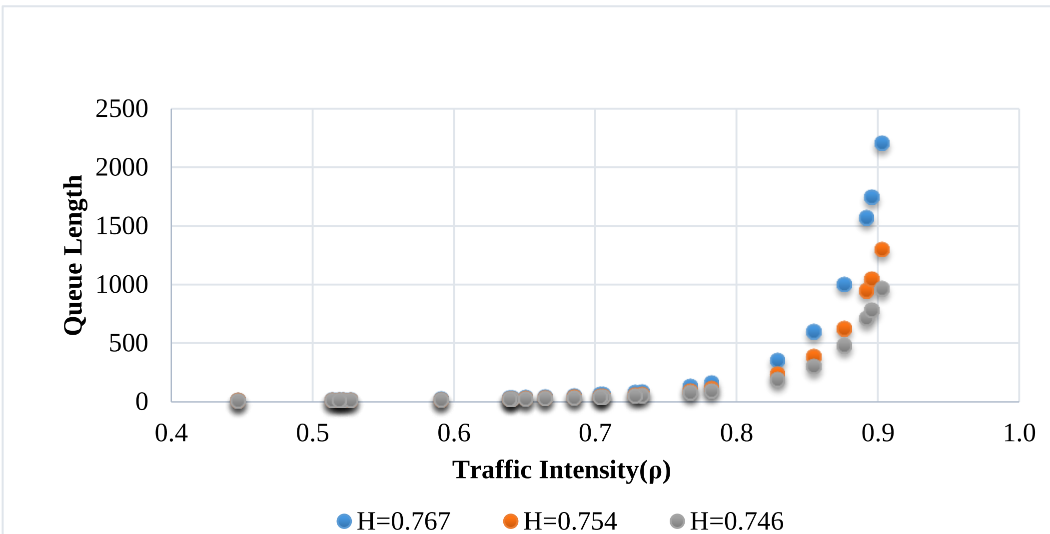

Figure 6(a): Queue length V/s Traffic intensity of traffic data on day 5

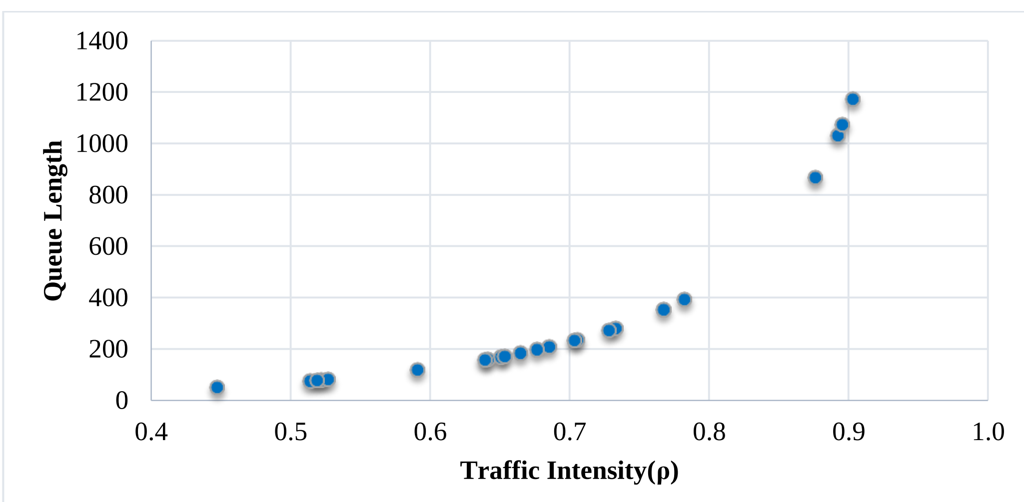

Figure 6(b): Queue length V/s Traffic intensity of traffic data on day 5 (M/M/I)

