IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Issue-1, Jan 2019 E-ISSN: 2347-2693

Reduction Method Using Minimum Supply And Demand Method to Find an Initial Basic Feasible Solution of Transportation Problem

N.M. Morade

Dept. of Mathematics, MVP Samsj's, Arts, Commerce and Science College, Dindori, Savitribai Phule Pune University, Nashik, India.

Corresponding Author: madhu_nirmala@rediffmail.com

Available online at: www.ijcseonline.org

Accepted:23/Jan/2019, Published:31/Jan/2019

Abstract— Transportation Problem plays an important role in our economy and managerial decision- making. The main objective of transportation problem solving method is to obtain an optimal solution. An initial basic feasible solution is the first step to obtain an optimal solution for the transportation problems. Among the existing methods, Vogel's Approximation Method gives an initial basic feasible solution near to the optimal solution, but it is very expansive in term of the execution of time. This paper introduces a new method, Reduction Method using minimum supply & demand method, to find an initial basic feasible solution of Transportation Problem. This method is easy to apply and fast compared to Vogel's Approximation Method. It gives better initial basic feasible solution compared to all existing prominent methods. The method is also illustrated with numerical examples.

Keywords— Linear Programming, Assignment, Transportation problem, initial basic feasible solution, Reduction method using Supply & Demand method.

I. INTRODUCTION

Transportation problem (TP) is a subclass of Linear Programming Problem (LPP) and Assignment Problem (AP) is a subclass of TP. Hungarian Method is used to solve AP. The origin of the Transportation methods dates back to 1941 when F.L. Hitchcock presented a study entitled "The distribution of a product from several sources to Numerous Localities". In 1947, T.C. Koopmans presented an independent study not related to Hitchcock's called "optimal utilization of the Transportation system". These two contributions are mainly responsible for the developments of Transportation models which involve a number of shipping sources and a number of destinations [1], [4], [5],[6]. Finally, placed in the framework of linear programming and solved by the simplex method by G. B. Dantzig in 1951. The solution of TP consists of two phases, first, obtain an IBFS by various methods and then the solution is tested for optimality and revised if necessary by using MODI Method (Modified Distribution Method) or the Stepping Stone method. The existing prominent methods to find an IBFS are the North-West Corner Rule (NWCR), Least Cost Method (LCM), and Vogel's Approximation Method (VAM). These methods are simple and reliable. They are easy to compute, understand & interpret. But among these methods, only VAM gives better IBFS but it is expensive in terms of execution of time.

In this paper, a new method, reduction method using minimum supply & demand method is proposed to find IBFS. The proposed method gives IBFS in minimum computation time compared to VAM. Also it gives directly optimal solution or a solution very close to optimal solution. As well as the obtained IBFS is compared with the existing prominent methods.

II. SOME DEFINITIONS

Balanced TP: If the total supply =total demand then the TP is called balanced otherwise TP is unbalanced.
Feasible Solution (FS): A set of non–negative individual allocations ($x_{ij} \geq 0$) which simultaneously removes deficiencies is called a feasible solution.
Basic Feasible Solution (IBFS): A feasible solution to m origins and n destinations problem is said to be basic if the number of positive allocations is m+n-1.
Optimal solution: A feasible solution is said to be optimal if it minimizes the total transportation cost [1], [4], [5],[6].

III. MINIMUM SUPPLY & DEMAND METHOD

Steps:
1. Make the TP balanced if it is not.
2. In balanced TP, select the row or column with minimum supply or demand. If there is a tie, select the row or

column that contains a cell with minimum transportation cost (TC). In selected row or column, select a cell say (i, j) with minimum TC.

3. Allocate x_{ij}= minimum (a_i, b_j) in the (i, j) cell.

I) If x_{ij} = b_j, cross out the j^{th} column of Transportation table and decrease a_i by b_j. Go to Step 2.

