

# Robustness Analysis of GWO/PID Approach in Control of Ball Hoop System with ITAE Objective Function

Mridul Porwal<sup>1\*</sup>, Girish Parmar<sup>2</sup>, Rajesh Bhatt<sup>3</sup>

<sup>1,2,3</sup>Department of Electronics Engineering, Rajasthan Technical University, Kota, Rajasthan, India

\*Corresponding author email: girish\_parmar2002@yahoo.com

Available online at: [www.ijcseonline.org](http://www.ijcseonline.org)

Accepted: 15/Aug/2018, Published: 31/Aug/2018

**Abstract**— This work deals with robustness analysis of Grey Wolf Optimization (GWO) / Proportional-Integral-Derivative (PID) approach in control of ball hoop (BH) system with integral of time multiplied absolute error (ITAE) objective function. The robustness analysis of GWO/PID approach has been carried out with  $\pm 5\%$  perturbation in the locations of the poles of the BH system. It has been observed that proposed GWO/PID approach with ITAE objective function gives satisfactory performance with  $\pm 5\%$  perturbation.

**Keywords**— BH System, PID, GWO, ITAE, Robustness.

## I. INTRODUCTION

Proportional-integral-derivative (PID) controllers are the most widely used controllers in industries [1-5]. Proportional gain ( $K_p$ ), integral gain ( $K_i$ ) and derivative gain ( $K_d$ ) are three parameters of PID controllers and adjustment of these parameters to achieve the desired response is called tuning of the PID controller. Ziegler-Nichols (Z-N) [1] and Cohen-Coon (C-C) [2] are two classical methods which have been used from several years for tuning of PID controller. Since last decade, due to complex processes in industries, tuning of parameters of PID controller has become a challenging task for researchers. Now a day's meta-heuristic algorithms for optimization have become highly popular to solve complex problems by several researchers. Some properties of meta-heuristic algorithms are simplicity, flexibility, random search and avoidance of local optima, etc. [6-8].

For the ball hoop system, various meta-heuristic algorithms are already available in the literature to tune PID controller such as; Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Chaotic Particle Swarm Optimization (CPSO), Adaptive Hybrid PSO (AHP SO) [11], Chaos driven Differential Evolution algorithm ( $DE_{Chaos}$ ) and Self-Organizing Migrating Algorithm ( $SOMA_{Chaos}$ ), Artificial Bee Colony Optimization (ABC), Bacterial Foraging Optimization (BFO), etc. [8-13, 18].

## II. BALL HOOP SYSTEM

The construction of ball hoop system (BH system) is easy, due to which it is preferred by control engineers for research.

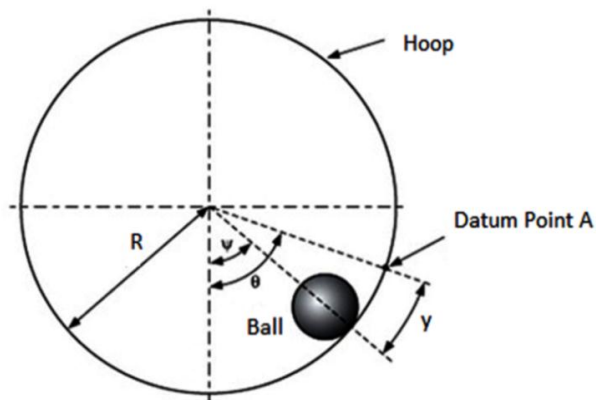


Figure 1: Dynamics of Ball hoop system [11-12]

Figure 1 shows schematic of the BH system. The main variables of BH system are: hoop radius:  $R$ , ball radius:  $r$ , ball mass:  $m$ , hoop angle:  $\theta$ , ball angles with vertical (slosh angle):  $\psi$ , ball position on the hoop:  $y$ , input torque to the hoop:  $T(t)$ .

The BH system illustrates the dynamics of a steel ball that is free to roll on the inner surface of a rotating circular hoop. The ball has oscillatory motion when moving inside a hoop. The transfer function of BH system is given by [13]:

$$G_{BH}(s) = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s} \quad (1)$$

In present work, equation (1) has been obtained by linearizing the equations of BH system [17].