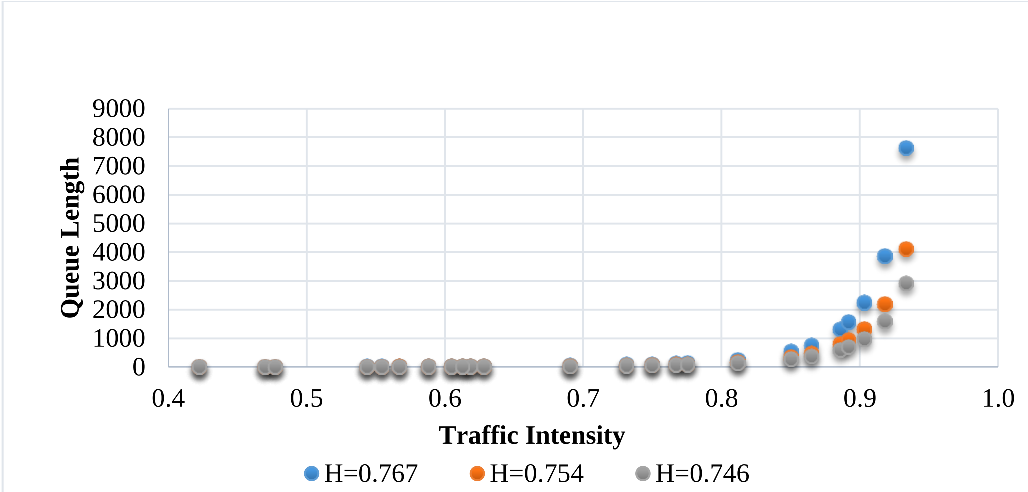

Figure 7(a): Queue length V/s Traffic intensity of traffic data on day 6

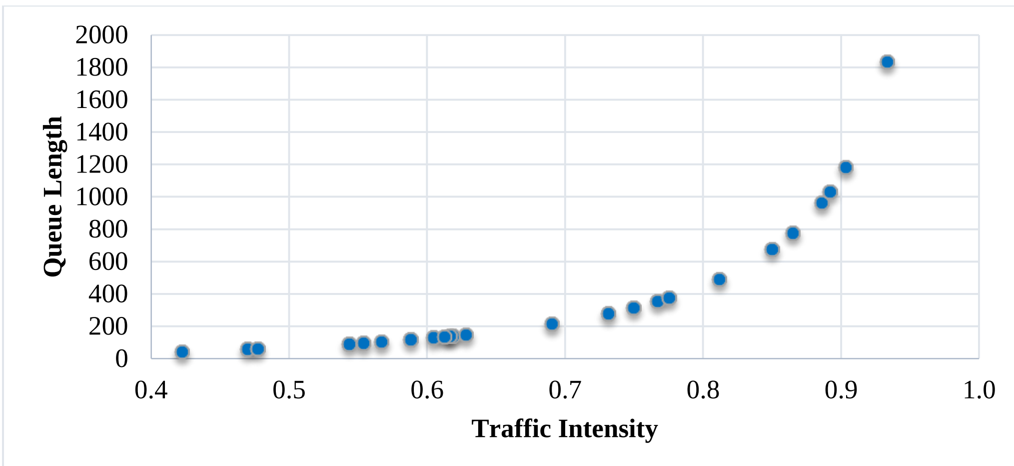

Figure 7(b): Queue length V/s Traffic intensity of traffic data on day 6 (M/M/I)

