IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol. 9, Issue.1, January 2021 E-ISSN: 2347-2693

Φ −Graceful Labeling In the Perspective of Graph Operations on Bistar

Mehul Chaurasiya[1*], Mehul Rupani[2]

[1,2]Department Of Mathematics, Shree H.N.Shukla College Of Science, Rajkot, India

*Corresponding Author:mehulchaurasiya724@gmail.com, Mob No: 9879282804

DOI: https://doi.org/10.26438/ijcse/v9i1.3639 | Available online at: www.ijcseonline.org

Received: 31/Dec/2020, Accepted: 13/Jan/2021, Published: 31/Jan/2021

Abstract: A Φ −graceful labelling of a graph G $= (V, E)$, if a function $f: V(G) \to \{0,1,2, \dots, n-1\}$ is an injective function and the induced function $f^*: E(G) \to N$ is defined by $f^*(e = mn) = 2\{f(m) + f(n)\}$;where $\forall mn \in E(G)$, then the resultant edge labels are distinct[3]. Here we discuss about some basic graph like as splitting graph of bistar$B_{n,m}$,degree splitting graph of bistar$B_{n,m}$,shadow graph of bistar$B_{n,m}$,restricted square graph of bistar $B_{n,m}$, barycentric sub division of bistar$B_{n,m}$and corona product ofbistar$B_{n,m}$with K_1 admits$\Phi -$ graceful labelling.

Keywords: Graceful labeling, Bistar Graph, Injective function

I. INTRODUCTION

Graph labeling is an effective region in graph theory. In graph labeling every vertices are assigned to it's appropriate values for some certain condition. Labeling of vertices and edges play fundamental appearance in graph theory. We make simple, finite graph G with $'p'$ vertices and $'q'$ edges. Some basic information for this graph like as definitions and other things which are used for the current analysis, which are given below.

II.BASIC TERMINOLOGY OF Φ −GRACEFUL LABELING

Definition2.1([3]): If the vertices are attached values to assured conditions then is called as graph labeling. A dynamic survey on different graph labeling methods is regularly updated by Gallian([3]) .

Definition2.2([3]): A Φ −graceful labelling of a graph $G = (V, E)$, if a function $f: V(G) \to \{0,1,2, \dots, n-1\}$ is an injective function and the induced function $f^*: E(G) \to N$ is defined by $f^*(e = mn) = 2\{f(m) + f(n)\}$; where $\forall mn \in E(G)$, then the resultant edge labels are distinct.

Definition2.3([6]): The splitting graph $S'(G)$ of a graph G is getting by add a new vertex u' corresponding to each vertex u of G such that $N(u) = N(u')$.

Definition2.4([6]): Let G be a graph with $V = u_1 \cup u_2 \cup \dots \cup u_m \cup T$, where each u_i is a set of vertices possessing at least two vertices of the equivalent degree and $T = V - \cup_{i=1}^{m} u_i$.The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, \dots, w_n$and attaching to each vertex of u_i for $i \in \{1,2, \dots, m\}$.

Definition2.5([6]): The shadow graph $D_2(G)$ of a connected graph G is composed by taking two copies of G say G' and G''. Attach each vertex u' in G' to the neighbourhood of the comparing vertex u'' in G''.

Definition2.6([6]): The square graph of G for the simple connected graph is denoted by G^2 and defined in the same vertices of graph G and two vertices are adjacent in G^2 if they are distance I or 2 apart in G. Here it is clear that restricted square graph of bistar $B_{n,m}^2$ by attaching all the pendent vertices of $K_{1,n}$with the apex vertex of $K_{1,m}$ and all the pendent vertices of the $K_{1,m}$ with the apex vertex of $K_{1,n}$.

Definition2.7([6]): Let G be a graph and suppose $e = uv$ be an edge of G and m is not a vertex of G. The edge $'e'$ is subdivided when it is succeed by the edge $e' = um$ and $e'' = vm$.

Definition2.8([6]): Let $G = (V, E)$ be a graph, if every edge of graph G is separate, then the resulting graph is called barycentric subdivision of graph G.

Definition2.9([6]): If G is graph of order m, the corona of G with different graph H, $G \odot H$ is the graph obtained by taking are copy of G and m copies of H and joining the j^{th} vertex of G with all edges to all the vertices in the j^{th} copy of H.

