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Analytical Treatment for Solving a Class of Non Linear Fractional Differential Equations

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Abstract— In the present paper, generalized differential transform method is used for obtaining the approximate analytic solutions of non-linear partial differential equations of fractional order. The fractional derivatives are described in the Caputo sense.

Keywords— Fractional differential equations; Caputo fractional derivative; Generalized Differential transform method; Analytic solution .

Mathematical Subject Classification (2010) — 26A33, 34A08, 35A22, 35R11, 35C10, 74H10.

I. INTRODUCTION

The fractional order Differential equations are generalizations of integer order classical differential equations and it is valuable tools in the modeling of many physical phenomena in various fields of science and engineering.

Recently, various analytical and numerical methods have been employed to solve linear and nonlinear fractional differential equations. The differential transform method (DTM) was proposed by Zhou [1] to solve linear and nonlinear initial value problems in electric circuit analysis. DTM constructs an analytical solution in the form of a polynomial and different from the traditional higher order Taylor series method. For solving two-dimensional linear and nonlinear partial differential equations of fractional order DTM is further developed as Generalized Differential Transform Method (GDTM) by Momani, Odibat, and Erturk in their papers [2-4].

II. GENERALIZED DIFFERENTIAL TRANSFORM METHOD

Consider a function of two variables $u(x,y)$ be a product of two single-variable functions, i.e.

$$u(x,y)=f(x)g(y),$$

which is analytic and differentiated continuously with respect to x and y in the domain of interest. Then the generalized two-dimensional differential transform [2-4] of the function $u(x,y)$ is

$$U_{\alpha,\beta}(k,h)=\frac{1}{\Gamma(\alpha k+1)\Gamma(\beta h+1)}\left[\left(D_{x_0}^{\alpha}\right)^k\left(D_{y_0}^{\beta}\right)^h u(x,y)\right]_{(x_0,y_0)} \quad (1)$$

where $0<\alpha,\beta\le 1; U_{\alpha,\beta}(k,h)=F_\alpha(k)G_\beta(h)$ is called the spectrum of $u(x,y)$ and

$$\left(D_{x_0}^{\alpha}\right)^k = D_{x_0}^{\alpha}, D_{x_0}^{\alpha}, \ldots\ldots\ldots, D_{x_0}^{\alpha} \quad (k-\text{ times})$$

The inverse generalized differential transform of $U_{\alpha,\beta}(k,h)$ is given by

$$u(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U_{\alpha,\beta}(k,h)(x-x_0)^{k\alpha}(y-y_0)^{h\beta} \quad (2)$$

It has the following properties:

I. if $u(x,y)=v(x,y)\pm w(x,y)$ then $U_{\alpha,\beta}(k,h)=V_{\alpha,\beta}(k,h)\pm W_{\alpha,\beta}(k,h)$

II. if $u(x,y)=av(x,y), a\in \mathrm{R}$ then $U_{\alpha,\beta}(k,h)=aV_{\alpha,\beta}(k,h)$

III. if $u(x,y)=v(x,y)w(x,y)$ then

$$U_{\alpha,\beta}(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}V_{\alpha,\beta}(r,h-s)W_{\alpha,\beta}(k-r,s)$$

IV. if $u(x,y)=v(x,y)w(x,y)q(x,y)$ then

$$U_{\alpha,\beta}(k,h)=\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}U_{\alpha,\beta}(r,h-s-p)$$
$$\times W_{\alpha,\beta}(t,s)Q_{\alpha,\beta}(k-r-t,p)$$

V. if $u(x,y)=(x-x_0)^{n\alpha}(y-y_0)^{m\beta}$ then

$$U_{\alpha,\beta}(k,h)=\delta(k-n)\delta(h-m)$$

VI. if $u(x,y)=D_{x_0}^{\alpha}v(x,y), 0<\alpha\le 1$ then

$$U_{\alpha,\beta}(k,h)=\frac{\Gamma(\alpha(k+1)+1)}{\Gamma(\alpha k+1)}V_{\alpha,\beta}(k+1,h)$$