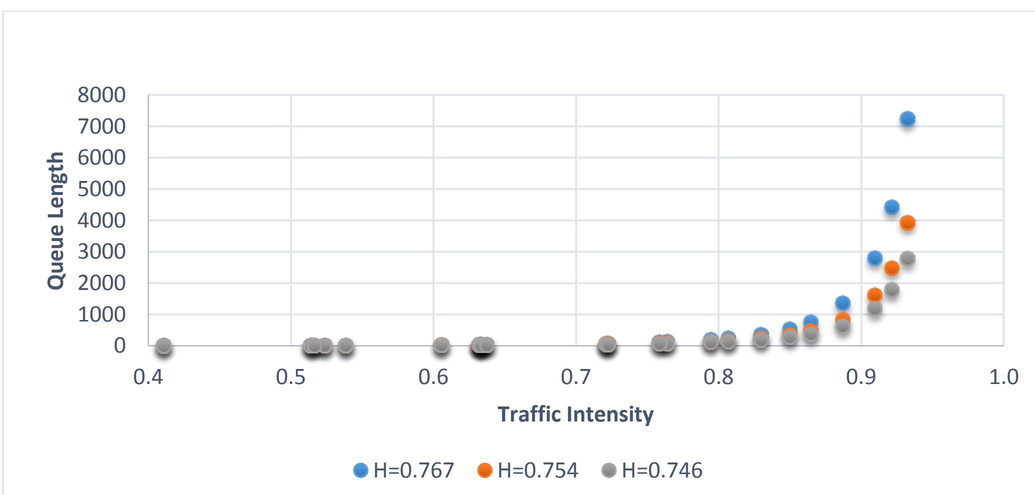

Figure 8(a): Queue length V/s Traffic intensity of traffic data on day 7

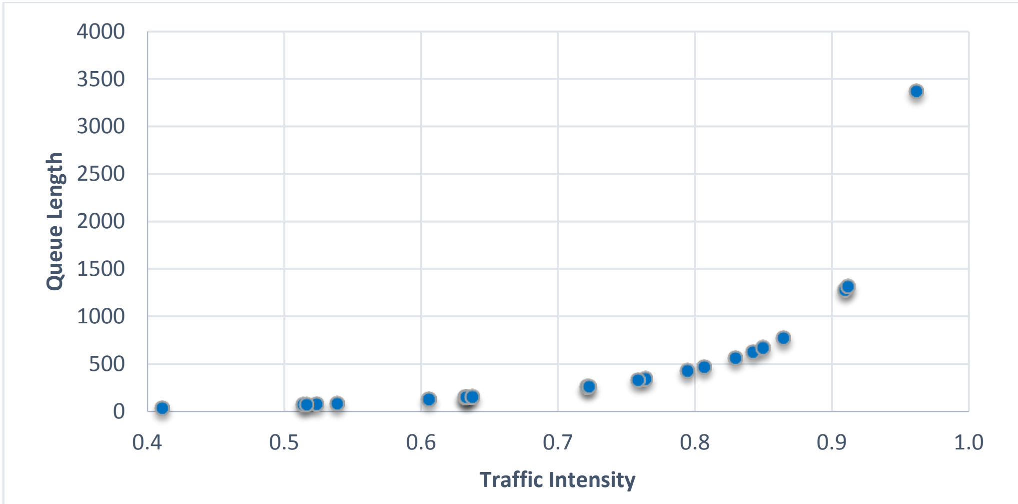

Figure 8(b): Queue length V/s Traffic intensity of traffic data on day 7 (M/M/I)

In the equation (4.3, 4.6) ρ is traffic intensity. Results are depicted in from Fig.2(a)-8 (b), we conclude that as ρ increases mean queue length increases which is expected. Further, as H increases, mean queue length increases by using M/M/1 model than and as well as Queue Length with Hurst Index formulae. This tendency agrees with our intuition.

VI. CONCLUSIONS

In this paper real time highway road traffic on a busy National Highway has been proved to be self-similar. Data intended for the study has been provided by a leading consulting company in India. Different methods to test the self-similarity have been used. The acquired values of Hurst parameter H are reasonably close to each other. Using an empirical formula, mean queue length has been computed against traffic intensity. Numerical results reveal that average queue length increases since ρ and H increase. This sort of study is helpful in development of plans of highways and toll plazas.