II) If x_{ij} = a_i, cross out the i^{th} row of Transportation table and decrease b_j by a_i. Go to Step 2.III) If x_{ij} = a_i = b_j, cross out either the i^{th} row or j^{th} column or both of Transportation table and decrease a_i by bj. Repeat steps 2 and 3 for reduced TP until all the requirements are satisfied.

Note: Whenever the minimum cost is not unique, make an arbitrary choice among the minimum cost [3].

Numerical Example: Consider the transportation problem.

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Demand	5	8	7	14	34

Transportation costs per unit of production from particular three origins to different four destinations are given in the above Table.

Solution by Minimum Supply & Demand Method:

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	[5] 19	30	50	[2] 10	7-5=2←
F2	70	[2] 30	[7] 40	60	9-7=2←
F3	40	[6] 8	70	[12] 20	18-6=12 12-12=0
Demand	5 ↑	8-2=6 ↑	7 ↑	14-2 =12↑	34

So, total TC =5x19+2x10+2x30+7x40+6x8+12x20=743. [3]

IV. NEW METHOD: Reduction Method using minimum supply and Demand method

Steps:

1) First, make TP balanced if not.
2) To obtain the I^{st} reduced matrix, subtract the minimum element (cost) of each row of the effectiveness matrix (balanced TP) from all the elements of the respective rows.
3) To obtain the II^{nd} reduced matrix, Subtract the minimum element (cost) of each column of the I^{st} reduced matrix from all elements of the respective columns.
4) Due do Steps 2) and 3) each row and each column of the II^{nd} reduced matrix has at least one zero entry.
5) In II^{nd} reduced matrix find IBFS using Minimum Supply & Demand method. The IBFS so obtained is optimal solution if it is the unique solution. Otherwise, we get more than one solution depending upon the choice of selection of cell which is near to an optimal solution. To find total transportation cost consider original transportation cost.

V. NUMERICAL EXAMPLE

Example: 1. Consider the transportation problem.

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Demand	5	8	7	14	34

Solution by Reduction method using Minimum Supply & Demand Method

Step i) I^{st} Reduced Matrix is

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	9	20	40	0	7
F2	40	0	10	30	9
F3	32	0	62	12	18
Demand	5	8	7	14	34

Step ii) II^{nd} Reduced Matrix is

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	0	20	30	0	7
F2	31	0	0	30	9
F3	23	0	52	12	18
Demand	5	8	7	14	34

Step iii) Find IBFS by Minimum Supply & Demand Method using second reduced matrix.

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	[5] 0	20	30	[2] 0	7-5=2 ←
F2	31	[2] 0	[7] 0	30	9-7=2←
F3	23	[6] 0	52	[12] 12	18-6=12←
Demand	5 ↑	8-2=6 ↑	7 ↑	14-2→ 12-12=0	34

Therefore, IBFS is

Factory (origins)	Destinations (warehouse)				Supply
	W1	W2	W3	W4	
F1	[5] 19	30	50	[2] 10	7
F2	70	[2] 30	[7] 40	60	9
F3	40	[6] 8	70	[12] 20	18
Demand	5	8	7	14	34

So, Total TC =5x19+2x10+2x30+7x40+6x8+12x20=743.

Example: 2

Destinations → Origins↓	1	2	3	Supply ↓
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Requirement →	7	9	18	34

Solution by Reduction method using Minimum Supply & Demand Method:

Step i) Ist Reduced Matrix is

Destinations → Origins↓	1	2	3	Supply ↓
1	0	5	2	5
2	2	2	0	8
3	1	0	3	7
4	0	5	1	14
Requirement →	7	9	18	34

Step ii) IInd Reduced Matrix is

Destinations → Origins↓	1	2	3	Supply ↓
1	0	5	2	5
2	2	2	0	8
3	1	0	3	7
4	0	5	1	14
Requirement →	7	9	18	34

Step iii) Find IBFS by Minimum Supply & Demand Method using second reduced matrix.