VII. if $u(x,y)=D_{x_0}^{\gamma}v(x,y), 0<\gamma\le 1$ then

$$U_{\alpha,\beta}(k,h)=\frac{\Gamma(\alpha k+\gamma+1)}{\Gamma(\alpha k+1)}V_{\alpha,\beta}\left(k+\frac{\gamma}{\alpha},h\right)$$

VIII. if $u(x,y)=D_{y_0}^{\gamma}v(x,y), 0<\gamma\le 1$ then

$$U_{\alpha,\beta}(k,h)=\frac{\Gamma(\beta h+\gamma+1)}{\Gamma(\beta h+1)}V_{\alpha,\beta}\left(k,h+\frac{\gamma}{\beta}\right)$$

IX. if $u(x,y)=f(x)g(y)$ and the function $f(x)=x^{\lambda}h(x)$ where $\lambda>-1$, $h(x)$ has the generalized Taylor series expansion

$$h(x)=\sum_{n=0}^{\infty}a_n(x-x_0)^{\alpha k}$$ and

(a) $\beta<\lambda+1$ and α is arbitrary or

(b) $\beta\ge\lambda+1$, α is arbitrary and $a_n=0$ for $n=0,1,2,.....m-1$, here $m-1<\beta\le m$.

Then (1) becomes

$$U_{\alpha,\beta}(k,h)=\frac{1}{\Gamma(\alpha k+1)\Gamma(\beta h+1)}$$
$$\times\left[D_{x_0}^{\alpha k}\left(D_{y_0}^{\beta}\right)^{h}u(x,y)\right]_{(x_0,y_0)}$$

X. if $v(x,y)=f(x)g(y)$, the function $f(x)$ satisfies the conditions given in (IX) and $u(x,y)=D_{x_0}^{\gamma}v(x,y)$,then

$$U_{\alpha,\beta}(k,h)=\frac{\Gamma(\alpha(k+1)+\gamma)}{\Gamma(\alpha k+1)}V_{\alpha,\beta}\left(k+\frac{\gamma}{\alpha},h\right)$$

where $U_{\alpha,\beta}(k,h), V_{\alpha,\beta}(k,h)$ and $W_{\alpha,\beta}(k,h)$ are the differential transformations of the functions $u(x,y), v(x,y)$ and $w(x,y)$ respectively and

$$\delta(k-n)=\begin{cases}1 & ; \quad k=n\\ 0 & ; \quad k\ne n\end{cases}$$

III. MATHEMATICAL PRELIMINARIES ON FRACTIONAL CALCULUS

In the present analysis we introduce the following definitions [5, 6].

3.1 Definition: Let $\alpha\in R^{+}$ On the usual Lebesgue space $L(a,b)$ integral operator I^{α} defined by

$$I^{\alpha}f(x)=\frac{d^{-\alpha}f(x)}{dx^{-\alpha}}=\frac{1}{\Gamma(\alpha)}\int_{0}^{x}(x-t)^{\alpha-1}f(t)dt$$

and

$$I^{0}f(x)=f(x)$$

is called Riemann-Liouville fractional integral operator of order $\alpha\ge 0$ and $a\le x<b$

It has the following properties:

I. $I^{\alpha}f(x)$ exists for any $x\in[a,b]$

II. $I^{\alpha}I^{\beta}f(x)=I^{\alpha+\beta}f(x)$

III. $I^{\alpha}I^{\beta}f(x)=I^{\beta}I^{\alpha}f(x)$

IV. $I^{\alpha}x^{\gamma}=\frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma}$

where $f(x)\in L[a,b]$, $\alpha,\beta\ge 0$, $\gamma>-1$

3.2 Definition: The Riemann-Liouville definition of fractional order derivative is

$${}_{0}^{RL}D_{x}^{\alpha}f(x)=\frac{d^{n}}{dx^{n}}\,{}_{0}I_{x}^{n-\alpha}f(x)$$
$$=\frac{1}{\Gamma(n-\alpha)}\frac{d^{n}}{dx^{n}}\int_{0}^{x}(x-t)^{n-\alpha-1}f(t)dt,$$

where n is an integer that satisfies $n-1<\alpha<n$.

