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Estimation of Ratio of Two Population Means in case of Post-Stratification

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Abstract- Problem of estimation of ratio of two population means has been discussed resulting with a ratio and product type estimators. Expressions for the bias and mean squared error have been derived. Developed estimator under study has been compared with usual estimator theoretically as well as empirically.

Keywords- Two population means, Post-stratification, Bias and MSE.

I. INTRODUCTION

In addition to estimation of finite population mean, ratio of two population means is also demanding in various fields such as in economics for the estimation of per capita income, in marketing, per person consumption of any commodity etc. Present study develops a ratio and a product type estimators for ratio of two population means in case of post-stratification.

Initially, Hansen et al. (1953) discussed the post-stratification concept. Later Ige and Tripathi (1989) studied the classical Cochran (1940) ratio estimator in case of post-stratification. Recently, Tailor et al. (2017) studied the properties of Bahl and Tutaja (1991) ratio and product type exponential estimators in post-stratification.

Here objective is to estimate ratio of two populations means of study variates i.e. $\hat{R}=\frac{\bar{Y}_0}{\bar{Y}_1}$. Where $\bar{Y}_0$ and $\bar{Y}_1$ are the population means of study variates y_0 and y_1 respectively.

From a population of size N a sample of size n is drawn using post-stratification technique. Let y_0, y_1 be the study variables making ratio of means to be estimated with the help of information available on auxiliary variate x_1 and x_2.

The usual estimator for ratio of two population means in post-stratification is expressed as

$$\hat{R}_{ps}=\frac{\bar{y}_{0ps}}{\bar{y}_{1ps}} \tag{1.1}$$

where $\bar{y}_{0ps}=\sum_{h=1}^{L}W_h\bar{y}_{0h}$ and $\bar{y}_{1ps}=\sum_{h=1}^{L}W_h\bar{y}_{1h}$ are the unbiased estimates of population means $\bar{Y}_0$ and $\bar{Y}_1$ respectively.

Upto the thre first degree of approximation the bias and mean squared error of $\hat{R}_{ps}$ are given as

$$B\left(\hat{R}_{ps}\right)=\hat{R}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_1h}^2}{\bar{Y}_1^2}-\frac{S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}\right) \tag{1.2}$$

and

$$MSE\left(\hat{R}_{ps}\right)=\hat{R}^2\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2}+\frac{S_{y_1h}^2}{\bar{Y}_1^2}-\frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}\right). \tag{1.3}$$

where $S_{y_0h}^2=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{0hi}-\bar{Y}_{0h}\right)^2$, $S_{y_1h}^2=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{1hi}-\bar{Y}_{1h}\right)^2$, $S_{x_1h}^2=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(x_{1hi}-\bar{X}_{1h}\right)^2$,

$S_{x_2h}^2=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(x_{2hi}-\bar{X}_{2h}\right)^2$, $S_{y_0y_1h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{0hi}-\bar{Y}_{0h}\right)\left(y_{1hi}-\bar{Y}_{1h}\right)$, $S_{y_0x_1h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{0hi}-\bar{Y}_{0h}\right)\left(x_{1hi}-\bar{X}_{1h}\right)$,

$$S_{y_0x_2h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{0hi}-\bar{Y}_{0h}\right)\left(x_{2hi}-\bar{X}_{2h}\right),\quad S_{y_1x_1h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{1hi}-\bar{Y}_{1h}\right)\left(x_{1hi}-\bar{X}_{1h}\right),$$

$$S_{y_1x_2h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{1hi}-\bar{Y}_{1h}\right)\left(x_{2hi}-\bar{X}_{2h}\right),\quad S_{x_1x_2h}=\frac{1}{N_h-1}\sum_{i=1}^{N_h}\left(x_{1hi}-\bar{X}_{1h}\right)\left(x_{2hi}-\bar{X}_{2h}\right).$$

II. DEVELOPED ESTIMATORS

Motivated by Ige and Tripathi (1989) , a ratio and a product type estimators for ratio of two population means in case of post-stratification are developed as

$$\hat{R}_{Rps}=\hat{R}_{ps}\left(\frac{\bar{X}_1}{\bar{x}_{1ps}}\right) \qquad (2.1)$$

and

$$\hat{R}_{Pps}=\hat{R}_{ps}\left(\frac{\bar{x}_{2ps}}{\bar{X}_2}\right). \qquad (2.2)$$