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APPENDIX

Table .1: Collection of data on day 1:

Vehicle arrival data and mean queue length with various H values

Vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	1782	2654	0.6714	11.2087	10.4098	9.9608	10.7706
01-02	988	1543	0.6875	11.5756	9.8382	9.7091	8.8732
02-03	703	1322	0.5531	9.3240	8.6723	8.3058	8.6980
03-04	654	771	0.8487	17.0951	17.6020	17.0564	18.3749
04-05	1164	1632	0.7134	11.4898	11.3640	10.8672	11.7761
05-06	2049	3123	0.6214	10.2648	10.0082	9.5789	8.5212
06-07	2918	3846	0.7587	12.6785	12.7373	12.1697	13.2255
07-08	6561	9614	0.5192	8.9657	8.3401	7.9882	20.3434
08-09	12910	15402	0.8382	16.5681	16.7867	16.0002	17.5105
09-10	11500	12904	0.9534	24.6317	24.3172	24.5623	23.3557
10-11	10169	2561	0.397	8.0959	7.5268	7.2069	7.7493
11-12	7796	12341	0.601	9.9481	9.7011	9.2865	10.0248
12-13	6636	10816	0.5768	9.6137	9.4243	9.0230	9.5644
13-14	6478	8253	0.7849	13.6690	13.7712	13.1490	14.3181
14-15	6806	7573	0.9865	30.1234	29.8235	29.7634	24.6144
15-16	6919	11379	0.423	8.2336	7.6568	7.3325	7.8840
16-17	8232	12633	0.6516	10.8028	10.0366	9.6059	9.8765
17-18	10801	14209	0.7601	13.8028	12.7874	13.0347	13.2784
18-19	12091	14319	0.9015	20.5213	20.4321	20.6421	18.0083
19-20	11341	17643	0.4672	8.5234	7.9283	7.5937	21.7645
20-21	8396	10346	0.8115	15.4552	15.0842	15.0724	15.7071
21-22	4956	6721	0.7372	12.0926	12.0310	12.0718	12.4797
22-23	3896	4859	0.8017	14.2016	14.5635	14.3910	26.8710
23-24	2859	3599	0.7943	14.6323	14.2001	13.5549	14.7716

Table 2: Collection of data on day 2

Vehicle arrival data and mean queue length with various H values

vehicles				H=0767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	1667	2814	0.5923	9.8230	9.1336	8.7459	9.6225
01-02	1024	1654	0.6189	10.2241	9.5036	9.0986	10.0236
02-03	916	1206	0.9476	41.7982	38.0738	35.9994	13.8023
03-04	952	1301	0.7314	12.7876	11.8583	11.3363	13.0912
04-05	1282	1930	0.6641	11.0536	10.2673	9.8253	10.8531
05-06	2195	3189	0.6883	11.5947	10.7644	10.2978	22.1387
06-07	2821	3901	0.7231	12.5307	11.6230	11.1130	12.3302
07-08	6799	8180	0.8592	20.1041	18.5260	17.6418	18.5378
08-09	13251	16335	0.8576	19.9395	18.3766	17.5009	32.9641
09-10	11793	14897	0.9263	32.3723	29.6101	28.0682	36.1093
10-11	8908	16356	0.5446	9.2283	8.5837	8.2211	9.0278
11-12	7272	14801	0.4563	8.4451	7.8551	7.5234	8.2446
12-13	7075	8612	0.9067	27.1708	24.9217	23.6646	26.9703
13-14	6233	7839	0.7951	15.3914	14.2382	13.5910	15.1909
14-15	6671	9560	0.7864	14.9521	13.8373	13.2116	11.0452
15-16	6818	11605	0.5875	9.7564	9.0721	8.6873	7.9346
16-17	8382	9616	0.432	8.2868	7.7068	7.3806	16.8870
17-18	10335	17262	0.5987	9.9144	9.2180	8.8264	9.7139
18-19	11683	17557	0.6654	11.0808	10.2922	9.8490	10.8802
19-20	11148	15994	0.697	11.8096	10.9617	10.4852	41.8955
20-21	8191	15210	0.9547	46.6436	46.2098	45.9853	43.7892
21-22	5667	9055	0.6258	10.3377	9.6084	9.1983	10.1372
22-23	4111	8539	0.4814	8.6325	8.0302	7.6914	8.4320
23-24	2945	3184	0.4235	8.2365	7.6595	7.3351	8.0360