III Φ −GRACEFUL LABELING IN THE CONTEXT OF BISATR

Theorem 3.1:$S'(B_{n,m})$ *is a $\Phi -$ graceful graph.*
Proof: Let $B_{n,m}$ be the bistar with vertex set,
$V(B_{n,m}) = \{u_0, v_0, u_j, v_i | 1 \le j \le n, 1 \le i \le m\}$.
Where u_j and v_i are pendent vertices. Let $u_0', v_0', u_j', v_i{}'$ be the newly added vertices in order to obtain

$G = S'(B_{n,m})$. Where $j \in \{1,2,\dots,n\}$ and $i \in \{1,2,\dots,m\}$. We observed that total number of vertices is $2n+2m+4$ and total number of edges is $3n+3m+3$.Without loss of generality assume that $n \le m$ because $S'(B_{n,m})$ and $S'(B_{m,n})$ are isomorphic graphs. Define vertex labeling

$f: V(G) \to \{1,2,\dots,2n+2m+4\}$as follows:

$f(u_0) = 1$

$f(v_0) = 2$

$f(u_0') = 2n+2m+3 \qquad f(v_0') = 4$

$$f(v_i) = \begin{cases} 4i+2, & 1 \le i \le \left\lceil \frac{n+m}{2} \right\rceil \\ 4\left(i - \left\lceil \frac{n+m}{2} \right\rceil\right) + 4, & \left\lceil \frac{n+m}{2} \right\rceil < i \le m \end{cases}$$

$f(v_i') = 2i+1; 1 \le i \le m$

$f(u_j) = 2m+1+2j; 1, \le j \le n$

$f(u_j') = 4\left(m+1-\left\lceil \frac{n+m}{2} \right\rceil\right) + 4j; 1 < j \le n$

So, from above defined function f,the induced function$f^*: E(G) \to N$, which is defined by

Case1: Now for $1 \le j \le n$we have

$f^*(u_0u_j') = 2\{f(u_0) + f(u_j')\}$,

$f^*(u_0u_j) = 2\{f(u_0) + f(u_j)\}$,

$f^*(u_0'u_j) = 2\{f(u_0') + f(u_j)\}$

Again, $1 \le i \le m$ we have

$f^*(v_0v_i') = 2\{f(v_0) + f(v_i')\}$,

$f^*(v_0v_i) = 2\{f(v_0) + f(v_i)\}$,

$f^*(v_0'v_i) = 2\{f(v_0') + f(v_i)\}$and

$f^*(u_0v_0) = 2\{f(u_0) + f(v_0)\}$,

$f^*(u_0v_0') = 2\{f(u_0) + f(v_0')\}$,

$f^*(u_0'v_0) = 2\{f(u_0') + f(v_0)\}$

From above mentioned edge function, it is clear that all edges are getting different edge labels. Hence the $S'(B_{n,m})$is $\Phi-$graceful graph

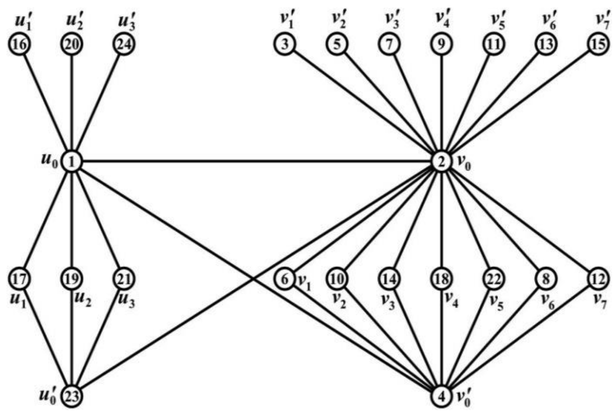

Figure1:$\Phi-$ graceful labeling of $S'(B_{3,7})$

Theorem 3.2:$DS(B_{n,m})$ *is a* $\Phi-$ *graceful graph.*

Proof: Let $B_{n,m}$ be the bistar graph with vertex set

$$V(B_{n,m}) = \{u_0, v_0, u_j, v_i \mid 1 \le j \le n, 1 \le i \le m\}.$$

Where u_j and v_i both are pendent vertices.

Here $V(B_{n,m}) = V_1 \cup V_2$.