To obtain the bias and mean squared error of the developed estimators $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$, it is assumed as $\bar{y}_{0h}=\bar{Y}_{0h}\left(1+e_{0h}\right)$, $\bar{y}_{1h}=\bar{Y}_{1h}\left(1+e_{1h}\right)$, $\bar{x}_{1h}=\bar{X}_{1h}\left(1+e_{2h}\right)$ and $\bar{x}_{2h}=\bar{X}_{2h}\left(1+e_{3h}\right)$ such that $E\left(e_{0h}\right)=E\left(e_{1h}\right)=E\left(e_{2h}\right)=E\left(e_{3h}\right)=0$ and

$$E\left(e_{0h}^2\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{y_0h}^2,\quad E\left(e_{1h}^2\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{y_1h}^2,\quad E\left(e_{2h}^2\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{x_1h}^2,\ E\left(e_{3h}^2\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{x_2h}^2,$$

$$E\left(e_{0h}e_{1h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{y_0y_1h}C_{y_0h}C_{y_1h},\ E\left(e_{0h}e_{2h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{y_0x_1h}C_{y_0h}C_{x_1h},$$

$$E\left(e_{0h}e_{3h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{y_0x_2h}C_{y_0h}C_{x_2h},E\left(e_{1h}e_{2h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{y_1x_1h}C_{y_1h}C_{x_1h},\ E\left(e_{1h}e_{3h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{y_1x_2h}C_{y_1h}C_{x_2h},$$

$$E\left(e_{2h}e_{3h}\right)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{x_1x_2h}C_{x_1h}C_{x_2h}.$$

Expressing $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ in terms of e_{ih} provides

$$\hat{\mathrm{R}}_{\mathrm{Rps}}=\hat{R}\left(1+e_0\right)\left(1+e_1\right)^{-1}\left(1+e_2\right)^{-1} \qquad (2.3)$$

$$\hat{R}_{Pps}=\hat{R}\left(1+e_0\right)\left(1+e_1\right)^{-1}\left(1+e_3\right) \qquad (2.4)$$

where $e_0=\frac{\sum_{h=1}^{L}W_h\bar{Y}_{0h}e_{0h}}{\bar{Y}_0}$, $e_1=\frac{\sum_{h=1}^{L}W_h\bar{Y}_{1h}e_{1h}}{\bar{Y}_1}$, $e_2=\frac{\sum_{h=1}^{L}W_h\bar{X}_{1h}e_{2h}}{\bar{X}_1}$ and $e_3=\frac{\sum_{h=1}^{L}W_h\bar{X}_{2h}e_{3h}}{\bar{X}_2}$

such that $E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=E\left(e_3\right)=0$ and $E\left(e_0^2\right)=\frac{1}{\bar{Y}_0^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_0h}^2$, $E\left(e_1^2\right)=\frac{1}{\bar{Y}_1^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_1h}^2$,

$$E\left(e_2^2\right)=\frac{1}{\bar{X}_1^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{x_1h}^2,\ E\left(e_3^2\right)=\frac{1}{\bar{X}_2^2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{x_2h}^2,$$

$$E\left(e_0e_1\right)=\frac{1}{\bar{Y}_0\bar{Y}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_0y_1h}^2,E\left(e_0e_2\right)=\frac{1}{\bar{Y}_0\bar{X}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_0x_1h}^2,\quad E\left(e_0e_3\right)=\frac{1}{\bar{Y}_0\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_0x_2h}^2,$$

$$E\left(e_1e_2\right)=\frac{1}{\bar{Y}_1\bar{X}_1}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_1x_1h}^2,,E\left(e_1e_3\right)=\frac{1}{\bar{Y}_1\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{y_1x_2h}^2,\ E\left(e_2e_3\right)=\frac{1}{\bar{X}_1\bar{X}_2}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{x_1x_2h}^2.$$

Finally, the bias and mean squared error of $\hat{R}_{Rps}$ are obtained as

$$B\left(\hat{R}_{Rps}\right)=\hat{R}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_1h}^2}{\bar{Y}_1^2}+\frac{S_{x_1h}^2}{\bar{X}_1^2}-\frac{S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}-\frac{S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1}+\frac{S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}\right) \quad (2.4)$$

and

$$MSE\left(\hat{R}_{Rps}\right)=\hat{R}^2\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2}+\frac{S_{y_1h}^2}{\bar{Y}_1^2}+\frac{S_{x_1h}^2}{\bar{X}_1^2}-\frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}+\frac{2S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}-\frac{2S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1}\right) \quad (2.5)$$