Table 3: Collection of data on day 3

Vehicle arrival data and mean queue length with various H values

vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	1941	3044	0.6641	11.0536	10.2673	9.8253	11.0215
01-02	1080	1892	0.8775	19.6451	20.4656	19.4705	34.5460
02-03	950	2072	0.4378	8.3226	7.7404	7.4130	8.2904
03-04	866	1607	0.553	9.3229	8.6712	8.3047	9.2907
04-05	1328	2714	0.4915	8.7153	8.1073	7.7653	8.6831

05-06	2075	2632	0.8312	17.6477	16.2937	15.5346	17.6155
06-07	2678	4377	0.646	10.6961	9.9384	9.5125	32.8977
07-08	6422	11281	0.9193	30.2572	27.7054	26.2802	30.7860
08-09	12864	16515	0.7824	14.7609	13.6628	13.0464	14.7288
09-10	12067	19307	0.6134	10.1365	9.4229	9.0216	25.0976
10-11	9446	13454	0.6962	11.7893	10.9431	10.4675	11.7572
11-12	7736	10677	0.7102	12.1589	11.2822	10.7895	12.1268
12-13	6589	11138	0.5984	9.9101	9.2140	8.8226	9.8779
13-14	6861	10547	0.6121	10.1161	9.4041	9.0037	27.8976
14-15	6956	8041	0.9076	25.7608	23.0912	26.4519	20.2299
15-16	7042	9283	0.7562	13.6519	12.6493	12.0863	13.6197
16-17	8217	13413	0.4092	8.1578	7.5854	7.2635	8.1256
17-18	10752	11534	0.9373	36.5299	33.3479	31.5735	36.4977
18-19	11711	21006	0.5676	9.4971	8.8324	8.4587	9.4649
19-20	11042	14208	0.7725	14.3143	13.2548	12.6600	14.2821
20-21	8323	10316	0.8175	16.6963	15.4277	14.7161	16.6642
21-22	5864	5794	0.948	42.0399	38.2903	36.2020	42.0077
22-23	4536	7947	0.6232	10.2944	9.5685	9.1603	22.8935
23-24	2875	5068	0.6361	10.5155	9.7721	9.3542	10.4833

Table 4: Collection of data on day 4

Vehicle arrival data and mean queue length with various H values

vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	2022	3044	0.918	29.8999	27.3834	25.9778	29.7437
01-02	1168	1892	0.9206	30.6249	28.0367	26.5912	30.4687
02-03	1015	2072	0.4897	8.7002	8.0932	7.7519	8.9254
03-04	889	1607	0.9097	27.8359	25.5221	24.2289	27.6797
04-05	1334	2714	0.4915	8.7153	8.1073	7.7653	8.5591
05-06	2188	2632	0.8312	17.6477	16.2937	15.5346	17.4915
06-07	2828	4377	0.646	10.6961	9.9384	9.5125	10.5399
07-08	6648	11281	0.5893	9.7812	9.0950	8.7091	9.6249
08-09	12922	16515	0.7824	14.7609	13.6628	13.0464	14.6047
09-10	11843	19307	0.8867	23.5501	21.6491	20.5853	26.3184
10-11	9367	13454	0.6962	11.7893	10.9431	10.4675	11.6331
11-12	7583	10677	0.7102	12.1589	11.2822	10.7895	12.0027
12-13	6665	11138	0.4065	8.1435	7.5719	7.2505	9.8726
13-14	6456	10547	0.879	22.4441	20.6477	19.6421	22.2879
14-15	6779	8041	0.843	18.5863	17.1473	16.3407	18.4301
15-16	7020	9283	0.7562	13.6519	12.6493	12.0863	13.4957