### III. PROBLEM STATEMENT

The present work deals with robustness analysis of GWO/PID approach in control of ball hoop (BH) system with integral of time multiplied absolute error (ITAE) objective function.

Designing a system using ITAE objective function has small overshoot and well-damped oscillations. This ITAE is given by equation:

$$ITAE = \int_0^{\infty} t |e(t)| dt \tag{2}$$

The simulink model of ITAE in Matlab is shown in Figure 2.

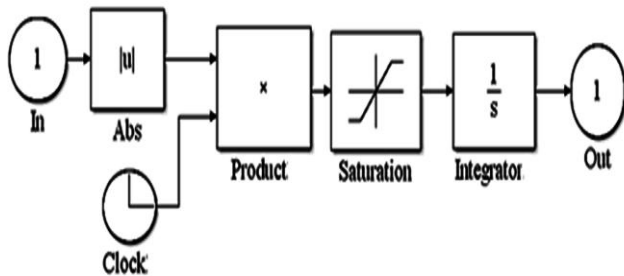


Figure 2: Simulink model representation of ITAE

### IV. GREY WOLF OPTIMIZATION

The GWO has recently been proposed bio inspired meta-heuristic algorithm inspired by social hierarchy and hunting behaviour of grey wolves. During hunting process, these wolves are divided in four groups and estimate the prey's (optimum) location through an iterative procedure [14, 16].

Each group functions are as follows:

- The alphas are leaders and responsible for making decisions about hunting, sleeping, time to wake. Interestingly, the alpha is not necessarily the strongest member of the pack but the best in terms of managing the pack.
- The betas are in second level in the hierarchy of grey wolves. The betas help the alpha in decision-making or other pack activities.
- The omegas are lowest ranking grey wolf. The omega plays the role of scapegoat. Omega wolves always have to submit to all the other dominant wolves.
- Delta wolves have to submit to alphas and betas, but they dominate the omega. Scouts, sentinels, elders, hunters, and caretakers belong to this category.

The functions of each group have also been shown in Figure 3.

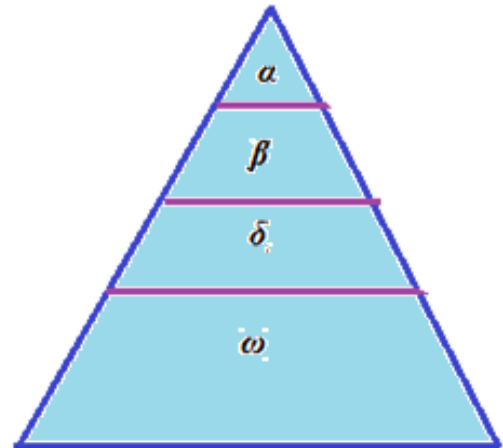


Figure 3: Social hierarchy of GWO and functions of each group [13]

The pseudo codes of the GWO algorithm are as follows [14]:  
 Initialize the algorithm parameters and generate the initial populations (positions of the wolves or agents)  
 Determine the fitness of each agent  
 Estimate  $X_{\alpha}$ ,  $X_{\beta}$  and  $X_{\delta}$ , the position of  $\alpha$ ,  $\beta$  and  $\delta$  wolves (the three best search agents)  
 While ( $t < \text{Max number of iterations}$ )  
     for each search agent  
         Update the position of current search agent  
     end for  
     Update search agents  
     Calculate the fitness of all search agents  
     Update the position of  $\alpha$ ,  $\beta$  and  $\delta$  wolves  
     Increase the iteration count  
 end while  
 Display the best wolves X.

### V. ROBUSTNESS ANALYSIS OF GWO/PID APPROACH

The Simulink model of the BH systems with ITAE objective function is shown in Figure 4.