3.3 Definition: A modified fractional differential operator ${}_0^cD_x^\alpha$ proposed by Caputo is given by

$$ {}_0^cD_x^\alpha f(x) = {}_0I_x^{n-\alpha} \frac{d^n}{dx^n} f(x) $$
$$ = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t)\,dt, $$

where $\alpha\left(\alpha \in R^+\right)$ is the order of operation and n is an integer that satisfies $n-1<\alpha<n$.

It has the following two basic properties [7]:

I. If $f \in L_\infty(a,b)$ or $f \in C[a,b]$ and $\alpha>0$ then ${}_0^cD_x^\alpha\, {}_0I_x^\alpha f(x) = f(x)$.

II. If $f \in C^n[a,b]$ and if $\alpha>0$ then

$$ {}_0I_x^\alpha\, {}_0^cD_x^\alpha f(x) = f(x) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0^+)}{k!} x^k \; ; $$
$$ n-1<\alpha<n. $$

3.4 Definition: For m being the smallest integer that exceeds α, the Caputo time-fractional derivative operator of order $\alpha>0$, is defined as [8]

$$ D_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} $$
$$ = \begin{cases} \dfrac{\partial^m u(x,\xi)}{\partial \xi^m} & ; \quad \alpha = m \in N \\ \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t (t-\xi)^{m-\alpha-1} \frac{\partial^m u(x,\xi)}{\partial \xi^m} d\xi & ; \quad m-1 \le \alpha < m \end{cases} $$

Relation between Caputo derivative and Riemann-Liouville derivative:

$$ {}_0^cD_x^\alpha f(x) = {}_0^{RL}D_t^\alpha f(x) - \sum_{k=0}^{m-1} \frac{f^{(k)}(0^+)}{\Gamma(k-\alpha+1)} x^{k-\alpha} \; ; $$
$$ m-1<\alpha<m $$

Integrating by parts, we get the following formulae as given by [9]

I.
$$ \int_a^b g(x)\, {}_a^cD_x^\alpha f(x)\,dx = \int_a^b f(x)\, {}_x^{RL}D_b^\alpha g(x)\,dx $$
$$ + \sum_{j=0}^{n-1} \left[{}_x^{RL}D_b^{\alpha+j-n} g(x)\, {}_x^{RL}D_b^{n-j-1} f(x) \right]_a^b $$

II. For $n=1$

$$ \int_a^b g(x)\, {}_a^cD_x^\alpha f(x)\,dx = \int_a^b f(x)\, {}_x^{RL}D_b^\alpha g(x)\,dx $$
$$ + \left[{}_xI_b^{1-\alpha} g(x).f(x) \right]_a^b $$

IV. TEST PROBLEMS

In this section, we present four examples [10] to illustrate the applicability of Generalized Differential Transform Method (GDTM) to solve non linear time fractional differential equations.

4.1 Example: We consider the following non-linear time fractional partial differential equation

$$ \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - 6u(x,t)\frac{\partial u(x,t)}{\partial x} + u^2(x,t)\frac{\partial^3 u(x,t)}{\partial x^3} = 0 \; ; \; t \ge 0 $$

subject to initial condition $u(x,0)=6x\,; x \in \mathrm{R}$ (3)

where $\frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional differential operator(Caputo derivative) of order $0<\alpha\le1$.

Applying (1) with $(x_0,t_0)=(0,0)$ on (3) we obtain

$$ U_{1,\alpha}(k,h) = \frac{\Gamma(\alpha(h-1)+1)}{\Gamma(\alpha h+1)} \Big\{ 6\sum_{r=0}^{k}\sum_{s=0}^{h-1} U_{1,\alpha}(r,h-s-1) \times (k-r+1) U_{1,\alpha}(k-r+1,s) $$
$$ - \sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h-1}\sum_{p=0}^{h-s-1} U_{1,\alpha}(r,h-s-p-1) U_{1,\alpha}(t,s) \times (k-r-t+3)(k-r-t+2) \times (k-r-t+1) U_{1,\alpha}(k-r-t+3,p) \Big\} $$