Destinations → Origins↓	1	2	3	Supply ↓
1	[5] 0	5	2	5←
2	2	[2] 2	[6] 0	8-2=6←
3		[7] 0	3	7←
4	[2] 0	5	[12] 1	14-2=>12-12=>0
Requirement →	7-5= 2↑	9-7=2↑	18-6=12↑	34

So, Total TC =2x5+3x2+1x6+4x7+1x2+2x12=76.

Example: 3.

Destinations → Origins↓	1	2	3	4	Supply ↓
1	1	2	1	4	30
2	3	3	2	1	50
3	4	2	5	9	20
Requirement →	20	40	30	10	

Solution by Reduction method using Minimum Supply & Demand Method:

Step i) Ist Reduced Matrix is

Destinations → Origins↓	1	2	3	4	Supply ↓
1	0	1	0	3	30
2	2	2	1	0	50
3	2	0	3	7	20
Requirement →	20	40	30	10	

Step ii) IInd Reduced Matrix is

Destinations → Origins↓	1	2	3	4	Supply ↓
1	0	1	0	3	30
2	2	2	1	0	50
3	2	0	3	7	20
Requirement →	20	40	30	10	

Step iii) Find IBFS by Minimum Supply &Demand Method using second reduced matrix.

Destinations → Origins↓	1	2	3	4	Supply ↓
1	[20]0	1	[10] 0	3	30-20=10 ←
2	2	[20] 2	[20] 1	[10] 0	50-10=40 ← 40-20=20 ←
3	2	[20] 0	3	6	20←
Requirement →	20 ↑	40-20=20 →20-20=0	30-10 =20↑	10↑	

So, Total TC =20x1+10x1+20x3+20x2+10x1+20x2=180

Example: 4.

Destinations → Origins↓	1	2	3	Supply ↓
1	50	30	220	1
2	90	45	170	3
3	250	200	50	4
Requirement →	4	2	2	

Solution by Reduction method using Minimum Supply & Demand Method: **Step i) I^st Reduced Matrix is**

Destinations → Origins↓	1	2	3	Supply ↓
1	20	0	190	1
2	45	0	125	3
3	200	150	0	4
Requirement →	4	2	2	

Step ii) II^nd Reduced Matrix is

Destinations → Origins↓	1	2	3	Supply ↓
1	0	0	190	1
2	15	0	125	3
3	180	150	0	4
Requirement →	4	2	2	

Step iii) Find IBFS by Minimum Supply and Demand Method using second reduced matrix.

Destinations → Origins↓	1	2	3	Supply ↓
1	0	[1] 0	190	1←
2	[2] 15	0	125	3-1=2←
3	[2] 180	150	[2] 0	4-2=2→2-2=0
Requirement →	4-2=2 ↑	2-1=1 ↑	2 ↑	

So, Total TC =30x1+90x2+45x1+250x2+50x2=855

OR

Destinations → Origins↓	1	2	3	Supply ↓
1	[1] 0	0	190	1←
2	[1] 15	[2] 0	125	3-2=1←
3	[2] 180	150	[2]0	4-2=2←
Requirement →	4-1=3→3-1 = 2 →2-2=0	2 ↑	2 ↑	

So, Total TC =50x1+90x1+45x2+250x2+50x2=830.

Example: 5.

Destinations → Origins↓	1	2	3	4	Supply ↓
1	1	2	3	4	6
2	4	3	2	0	8
3	0	2	2	1	10
Requirement →	4	6	8	6	24

Solution by Reduction method using Minimum Supply & Demand Method:**Step i) I^st Reduced Matrix is**

Destinations → Origins↓	1	2	3	4	Supply ↓
1	0	1	2	3	6
2	4	3	2	0	8
3	0	2	2	1	10
Requirement →	4	6	8	6	24

Step ii) II^nd Reduced Matrix is

Destinations → Origins↓	1	2	3	4	Supply ↓
1	0	0	0	3	6
2	4	2	0	0	8
3	0	1	0	1	10
Requirement →	4	6	8	6	24

Step iii) Find IBFS by Minimum Supply & Demand Method using second reduced matrix.