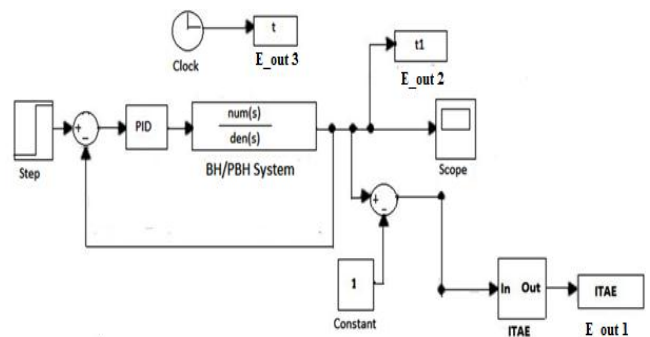


Figure 4: Complete simulink model of BH system with PID controller and ITAE objective function

The GWO algorithm has been run 5 times in Matlab for the simulink model shown in Figure 4 and obtained parameters of PID controller are given by:

$$K_p = 3.9920; \quad K_i = 0.0010; \quad K_D = 4.4359 \quad (3)$$

Therefore, the PID controller is given by:

$$G_c = 3.9920 + \frac{0.0010}{s} + 4.4359s \quad (4)$$

With +5% change in the location of the poles of BH system, the four poles of the perturbed BH system will be located at  $s = 0, s = -1.0500, s = -2.100$  &  $s = -3.1500$  and the transfer function of perturbed BH system will be:

$$G_{PBH}(s) = \frac{1}{s^4 + 6.3s^3 + 12.127s^2 + 6.9457s} \quad (5)$$

With -5% change in the location of the poles, the four poles of the perturbed BH system will be located at  $s = 0, s = -0.9500, s = -1.900$  &  $s = -2.8500$  and the transfer function of perturbed BH system will be:

$$G'_{PBH}(s) = \frac{1}{s^4 + 5.7s^3 + 9.927s^2 + 5.1442s} \quad (6)$$

In order to validate the robustness of proposed GWO/PID approach, the PID controller given by (4) has also been applied to the perturbed systems and the simulation results have been given. The closed loop transfer functions of the BH, PBH (+5%) & PBH (-5%) systems with PID controller and unity feedback have been given in Table 1.

Table 1: Closed loop transfer function of the BH, PBH (+5%) and PBH (-5%) systems

System	Closed loop transfer function ( $G_{CL}$ )
BH	$\frac{4.4359s^2 + 3.9920s + 0.0010}{s^2 + 6s^4 + 11s^2 + 10.4359s^2 + 3.9920s + 0.0010}$
PBH (+5%)	$\frac{4.4359s^2 + 3.9920s + 0.0010}{s^2 + 6.3s^4 + 12.127s^2 + 11.3816s^2 + 3.9920s + 0.0010}$
PBH (-5%)	$\frac{4.4359s^2 + 3.9920s + 0.0010}{s^2 + 5.7s^4 + 9.927s^2 + 9.5801s^2 + 3.9920s + 0.0010}$

Figure 5 shows that the GWO/PID approach operates in a satisfactory manner on BH, PBH (+5%) and PBH (-5%) systems. Furthermore, the transient response parameters, i.e., rise, settling times and overshoot of closed loop BH, PBH (+5%) and PBH (-5%) systems have also been calculated and given in tabular and graphical forms.

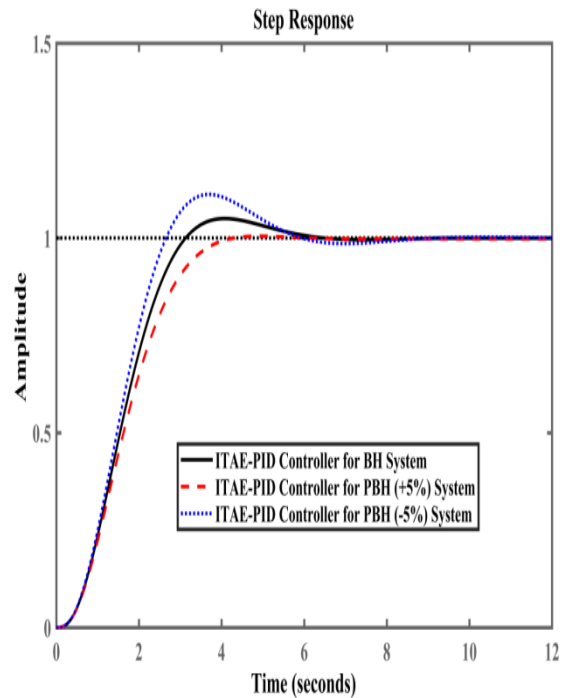


Figure 5: Step responses of GWO/PID approach with ITAE objective function for BH, PBH (+5%) and PBH (-5%) systems