16-17	8429	13413	0.8245	17.1657	15.8550	15.1200	25.0900
17-18	10811	11534	0.9373	36.5299	33.3479	31.5735	36.3737
18-19	11797	21006	0.5676	9.4971	8.8324	8.4587	9.3409
19-20	10976	14208	0.7725	14.3143	13.2548	12.6600	14.1581
20-21	8434	10316	0.9313	34.1155	31.1782	29.5393	33.9592
21-22	5493	5794	0.948	42.0399	38.2903	36.2020	41.8837
22-23	4953	7947	0.6232	10.2944	9.5685	9.1603	9.5389
23-24	3224	5068	0.6361	10.5155	9.7721	9.3542	10.3593

Table 5: Collection of data on day 5

Vehicle arrival data and mean queue length with various H values

vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	1917	4895	0.3916	8.0702	7.5025	7.1833	7.9465
01-02	1159	2256	0.5137	8.9131	8.2912	7.9415	8.7894
02-03	1035	1412	0.7325	12.8228	11.8905	11.3668	12.6991
03-04	941	1416	0.8523	19.4205	17.9053	17.0562	22.7840
04-05	1372	1566	0.876	22.0466	20.2875	19.3027	21.9229
05-06	2132	2390	0.8918	24.3596	22.3815	21.2749	24.2359
06-07	2816	3992	0.8472	18.9517	17.4794	16.6543	18.8280
07-08	6495	12450	0.5213	8.9861	8.3590	8.0064	8.8624
08-09	13387	20881	0.6411	10.6055	9.8550	9.4331	10.4818
09-10	11031	16955	0.6506	10.7835	10.0188	9.5890	10.6598
10-11	9409	12028	0.7822	14.7516	13.6542	13.0382	14.6279
11-12	7902	11234	0.8586	20.0414	18.4690	17.5881	19.9177
12-13	6823	12951	0.5268	9.0408	8.4098	8.0549	8.9171
13-14	6743	12999	0.5187	8.9608	8.3356	7.9839	8.5860
14-15	7116	15915	0.4471	8.3825	7.7966	7.4671	8.2588
15-16	7223	9414	0.7672	14.0896	13.0495	12.4655	13.9659
16-17	8672	13274	0.6533	10.8359	10.0670	9.6348	10.7122
17-18	11163	16293	0.8126	16.3867	15.1456	14.4494	17.0453
18-19	12162	20585	0.5908	9.8020	9.1142	8.7275	9.6783
19-20	11372	12597	0.9027	26.3409	24.1724	22.9599	26.2172
20-21	8810	9838	0.8955	24.9908	22.9523	21.8121	24.8671
21-22	6708	9214	0.728	12.6806	11.7603	11.2433	12.5569
22-23	4533	7091	0.6392	10.5710	9.8232	9.4028	15.2345
23-24	3184	4707	0.6764	11.3189	10.5111	10.0570	11.1952