Destinations → Origins↓	1	2	3	4	Supply ↓
1	[4] 0	[2] 0	0	3	6-4=2←
2	4	2	[2] 0	[6] 0	8-2=6←
3	0	[4] 1	[6] 0	1	10-4=6←
Requirement →	4 ↑	6-2=4 ↑	8-6=2 ↑	6	24

So, Total TC =4x1+2x2+2x2+6x0+4x2+6x2=32

OR

Destinations → Origins↓	1	2	3	4	Supply ↓
1	0	[6] 0	0	3	6-6=0
2	4	2	[2] 0	[6] 0	8-2=6←
3	[4] 0	1	[6] 0	1	10-4=6←
Requirement →	4 ↑	6 ↑	8-6=2 ↑	6-6=0	24

So, Total TC =6x2+2x2+6x0+4x0+6x2=28

Comparison Table:

Ex.	TC by				Optimal Solution by MODI
	VAM	NWCR	LCM	Reduction method using minimum supply &	

				demand method	
EX. 1	779	1015	814	743	743
EX. 2	80	102	83	76	76
EX. 3	180	310	200	180	180
EX. 4	820	820	855	830,855	820
EX. 5	28	32	28	28,32	28

VI. RESULTS AND DISCUSSION

The comparison table shows a comparison of the TC obtained by Reduction method using minimum supply & demand method and the existing methods along with the optimal solution by means of the above sample examples. The comparative study shows that the proposed method gives optimal solution or a solution near to optimal solution in comparison with the other existing methods. The Reduction method using minimum supply & demand method is simple and reliable. It is easy to compute, understand and thus saves time compared to VAM . If the initial basic feasible solution obtained by the reduction method using minimum supply & demand method is unique then it is optimal. But sometimes we get more than one initial basic feasible solution with different transportation cost depending on choice of selection or in case of a tie. If we get more than one IBFS, then the IBFS with minimum cost may be optimal or very close to it.

VII. CONCLUSION AND FUTURE SCOPE

This paper introduces a new method, Reduction method using Supply & Demand Method to find IBFS of TP. It is easy and simplest method compared to VAM. It gives directly optimal solution or a solution which is very close to the optimal solution. This method gives better solution and computational complexity is low compared to VAM. It is excellent method compared to the prominent methods. Therefore this method can be applied in real transportation problems.

REFERENCES

[1] KAPOOR V. K., "Operations Research (Quantitative Techniques for management)", Sultan Chand & Sons Publisher, India ,pp. 5.3-5.97, 2008.
[2] M. K. Hasan, "Direct Methods for Finding Optimal Solution of a Transportation Problem are not Always Reliable", International Refereed Journal of Engineering and Science , Vol. 1, Issue. 2, pp.46-52, 2012.
[3] N. M. Morade, "New Method to find initial basic feasible solution of Transportation Problem using MSDM" , International Journal of Computer Sciences and Engineering, Vol.5, issue.12, pp.223-226, 2017.
[4] P.K.Gupta and D.S.Hira, " Operations Research", Sultan Chand & Sons Publisher , India, pp. 148 -210,1997.
[5] P.K. Gupta and M. Mohan, "Problems in Operations Research", Sultan Chand & Sons Publisher, India, pp. 337 -400, 1997.
[6] S.D. Sharma, "Operations Research Theory, Methods and Applications", Kedar Nath Ram Nath & Co., India, pp. 347 -434, 2003.

Authors Profile

Smt.N M Morade pursed Bachelor of Science from University of Pune, India in 1997 and Master of Science from Pune University in year 1999. She is currently working as Assistant Professor in Department of Mathematics, Arts, Commerce and Science College, Dindori, Savitribai Phule Pune university, India since 1999. She has 19 years of teaching experience and 5 years of Research Experience.