Similarly, the bias and mean squared error of $\hat{R}_{Pps}$ are obtained as

$$B\left(\hat{R}_{Pps}\right)=\hat{R}\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2}-\frac{S_{y_0x_1h}}{\bar{Y}_0\bar{Y}_1}-\frac{S_{y_1x_2h}}{\bar{Y}_1\bar{X}_2}+\frac{S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right) \quad (2.6)$$

and

$$MSE\left(\hat{R}_{Pps}\right)=\hat{R}^2\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(\frac{S_{y_0h}^2}{\bar{Y}_0^2}+\frac{S_{y_1h}^2}{\bar{Y}_1^2}+\frac{S_{x_2h}^2}{\bar{X}_2^2}-\frac{2S_{y_0y_1h}}{\bar{Y}_0\bar{Y}_1}-\frac{2S_{y_1x_2h}}{\bar{Y}_1\bar{X}_2}+\frac{2S_{y_0x_2h}}{\bar{Y}_0\bar{X}_2}\right) \quad (2.7)$$

III. THEORETICAL COMPARISON OF ESTIMATORS

Comparison of (1.3), (2.5) shows that the developed ratio type estimator $\hat{R}_{Rps}$ would be more efficient than usual estimator $\hat{R}_{ps}$ if

(i) $MSE\left(\hat{R}_{Rps}\right)<MSE\left(\hat{R}_{ps}\right)$

$$\sum_{h=1}^{L}W_h\left(\frac{S_{x_1h}^2}{\bar{X}_1^2}+\frac{2S_{y_1x_1h}}{\bar{Y}_1\bar{X}_1}-\frac{2S_{y_0x_1h}}{\bar{Y}_0\bar{X}_1}\right)<0. \quad (3.1)$$

Comparison of (1.3) and (2.7) indicates that the developed product type estimator $\hat{R}_{Pps}$ would be more efficient than $\hat{R}_{ps}$ if

(ii) $MSE\left(\hat{R}_{Pps}\right)<MSE\left(\hat{R}_{ps}\right)$

$$\sum_{h=1}^{L}W_h\left(\frac{S_{x_2h}^2}{\bar{X}_2^2}+\frac{2S_{y_1x2h}}{\bar{Y}_0\bar{X}_2}-\frac{2S_{y_0x_2h}}{\bar{Y}_1\bar{X}_2}\right)<0. \quad (3.2)$$

IV. NUMERICAL ILLUSTRATION

To analyze the performance of the developed estimators in comparison to usual estimator, we consider a natural population data set.

Population Source: [Johnson and Wichern, P. No. 245]

y_0 : Voltage (volts)

y_1 : Current (amps)

x_1 : Feed speed (in/min)

x_2 : Gas flow (cfm)

Constant	Stratum I	Stratum II
N_h	30	10
n_h	4	4
$\bar{Y}_{0h}$	22.13	22.8
$\bar{Y}_{1h}$	272.66	272

$\overline{X}_{1h}$	288.38	289
$\overline{X}_{2h}$	52.09	52
S_{y0h}	0.3	0.1
S_{y1h}	5.60	2.45
S_{x1h}	1.07	1.25
S_{x2h}	1.03	0.64
S_{y0y1h}	-0.48	-0.03
S_{y0x1h}	-0.03	0.05
S_{y0x2h}	-0.04	-0.03
S_{y1x1h}	0.61	0.24
S_{y1x2h}	0.3	-0.12
S_{x1x2h}	0.06	-0.45

Table 4.1: The Mean Squared Error and Percent Relative Efficiencies of $\hat{R}_{ps}$, $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ with respect to $\hat{R}_{ps}$.

Estimator	MSE	PRE
$\hat{R}_{ps}$	9.632	100.00
$\hat{R}_{Rps}$	2.930	376.58
$\hat{R}_{Pps}$	7.421	232.26

V. CONCLUSION

Expressions (3.1) and (3.2) are the conditions under which the developed estimators $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ has less mean squared error in comparison to usual unbiased estimator $\hat{R}_{ps}$.

Table 4.1 shows that the developed ratio and product type estimators for ratio of two population means has better percent relative efficiency as compared to other usual estimator. Thus there is a substantial gain in efficiency by using developed estimators $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ over usual estimator $\hat{R}_{ps}$.

Therefore, the developed estimators $\hat{R}_{Rps}$ and $\hat{R}_{Pps}$ are recommended for the use in practice for the estimation of ratio of two population means when conditions obtained in section 3 are satisfied.

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