Where $V_1 = \{u_j, v_i \mid 1 \le j \le n, 1 \le i \le m\}$ and $V_2 = \{u_0, v_0\}$. Without loss of generality assume that $n \le m$ because $DS(B_{n,m})$ and $DS(B_{m,n})$ are isomorphic graphs. Now in order to obtain $G = DS(B_{n,m})$ from $B_{n,m}$. We consider following two cases…

Case1:$n = m$

We add w_1, w_2 corresponding to V_1, V_2.

Then $|V(G)| = 2m+4$, $|E(G)| = 4m+3$. And

$E(G) = E(B_{m,m}) \cup \{u_0w_2, v_0w_2, u_jw_1, v_jw_1 \mid 1 \le j \le m\}$.

Define vertex labeling $f: V(G) \to \{1,2,\dots,2m+4\}$ as follows: Let p be the largest prime number such that $p < 2m+4$.

$f(u_0) = 2 \qquad f(v_0) = 1$

$f(w_1) = p \qquad f(w_2) = 3$

$f(u_j) = 2j+2; j \in \{1,2,\dots,m\}$

Label the remaining vertices $v_1, v_2, \dots, v_n$ from the set $\{5,7,9,\dots,2m+3,2m+4\} - \{p\}$.

So, from above defined function f, the induced function $f^*: E(G) \to N$, this is defined by

Case A: Now for $1 \le j \le n$we have

$f^*(u_0u_j') = 2\{f(u_0) + f(u_j')\}$,

$f^*(u_0u_j) = 2\{f(u_0) + f(u_j)\}$,

$f^*(u_0'u_j) = 2\{f(u_0') + f(u_j)\}$

Again, $1 \le i \le m$ we have

$f^*(v_0v_i') = 2\{f(v_0) + f(v_i')\}$,

$f^*(v_0v_i) = 2\{f(v_0) + f(v_i)\}$,

$f^*(v_0'v_i) = 2\{f(v_0') + f(v_i)\}$ and

$f^*(u_0v_0) = 2\{f(u_0) + f(v_0)\}$,

$f^*(u_0v_0') = 2\{f(u_0) + f(v_0')\}$,

$f^*(u_0'v_0) = 2\{f(u_0') + f(v_0)\}$

From above mentioned edge function, it is clear that all edges are getting different edge labels. Hence the $S'(B_{n,m})$ is$\Phi-$graceful graph.

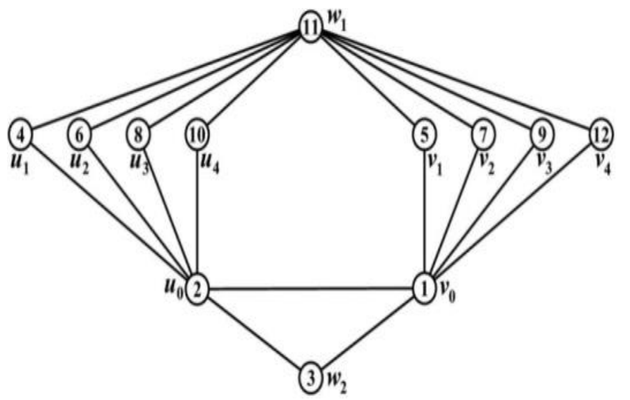

Figure 2:$\Phi-$ graceful labeling of *DS(*$B_{4,4}$)

Case2: $n < m$

We add w_1 to v_1.Then $|V(G)| = n+m+3$

And $|E(G)| = E(B_{n,m}) \cup \{u_jw_1, v_iw_1 \mid 1 \le j \le n, 1 \le i \le m\}$.

So, $|E(G)| = 2n+2m+1$.

Define vertex labeling $f: V(G) \to \{1,2,\dots,n+m+3\}$ as follows: Let q be the largest prime number such that

$q \leq n + m + 3$.

$f(u_0) = 2$ $\quad f(v_0) = 1$ $\quad f(w_1) = q$

$f(u_j) = 2j + 2; 1 \leq j \leq n$.

Label the remaining vertices $v_1, v_2, \ldots, v_m$ from the set $\{3,5,7, \ldots, 2n + 3, 2n + 4, \ldots, n + m + 3\} - \{q\}$

So from above defined function f is the induced function $f^*: E(G) \to N$, which is defined by

Case B: Now for $1 \leq j \leq n$,we have

$f^*(w_1 u_j) = 2\{f(w_1) + f(u_j)\}$,

$f^*(u_0 u_j) = 2\{f(u_o) + f(u_j)\}$

$f^*(w_1 v_i) = 2\{f(w_1) + f(v_i)\}$,

$f^*(v_0 v_i) = 2\{f(v_0) + f(v_i)\}$

and Once again, $f^*(u_0 v_0) = 2\{f(u_0) + f(v_0)\}$.