(4)

and $U_{1,\alpha}(k,0)=6\delta(k-1)\quad \forall k=0,1,2,3,.......$ (5)

Utilizing (4) and (5), we obtain after a little simplification the following values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$

$U_{1,\alpha}(0,1)=0$; $U_{1,\alpha}(1,1)=-\dfrac{216}{\Gamma(\alpha+1)}$;

$U_{1,\alpha}(2,1)=0$; $U_{1,\alpha}(3,1)=0$; $U_{1,\alpha}(2,2)=0$;

$U_{1,\alpha}(0,2)=0$; $U_{1,\alpha}(1,2)=\dfrac{216^3}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}$;

$$U_{1,\alpha}(1,3)=\frac{216^2\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)}\left\{\frac{216^5}{(\Gamma(2\alpha+1))^2}+1\right\}$$

and so on

Using the above values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$ in (2), the solution of (3) is obtained as

$$u(x,t)=\frac{216}{\Gamma(\alpha+1)}xt^{\alpha}+\frac{216^3}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}xt^{2\alpha}+\frac{216^2\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)}\left\{\frac{216^5}{(\Gamma(2\alpha+1))^2}+1\right\}xt^{3\alpha}+...... \quad (6)$$

4.2 Example: We consider the following non-linear time fractional partial differential equation

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}-6u^2(x,t)\frac{\partial u(x,t)}{\partial x}+x^2\frac{\partial^3 u(x,t)}{\partial x^3}=0\quad ;\ t\ge 0$$

subject to initial condition $u(x,0)=\frac{1}{6}(x-1)\,;\ x\in \mathrm{R}$ (7)

where $\frac{\partial^{\alpha}}{\partial t^{\alpha}}$ is the fractional differential operator(Caputo derivative) of order $0<\alpha\le 1$.

Applying (2) with $(x_0,t_0)=(0,0)$ on (7) we obtain

$$U_{1,\alpha}(k,h)=\frac{\Gamma(\alpha(h-1)+1)}{\Gamma(\alpha h+1)}\left\{6\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h-1}\sum_{p=0}^{h-s-1}U_{1,\alpha}(r,h-s-p-1)U_{1,\alpha}(t,s)(k-r-t+1)U_{1,\alpha}(k-r-t+1,p)\right.$$
$$\left.-\sum_{r=0}^{k}\sum_{s=0}^{h-1}\delta(r-2)\delta(h-s-1)(k-r+3)(k-r+2)\times(k-r+1)U_{1,\alpha}(k-r+3,s)\right\} \quad (8)$$

and $U_{1,\alpha}(k,0)=\frac{1}{6}(\delta(k-1)-1)$; $\forall k=0,1,2,3,.......$ (9)

Uutilizing (8) and (9), we obtain after a little simplification the following values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$

$U_{1,\alpha}(1,0)=0$; $U_{1,\alpha}(k,0)=-\dfrac{1}{6}\ \ \forall\ k=0,2,3,4,...$;

$U_{1,\alpha}(2,1)=\dfrac{11}{12\Gamma(\alpha+1)}$; $U_{1,\alpha}(1,1)=-\dfrac{1}{18\Gamma(\alpha+1)}$;

$U_{1,\alpha}(5,1)=-\dfrac{235}{12\Gamma(\alpha+1)}$; $U_{1,\alpha}(3,1)=\dfrac{34}{9\Gamma(\alpha+1)}$;

and so on

Using the above values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$ in (2) the solution of (7) is obtained as

$$u(x,t)=-\frac{1}{6}-\frac{1}{18\Gamma(\alpha+1)}xt^{\alpha}+\frac{1}{6}\left(-1+\frac{11}{2\Gamma(\alpha+1)}t^{\alpha}\right)x^2+\frac{1}{3}\left(-\frac{1}{2}+\frac{34}{3\Gamma(\alpha+1)}t^{\alpha}\right)x^3+\frac{1}{6}\left(-1-\frac{235}{2\Gamma(\alpha+1)}t^{\alpha}\right)x^5+... \quad (10)$$

4.3 Example: We consider the following non-linear time fractional partial differential equation

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}+u(x,t)\frac{\partial u(x,t)}{\partial x}-3\frac{\partial^3 u(x,t)}{\partial x^3}=2t^\alpha+x+t^{3\alpha}+xt^{2\alpha}\ ;\ t\ge 0$$

subject to initial condition $u(x,0)=1;\ x\in\mathrm{R}$ (11)

where $\frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional differential operator(Caputo derivative) of order $0<\alpha\le 1$.