Table 6: Collection of data on day 6

Vehicle arrival data and mean queue length with various H values

vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rate λ	Service rate μ	ρ	$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	2479	3590	0.6904	11.6455	10.8111	10.3421	11.5420
01-02	1653	3518	0.4698	8.5428	7.9464	7.6110	8.4392
02-03	1247	1705	0.731	12.7749	11.8467	11.3252	12.6713
03-04	1238	2001	0.7896	15.1099	13.9813	13.3479	11.0954
04-05	1569	3714	0.4224	8.2301	7.6536	7.3293	8.1266
05-06	2200	4047	0.5256	9.0287	8.3986	8.0442	8.9251
06-07	2379	3101	0.9265	32.4380	29.6692	28.1237	32.8957
07-08	3549	4006	0.8858	23.4140	21.5259	20.4693	23.3104
08-09	6550	10831	0.6047	10.0030	9.2997	8.9043	29.8710
09-10	6946	11265	0.6238	10.3044	9.5776	9.1690	31.8932
10-11	7160	12919	0.5542	9.3367	8.6840	8.3170	30.8945
11-12	6319	7304	0.8651	20.7362	19.0995	18.1827	20.6326
12-13	6417	13444	0.4773	8.6002	8.0000	7.6624	8.4966
13-14	6383	7066	0.9033	26.4615	24.2813	23.0623	26.3579
14-15	6459	7241	0.892	24.3927	22.4115	21.3031	24.2892
15-16	6684	11365	0.5881	9.7646	9.0797	8.6945	9.6610
16-17	6989	8608	0.8119	16.3437	15.1064	14.4123	16.2401
17-18	7643	10193	0.7498	13.4137	12.4314	11.8798	13.3101
18-19	7956	9358	0.8501	19.2148	17.7184	16.8799	19.1112
19-20	7066	9111	0.7755	14.4458	13.3750	12.7738	14.3423
20-21	6037	10647	0.567	9.4897	8.8256	8.4521	9.3861
21-22	4841	5186	0.9334	34.9165	31.8983	30.2147	34.8130
22-23	5473	8931	0.6509	10.7893	10.0242	9.5940	10.0654
23-24	3726	5931	0.6282	10.3783	9.6457	9.2339	10.2747

Table 7: Collection of data on day 7

Vehicle arrival data and mean queue length with various H values

vehicles				H=0.767	H=0.754	H=0.746	
Hours	Arrival rateλ	Service rateμ		$\bar{L}$	$\bar{L}$	$\bar{L}$	L_q
00-01	2194	2872	0.7638	13.9503	12.9222	12.3449	13.9376
01-02	1551	2454	0.6430	10.6403	10.2820	9.8871	10.6276
02-03	1043	2632	0.3962	8.0920	7.8214	7.5232	8.0793
03-04	926	1529	0.5887	9.7729	9.4468	9.0873	9.7602
04-05	1308	2546	0.9286	33.1472	31.7932	30.3073	33.1345
05-06	1777	2464	0.7021	11.9411	11.5325	11.0823	11.9284

06-07	1934	2296	0.9166	29.5262	28.3443	27.0466	26.9978
07-08	2358	2727	0.9576	40.9865	39.0787	38.0976	40.9738
08-09	3158	3472	0.9348	35.4760	34.0096	32.4011	36.4566
09-10	3665	8921	0.4256	8.2487	7.9739	7.6710	8.2360
10-11	4336	6848	0.6331	10.4627	10.1111	9.7235	10.4500
11-12	5237	7247	0.7226	12.5157	12.0844	11.6092	12.5030
12-13	5321	8406	0.6330	10.4609	10.1094	9.7219	10.4482
13-14	5050	5540	0.9115	28.2542	27.1320	25.8995	29.6750
14-15	4903	6077	0.8907	24.1793	23.2451	22.2184	24.1666
15-16	5145	6476	0.7944	15.3548	14.8075	14.2048	15.3421
16-17	5625	6619	0.8670	20.9507	20.1617	19.2941	20.9380
17-18	5855	11182	0.5236	9.0088	8.7098	8.3801	8.9961
18-19	6147	11905	0.4832	8.6470	8.3600	8.0436	8.6343
19-20	5729	8990	0.6372	10.5351	10.1807	9.7902	10.5224
20-21	5312	7004	0.7584	13.7365	13.2560	12.7267	46.9832
21-22	4887	5082	0.9615	50.5094	48.0963	46.0989	50.4967
22-23	4117	4962	0.8297	17.5368	16.8970	16.1928	17.5241
23-24	3287	6105	0.5384	9.1611	8.8568	8.5213	9.1484