From above mentioned edge function in both cases, it is clear that all edges are getting different edge labels. Hence $DS(B_{m,n})$ is a Φ −graceful graph.

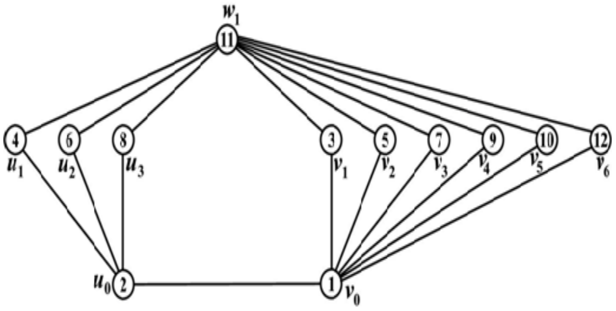

Figure 3:$\Phi -$ graceful labeling of *DS*($B_{3,6}$)

Theorem 3.3:*Restricted $B^2_{n,m}$ is a Φ −graceful graph.*

Proof: Let $B_{n,m}$ be the bistar graph with vertex set $V(B_{n,m}) = \{u_0, v_0, u_j, v_i | 1 \leq j \leq n, 1 \leq i \leq m\}$.

Where u_j and v_i are pendent vertices, let G be the restricted $B^2_{n,m}$ graph with $V(G) = V(B_{n,m})$ and $E(G) = E(B_{n,m}) \cup \{u_0 u_j, u_0 v_i | 1 \leq j \leq n, 1 \leq i \leq m\}$.

Note that $|V(G)| = n + m + 2$ and $|E(G)| = 2n + 2m + 1$.Without loss of generality we can assume that $n \leq m$ because restricted $B^2_{m,n}$ are isomorphic graphs.

Define vertex labeling $f: V(G) \to \{1,2, \ldots, n + m + 2\}$ as follows: Let p be the largest prime number such that $p \leq n + m + 2$.

$f(u_0) = p$ $\quad f(v_0) = 1$

$f(u_j) = 2j; 1 \leq j \leq n$.

Label the remaining vertices $v_1, v_2, \ldots, v_m$ from the set $\{3,5,7, \ldots, 2n + 1, 2n + 2, \ldots, n + m + 2\} - \{p\}$

So from above defined function f,is the induced function$f^*: E(G) \to N$, which is defined by

$f^*(v_0 v_i) = 2\{f(v_0) + f(v_i)\}$, where $1 \leq i \leq m$

$f^*(u_0 v_0) = 2\{f(u_0) + f(v_0)\}$,

Now for $1 \leq j \leq n$,

$f^*(v_0 u_j) = 2\{f(v_0) + f(u_j)\}$,

$f^*(u_0 u_j) = 2\{f(u_0) + f(u_j)\}$

From above mentioned edge function, it is clear that all edges are getting different edge labels.

Hence $(B^2_{n,m})$ is a Φ −graceful graph.

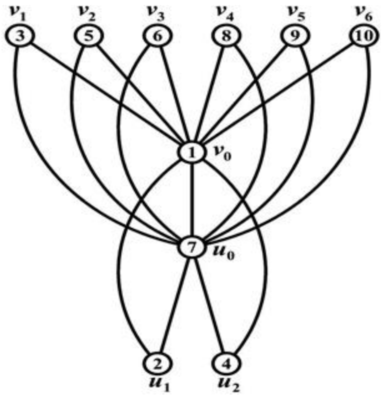

Figure 4:$\Phi -$ graceful labeling of $B^2_{2,6}$

Theorem 3.4:*The barycentric subdivision $S(B_{n,m})$ of bistar $B_{n,m}$ is a Φ −graceful graph.*

Proof: Let $B_{n,m}$ be the bistar graph with vertex Set $V(B_{n,m}) = \{u_0, v_0, u_j, v_i | 1 \leq j \leq n, 1 \leq i \leq m\}$. Where u_j and v_i both are pendent vertices and edge set $E(B_{n,m}) = \{u_0 v_0, u_0 u_j, v_0 v_i | 1 \leq j \leq n, 1 \leq i \leq m\}$.

