

# An Integral Solution Of Negative Pell’s Equation Involving Two Digit Sphenic Numbers

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**Abstract**— We look for figuring out non-trivial integral solutions of the negative Pell’s equation involving two digit sphenic numbers 30 and 66 for the choice of odd integers including zero. We find some interesting recurrence relations using the solutions.

**Keywords**—Pell equation, Integer Solutions, 2 digit sphenic numbers.

## I. INTRODUCTION

The Pell equation  $x^2 - Ny^2 = 1$  is one of the oldest equations in arithmetic and it is essential to take a look at of quadratic Diophantine equations [1]. This equation has an infinite wide variety of answers was conjectured by using Fermat in 1657 and finally solved through Lagrange [2]. There is no simple formula for the smallest natural number solution and it is not always trivial even to show that it exists.

Sphenic number is a positive integer that is the made from multiplication of three different prime numbers. In this communication we are using two digit sphenic numbers 30 and 66 to form a negative Pell’s equation  $x^2 = 66y^2 - 30^t$  and search for non-trivial integer solutions.

Section I contain the introduction of Pell’s equation and sphenic number, Section II explains the methodology of finding the integer solutions of the negative Pell’s equation  $x^2 = 66y^2 - 30^t$  with numerical examples and recurrence relation satisfied the integer solutions. Section III concludes the research work and future scope for improvement.

## II. METHOD OF ANALYSIS

Consider the negative Pell equation  $x^2 = 66y^2 - 30^t$

For the choice of  $t = 2k + 1$ ,

The Pell equation is

$$x^2 = 66y^2 - 30^{2k+1} \tag{1}$$

Let  $(x_0, y_0)$  be the initial solution of (1) given by

$$x_0 = 6 \cdot 30^k, \quad y_0 = 30^k$$

To find the other solutions of (1), consider the Pell equation

$$x^2 = 66y^2 + 1$$

Here  $\tilde{x}_0 = 65, \tilde{y}_0 = 8$

where initial solution  $(\tilde{x}_n, \tilde{y}_n)$  is given by

$$\tilde{x}_n = \frac{1}{2} f_n$$

$$\tilde{y}_n = \frac{1}{2\sqrt{66}} g_n$$

where

$$f_n = (65 + 8\sqrt{66})^{n+1} + (65 - 8\sqrt{66})^{n+1}$$

$$g_n = (65 + 8\sqrt{66})^{n+1} - (65 - 8\sqrt{66})^{n+1}$$

Implementing of Brahmagupta lemma between  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the sequence of non-zero distinct integer solutions are obtained as

$$x_{n+1} = \frac{30^k}{2} [6f_n + \sqrt{66}g_n]$$

$$y_{n+1} = \frac{30^k}{2\sqrt{66}} [\sqrt{66}f_n + 6g_n]$$

A few numerical examples of the above solutions are exhibited below.

**Table 1**

$n$	$x_{n+1}$	$y_{n+1}$
0	$30^k \cdot 918$	$30^k \cdot 113$
1	$30^k \cdot 119334$	$30^k \cdot 14689$
2	$30^k \cdot 15512502$	$30^k \cdot 1909457$

### Remarkable Observation

The recurrence relation satisfied by means of the solutions of (1) are given by

$$x_{n+3} - 130x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 130y_{n+2} + y_{n+1} = 0$$

### III. CONCLUSION and Future Scope

In this paper, we have presented infinitely many integer solutions for the negative Pell equation  $x^2 = 66y^2 - 30^t$  involving two digit sphenic numbers. As the binary quadratic Diophantine equations are wealthy in range, one may search for the alternative choices of negative Pell equation and determine their integral solutions.

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