Table 2: Comparison of rise time for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

System	Rise Time (Sec)
BH	1.93
PBH (+5%)	2.29
PBH (-5%)	1.67

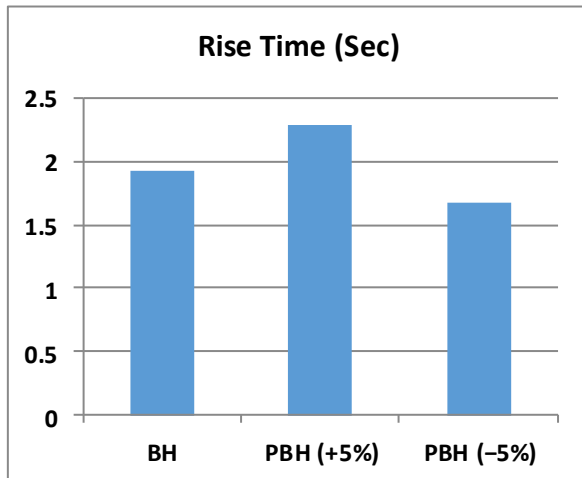


Figure 6: Bar chart comparison of rise time for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

Table 3: Comparison of settling time for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

System	Settling Time (Sec)
BH	5.41
PBH (+5%)	3.74
PBH (-5%)	5.46

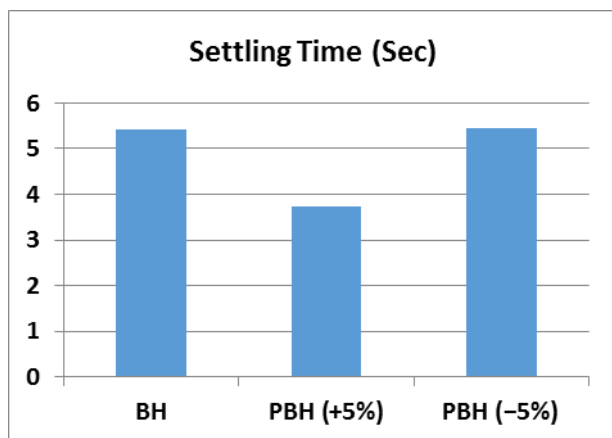


Figure 7: Bar chart comparison of settling time for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

Table 4: Comparison of overshoot for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

System	Overshoot (%)
BH	5
PBH (+5%)	0.473
PBH (-5%)	11.2

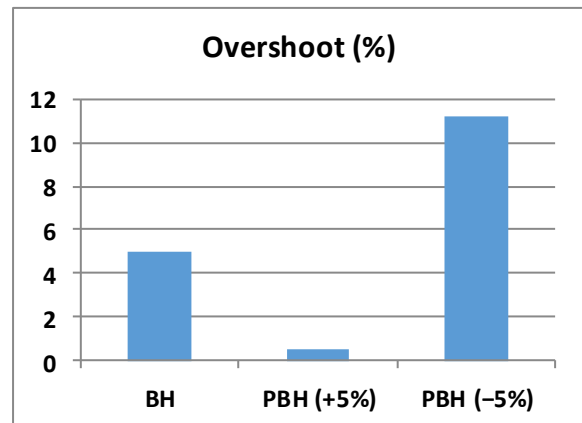


Figure 8: Bar chart comparison of Overshoot for GWO/PID (ITAE) approach for BH, PBH (+5%) and PBH (-5%) systems

## VI. CONCLUSIONS

The work deals with robustness analysis of GWO/PID approach in control of BH system with integral of time multiplied absolute error (ITAE) objective function. The robustness analysis has been carried out with  $\pm 5\%$  perturbation in locations of the poles of BH system. It can be observed from the simulation results that the transient response parameters of closed loop BH, PBH (+5%) & PBH (-5%) systems are comparable. The performance of the PID controller hardly alters with perturbations, once it is tuned by GWO.