Let $w_0, w_1, w_2, \ldots, w_n, w'_1, w'_2, \ldots, w'_m$ be the newly added vertices to obtain $G = S(B_{n,m})$.Where $w_0$is added between u_0 and v_0,w_j is added between u_0 and u_j for $j \in \{1,2, \ldots, n\}$ and w'_i is added between v_0 and v_i for $i \in \{1,2, \ldots, m\}$. Here note that $|V(G)| = 2n + 2m + 3$ and $|E(G)| = 2n + 2m + 2$.

Define vertex labeling $f: V(G) \to \{1,2, \ldots, 2n + 2m + 3\}$ as follows:

$f(u_0) = 2$ $\quad f(v_0) = 1$ $\quad f(w_0) = 3$.

for $j \in \{1,2, \ldots, n\}$;

$f(u_j) = 2j + 3$ and $\quad f(w_j) = 2j + 2$

Now for $i \in \{1,2, \ldots, m\}$;

$f(v_i) = 3 + 2n + 2i$and $f(w'_i) = 2 + 2n + 2i$.

So from above defined function f is the induced function$f^*: E(G) \to N$, which is defined by

$f^*(u_0 w_0) = 2\{f(u_0) + f(w_0)\}$,

$f^*(v_0 w_0) = 2\{f(v_0) + f(w_0)\}$

for $1 \leq j \leq n$, we have

$f^*(u_0 w_j) = 2\{f(u_0) + f(w_j)\}$,

$f^*(w_j u_j) = 2\{f(w_j) + f(u_j)\}$

for $1 \leq i \leq m$, we have

$f^*(v_0 w'_i) = 2\{f(v_0) + f(w'_i)\}$,

$f^*(w'_i v_i) = 2\{f(w'_i) + f(v_i)\}$

From above mentioned edge function, it is clear that all edges are getting different edge labels. Hence $S'(B_{n,m})$ is Φ −graceful graph.

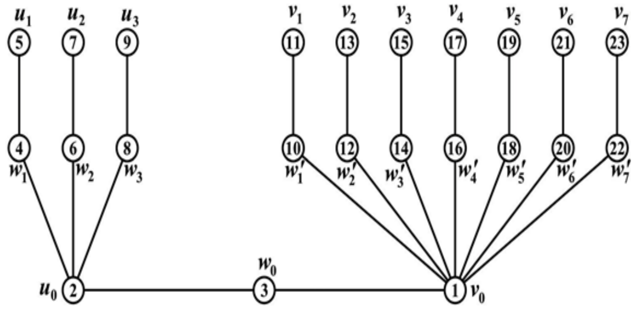

Figure 5: $\Phi -$ graceful labeling of *S*($B_{3,7}$).

Theorem 3.5*: $B_{n,m} \odot K_1$ is a Φ −graceful graph.*

Proof: Let $B_{n,m}$ be the bistar graph with vertex set $V(B_{n,m}) = \{u_0, v_0, u_j, v_i | 1 \le j \le n, 1 \le i \le m\}$.

Where u_j and v_i are pendent vertices, let $u'_0, u'_1, u'_2, \dots, u'_n, v'_0, v'_1, v'_2, \dots, v'_m$ be the newly added vertices to obtain the graph $G = B_{n,m} \odot K_1$.

We note that $V(G) = V(B_{n,m}) \cup \{u'_0, u'_j, v'_0, v'_i | 1 \le j \le n, 1 \le i \le m\}$, and $E(G) = E(B_{n,m}) \cup$
$\{u_0u'_0, v_0v'_0, u_ju'_j, v_iv'_i | 1 \le j \le n, 1 \le i \le m\}$.

Hence $|V(G)| = 2n + 2m + 4$ and

$|E(G)| = 2n + 2m + 3$.