Applying (1) with $(x_0,t_0)=(0,0)$ on (11) we obtain

$$U_{1,\alpha}(k,h)=\frac{\Gamma(\alpha(h-1)+1)}{\Gamma(\alpha h+1)}\Big\{-\sum_{r=0}^{k}\sum_{s=0}^{h-1}U_{1,\alpha}(r,h-s-1)\times(k-r+1)U_{1,\alpha}(k-r+1,s)+3(k+3)(k+2)(k+1)\times U_{1,\alpha}(k+3,h-1)+2\delta(k)\delta(h-2)+\delta(k-1)\delta(h-1)+2\delta(k)\delta(h-4)+\delta(k-1)\delta(h-3)\Big\} \quad (12)$$

and $U_{1,\alpha}(k,0)=1;\ \forall k=0,1,2,3,.......$ (13)

Uutilizing (12) and (13), we obtain after a little simplification the following values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$

$$U_{1,\alpha}(0,1)=\frac{17}{\Gamma(\alpha+1)};\ U_{1,\alpha}(1,1)=\frac{70}{\Gamma(\alpha+1)};$$

$$U_{1,\alpha}(2,1)=\frac{174}{\Gamma(\alpha+1)};\ U_{1,\alpha}(3,1)=\frac{350}{\Gamma(\alpha+1)};$$

$$U_{1,\alpha}(4,1)=\frac{615}{\Gamma(\alpha+1)};\ U_{1,\alpha}(5,1)=\frac{987}{\Gamma(\alpha+1)};$$

$$U_{1,\alpha}(6,1)=\frac{1484}{\Gamma(\alpha+1)};\ U_{1,\alpha}(7,1)=\frac{2124}{\Gamma(\alpha+1)};$$

$$U_{1,\alpha}(8,1)=\frac{2925}{\Gamma(\alpha+1)};\ U_{1,\alpha}(9,1)=\frac{3905}{\Gamma(\alpha+1)};$$

$$U_{1,\alpha}(10,1)=\frac{5082}{\Gamma(\alpha+1)};\ U_{1,\alpha}(11,1)=\frac{6474}{\Gamma(\alpha+1)}$$

and so on

Using the above values of $U_{1,\alpha}(k,h)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$ in (2) the solution of (11) is obtained as

$$u(x,t)=1+\frac{17}{\Gamma(\alpha+1)}t^\alpha+\left(1+\frac{70}{\Gamma(\alpha+1)}t^\alpha\right)x+\left(1+\frac{174}{\Gamma(\alpha+1)}t^\alpha\right)x^2+\left(1+\frac{350}{\Gamma(\alpha+1)}t^\alpha\right)x^3+\left(1+\frac{615}{\Gamma(\alpha+1)}t^\alpha\right)x^4+\left(1+\frac{987}{\Gamma(\alpha+1)}t^\alpha\right)x^5+\left(1+\frac{1484}{\Gamma(\alpha+1)}t^\alpha\right)x^6+\left(1+\frac{2124}{\Gamma(\alpha+1)}t^\alpha\right)x^7+\left(1+\frac{2925}{\Gamma(\alpha+1)}t^\alpha\right)x^8+\left(1+\frac{3905}{\Gamma(\alpha+1)}t^\alpha\right)x^9+\left(1+\frac{5082}{\Gamma(\alpha+1)}t^\alpha\right)x^{10}+\left(1+\frac{6474}{\Gamma(\alpha+1)}t^\alpha\right)x^{11}..... \quad (14)$$

4.4 Example: We consider the following non-linear time fractional partial differential equation

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}+u(x,t)\frac{\partial u(x,t)}{\partial x}=\frac{\partial^3 u(x,t)}{\partial x^3}+\frac{\partial^2 u(x,t)}{\partial x^2}+2x^2t^\alpha+2xt^{2\alpha}+2x^3t^{4\alpha}\ ;\ t\ge 0$$

subject to initial condition $u(x,0)=1;\ x\in\mathrm{R}$ (15)

where $\frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional differential operator(Caputo derivative) of order $0<\alpha\le 1$.