## REFERENCES

- [1] J. G. Ziegler, N. B. Nichols, "Optimum setting for automatic controllers", *Trans. ASME*, Vol. 64, pp. 7759-7768, 1942.
- [2] G.H. Cohen and G.A. Coon, "Theoretical Investigation of Retarded Control", *The American Society of Mechanical Engineer*, Vol. 75, pp. 827-834, 1953.
- [3] M. Zhuang and D.P. Atherton, "Automatic tuning of optimum PID controllers", *IEEE Proceedings*, Vol. 140, Issue 3, pp. 216-224, 1993.
- [4] K.J. Astrom and T. Hagglund, "The future of PID control", *Control Engineering Practice*, pp. 1163-1175, 2001.
- [5] K.H. Ang, and G. Chong, Y. Li, "PID control system analysis, design and technology", *IEEE Trans. Control System Technology*, Vol. 13, pp. 559-576, 2005.
- [6] O. Roeva and T. Slavov, "PID Controller tuning based on metaheuristic algorithms for bioprocess control", *Biotechnology & Biotechnological Equipment*, pp. 3267-3277, 2014.
- [7] S. Pareek, M. Kishnani and R. Gupta, "Optimal tuning of PID controller using meta heuristic algorithms", *IEEE International Conference on Advances in Engineering & Technology Research (ICAETR)*, 2014.
- [8] P. Sreekanth and A. Hari, "Genetic algorithm based self tuning regulator for ball and hoop system", *IEEE Conference on Emerging Devices and Smart Systems (ICEDSS)*, 2016.
- [9] M. El-Said El-Telbany, "Employing Particle Swarm Optimizer and Genetic Algorithms for Optimal Tuning of PID Controllers: A Comparative Study", *ICGST-ACSE Journal*, volume 7, Issue 2, pp. 49-54, 2007.
- [10] H. Mojallali, R. Gholipour, A. Khosravi and H. Babaee, "Application of chaotic particle swarm optimization to PID parameter tuning in ball and hoop system", *International Journal of Computer and Electrical Engineering*, Vol. 4, No. 4, pp. 452-457, 2012.
- [11] Morkos and H. Kamal, "Optimal tuning of PID controller using adaptive hybrid particle swarm optimization algorithm", *Int. J. of Computers, Communications & Control*, Vol VII, No.1, pp.101-114, 2012.
- [12] D. Davendra, I. Zelinka and R. Senkerik, "Chaos driven evolutionary algorithms for the task of PID Control", *Computers & Mathematics with Applications*, Vol. 60, No. 4, pp.1088-1104, 2010.
- [13] N. Jain, G. Parmar, R. Gupta and I. Khanam, "Performance evaluation of GWO/PID approach in control of ball hoop system with different objective functions and perturbation", *Cogent Engineering, Taylor & Francis*, Vol. 5, pp. 1-18, 2018.
- [14] S. Mirjalili, S. M. Mirjalili and A. Lewis, "Grey Wolf Optimizer", *Adv. Eng. Softw., ELSEVIER*, Vol. 69, pp. 46-61, 2014.
- [15] V. Soni, G. Parmar and M. Kumar, "A hybrid grey wolf optimisation and pattern search algorithm for automatic generation control of multi area interconnected power system", *Int. J. of Advanced Intelligence Paradigms, Inderscience* [In Press].
- [16] C. Muro, R. Escobedo, L. Spectoe and R. P. Coppinger, "Wolf-pack (Canis Lupus) hunting strategies emerge from simple rules in computational simulations", *ELSEVIER*, Vol. 88, pp. 192-197, 2011.
- [17] P. Wellstead, "The ball and hoop system", *Automatica*, Vol.19, No. 4, pp. 401-406, 1983.
- [18] G. Parmar, S. Mukharjee and R. Prasad, "Reduced order modelling of linear multivariable system using particle swarm optimization technique", *International Journal of Innovative Computing and Applications*, Vol.1, No. 2, pp. 128-137, 2007.