Define vertex labeling

$f: V(G) \to \{1,2, \dots, 2n + 2m + 4\}$as follows:

$f(u_0) = 1$ $\quad f(v_0) = 2$

$f(v'_0) = 3$ $\quad f(u'_0) = 2n + 2m + 4$

for$j \in \{1,2, \dots, n\}$

$f(u_j) = 2 + 2j$, $\quad f(u'_j) = 1 + 2j$

Now for $i \in \{1,2, \dots, m\}$, $f(v_i) = 2 + 2n + 2i$

And $f(v'_i) = 3 + 2n + 2i$

So from above defined function f, is the induced function $f^*: E(G) \to N$, which is defined by

$f^*(u'_0u_0) = 2\{f(u'_0) + f(u_0)\}$,

$f^*(u_0v_0) = 2\{f(u_0) + f(v_0)\}$,

$f^*(v'_0v_0) = 2\{f(v'_0) + (v_0)\}$

For $1 \le j \le n$, we have

$f^*(u_0u_j) = 2\{f(u_0) + f(u_j)\}$,

$f^*(u_ju'_j) = 2\{f(u_j) + f(u'_j)\}$

For $1 \le i \le m$, we have

$f^*(v_0v_i) = 2\{f(v_0) + f(v_i)\}$,

$f^*(v_iv'_i) = 2\{f(v_i) + f(v'_i)\}$

From above mentioned edge function, it is clear that all edges are getting different edge labels. Hence $S'(B_{n,m})$is Φ −graceful graph.

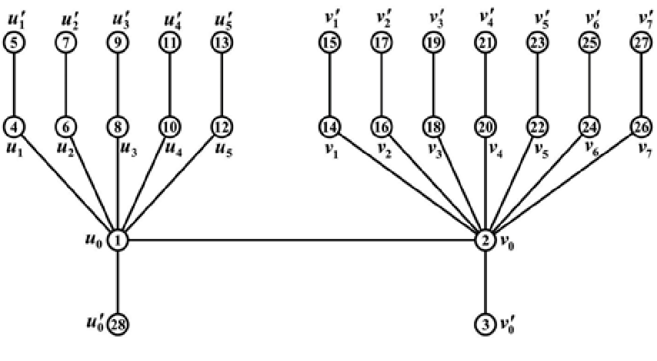

Figure 6:Φ − graceful labeling of $B_{5,7} \odot K_1$.

IV. CONCLUSION

In this paper we have to show that splitting graph of bistar graph, degree splitting graph of bistar graph, shadow graph of bistar graph, barycentric subdivision of bistar graph and corona product graph of bistar $B_{n,m}$ with K_1 admits a $\emptyset$ − graceful labeling.

REFERENCE

[1]. M.I.Bosamia, K.K.Kanani, "Divisor cordial labeling in the context of graph operations on Bistar", Global Journal of Pure and Applied Mathematics, **Volume 12, Number 3, pp.2605-2618, 2016.**

[2]. I.Cahit, Cordial Graphs, "Aweakerversion of graceful and harmonious graphs", Ars Combinatoria, **23, pp.201–207, 1987**

[3]. M.Chaurasiya, M.Rupani, S.Khunti, "Φ −graceful labeling of merge graph,bistar graph,friendship graph,C_d^n relation graph", International journal of Technical Innovation in Modern Engineering & Science,**Vol.6, Issue 11, pp.7-14, Nov-2020.**

[4]. J.A. Gallian, "A Dynamic Survey of graph labeling", The Electronics Journal of Combinatorics, **Dec 20, 2013.**

[5]. J. Gross and J. Yellen, Handbook of Graph Theory, CRC Press, **2004**.

[6]. R.Uma ,S.Divya , "Cube difference labeling of star related graphs", International Journal Of Scientific Research In Computer Science, Engineering and Information Technology, **2(4), pp.298- 301, 2017.**

[7]. R.Varatharajan, S.Navanaeethakrishnan, K.Nagarajan, "Divisor cordial graphs", International Journal of Mathematical Combinatorics, **4, pp.15–25, 2011.**

AUTHORS PROFILE

Mr.Mehul Chaurasiya pursed Bachelor of Science from University of Saurashtra, Rajkot in 2011,Master of Science from Saurashtra University in year 2013 and Master of Philosophy in 2016 from Saurashtra University. He is currently pursuing Ph.D. and currently working as Assistant Professor in Department of Mathematics, In Shree H.N.Shukla College Of Science Rajkot,Gujarat-India.

Dr. Mehul P Rupani pursed Bachelor of Science and Master of Science. And currently working as Dean Of Saurashtra in science faculty.And he is also syndicate member of Saurashtra University Rajkot,Gujarat-India.