Applying (1) with $(x_0,t_0)=(0,0)$ on (15) we obtain

$$U_{1,\alpha}(k,h)=\frac{\Gamma(\alpha(h-1)+1)}{\Gamma(\alpha h+1)}\Big\{-\sum_{r=0}^{k}\sum_{s=0}^{h-1}U_{1,\alpha}(r,h-s-1)\times(k-r+1)U_{1,\alpha}(k-r+1,s)$$

$$+\left(k+3\right)\left(k+2\right)\left(k+1\right)U_{1,\alpha}\left(k+3,h-1\right)$$
$$+\left(k+2\right)\left(k+1\right)U_{1,\alpha}\left(k+2,h-1\right)+2\delta\left(k-2\right)\delta\left(h-2\right)$$
$$+2\delta\left(k-1\right)\delta\left(h-3\right)+2\delta\left(k-3\right)\delta\left(h-5\right)\big\} \quad (16)$$

and $U_{1,\alpha}\left(k,0\right)=1;\ \forall k=0,1,2,3,.......$ (17)

Uutilizing (16) and (17), we obtain after a little simplification the following values of $U_{1,\alpha}\left(k,h\right)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$

$$U_{1,\alpha}\left(0,1\right)=\frac{4}{\Gamma\left(\alpha+1\right)};U_{1,\alpha}\left(1,1\right)=\frac{27}{\Gamma\left(\alpha+1\right)};$$

$$U_{1,\alpha}\left(2,1\right)=\frac{54}{\Gamma\left(\alpha+1\right)};\ U_{1,\alpha}\left(3,1\right)=\frac{130}{\Gamma\left(\alpha+1\right)};$$

$$U_{1,\alpha}\left(4,1\right)=\frac{225}{\Gamma\left(\alpha+1\right)};U_{1,\alpha}\left(5,1\right)=\frac{357}{\Gamma\left(\alpha+1\right)};$$

$$U_{1,\alpha}\left(6,1\right)=\frac{532}{\Gamma\left(\alpha+1\right)}\ ;\ U_{1,\alpha}\left(7,1\right)=\frac{756}{\Gamma\left(\alpha+1\right)}\ ;$$

$$U_{1,\alpha}\left(8,1\right)=\frac{1035}{\Gamma\left(\alpha+1\right)}\ ;\ U_{1,\alpha}\left(9,1\right)=\frac{1375}{\Gamma\left(\alpha+1\right)}\ ;$$

$$U_{1,\alpha}\left(10,1\right)=\frac{1782}{\Gamma\left(\alpha+1\right)}\ ;\ U_{1,\alpha}\left(11,1\right)=\frac{2262}{\Gamma\left(\alpha+1\right)}$$

and so on

Using the above values of $U_{1,\alpha}\left(k,h\right)$ for $k=0,1,2,3,...$ and $h=0,1,2,3,...$ in (2) the solution of (15) is obtained as

$$u\left(x,t\right)=1+\frac{4}{\Gamma\left(\alpha+1\right)}t^{\alpha}+\left(1+\frac{27}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x$$

$$+\left(1+\frac{54}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{2}+\left(1+\frac{130}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{3}$$
$$+\left(1+\frac{225}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{4}+\left(1+\frac{357}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{5}$$
$$+\left(1+\frac{532}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{6}+\left(1+\frac{756}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{7}$$
$$+\left(1+\frac{1035}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{8}+\left(1+\frac{1375}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{9}$$
$$+\left(1+\frac{1782}{\Gamma\left(\alpha+1\right)}\ t^{\alpha}\right)x^{10}+\left(1+\frac{2262}{\Gamma\left(\alpha+1\right)}t^{\alpha}\right)x^{11}+..... \quad (18)$$

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