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Slip Effects on Steady and Pulsatile Motion of Blood in a Circular Tube under Periodic Body Acceleration

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***Abstract*—** In the present paper, the Author has investigated the pulsatile flow of blood through an artery. The arterial vessel has been assumed to be small, rigid cylindrical tube with circular uniform cross-section. The blood flow will be steady and a slip velocity is imposed in the arterial wall. By using microcontinuum technique the solutions for variation of blood velocity and variation of cell rotational velocity are obtained for both steady and unsteady motion of the blood. The both Fourier-Bessel series and Laplace-Hankel transforms are applied for obtaining such solutions. The effect of the slip parameter on both steady and unsteady motion of the blood is shown in graphical forms. It is observed that the slip velocity bear the potential to influence the velocity distribution of blood to a considerable extent. Further, the imposition of the slip velocity leads to a growth in each of axial velocity of blood and the cell rotational velocity of blood.

Keywords— steady and pulsatile motion of blood, slip flow regime, blood velocity, Fourier-Bessel series, Laplace-Hankel transforms.

I. INTRODUCTION

Eringen first introduced about micropolar fluid which is a type of fluid exhibiting some microscopic effect that arises from microrotation of the fluid elements and the local elementary structure [1, 2]. It is interesting to note that blood is a micropolar fluid and, therefore, the principles can fruitfully be applied to study deacceleration of blood in human arterial system. To analyse the steady and pulsatile motion of blood in rigidly circular based tube by micro-continuum process, a mathematical model was developed by Ariman [3], while the unsteady motion of blood between parallel plates has been considered by Turk, et al [4]. Bugliarello, et al have shown that the red cells of blood rotate at the wall of the vessel while the rotational velocity is a function of the pressure gradient, system dimensions, the viscosity of the medium and the density of the packing of the cells [5]. The microcontinuum approach has been applied by Ariman et al to investigate the steady and pulsatile motion of blood through small, rigidly circular based tubes [6]. Later, Sanyal and Maitihave generalised this problem by taking blood to be a conducting micropolar fluid [7].

Now the existence of velocity slip at the flow boundaries has been reported by many authors both from theoretical [8 - 13] and experimental [14, 15] points of view. The aim of our analysis is to bridge this gap in the literature of steady and pulsatile motion of blood through a rigidly circular based tube by micro-continuum process. The effects of slip on the flow variables have been studied in the present paper and it is shown how these results obtained with the slip could be useful in the diseased circulatory system. The proposed problem can be solved for both the motions by using of Fourier-Bessel series and Laplace-Hankel transforms respectively.

II. BASIC EQUATIONS

The blood circulatory system in real situation consists elastic tubes of varying cross-sections. However, for mathematical convenience, wereplace the artery by a long cylindrical rigid circular tube. Wealso suppose that the blood is micropolar, viscous and incompressible fluid of constant density and viscosity.

Then the constitutive equations for the floware as follows [1, 2]:

Equation of continuity

$$\nabla.\mathbf{V} = 0 \tag{1}$$

The balance of momentum equation

$$(\lambda_s + 2\mu_s)\nabla(\nabla.\mathbf{V}) - (\mu_s + \mu_R)\nabla \times \nabla \times \mathbf{V} + 2\mu_R\nabla \times q - \nabla\pi_t + \rho F = \rho\left[\frac{\partial V}{\partial t} - \mathbf{V} \times (\nabla \times \mathbf{V}) + \frac{1}{2}\nabla(V^2)\right] \tag{2}$$

The balance of momentum equation

$$(\alpha' + \beta' + \gamma)\nabla(\nabla.q) - \gamma\nabla \times \nabla \times q + 2\mu_R\nabla \times \mathbf{V} - 4\mu_R\mathbf{q} + \rho\mathbf{I} = \rho j\frac{\partial q}{\partial t} \tag{3}$$

In the above, **V**, the velocity vector; **q**, the cell rotational vector; t, the thermodynamical pressure; **F**, the body force vector;ρ, be the density; I, the body couple vector; j, the micro-inertia coefficient and λ_s, μ_s,μ_R,α', β',γ arematerial and viscosity coefficients.

We assume the flow to be axisymmetric and the velocity and microrotation components are

$$\boldsymbol{V} = \big(0, 0, W(r,t)\big), \quad \boldsymbol{q} = (0, v(r,t), 0). \tag{4}$$

Also the body force and body couple vectors are taken to be zero, ie. **F** = **I** = 0.

Then, using the equation (1), the equations (2) and (3) with the help of (4) give

$$(\mu_s + \mu_R)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial W}{\partial r}\right) + 2\mu_R\frac{1}{r}\frac{\partial}{\partial r}(vr) - \frac{\partial p}{\partial z} = \rho\frac{\partial W}{\partial t}, \tag{5}$$

and

$$\gamma\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv)\right] - 2\mu_R\frac{\partial W}{\partial r} - 4\mu_R v = \rho j\frac{\partial v}{\partial t} \tag{6}$$

The boundary condition on velocity is

$$W(r,t) + \beta\frac{\partial}{\partial r}W(r,t) = 0 \; at \; r = R_0 \tag{7}$$

and the constant spin condition on the boundary is

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) = 0 \; at \; r = R_0 \tag{8}$$

where β is the slip parameter and R_0 is the tube radius.

III. SOLUATIONS

Case I - Steady flow.

In this case, W = W(r) and v = v(r). Then, introducing the following non-dimensional quantities forconvenience

$$\eta = \frac{r}{R_0}, \quad W_c = \frac{R_0^2}{4\mu_s}\left(-\frac{dp}{dz}\right), \quad \frac{vR_0}{W_c} = v^* \quad and \quad \alpha = \frac{\beta}{R_0}, \tag{9}$$

The equations (5) and (6) reduce to

$$\frac{\mu_s+\mu_R}{R_0^2}\frac{d}{d\eta}\left(\eta\frac{dW^*}{d\eta}\right) + \frac{2\mu_R}{R_0}\frac{d}{d\eta}(\eta W^*) = c\eta \tag{10}$$

and

$$\frac{\gamma}{2\mu_R R^2}\frac{d}{d\eta}\left\{\frac{1}{\eta}\frac{d}{d\eta}(\eta v^*)\right\} - \frac{dW^*}{d\eta} - 2v^* = 0 \tag{11}$$

while the boundary conditions (7) and (8) read

$$W^* + \alpha\frac{dW^*}{d\eta} = 0 \;\; at \;\; \eta = 1$$

and

$$\frac{1}{\eta}\frac{d}{d\eta}(\eta v^*) = 0 \quad at \quad \eta = 1 \tag{13}$$

In the above $c = \frac{\partial p}{\partial z}$

Solutions of the equations (10) and (11) subject to the boundary conditions (12) and (13) are

$$W^*(\eta) = V_c\left[1 + 2\alpha - \eta^2 + \frac{4\mu_R}{\mu_R+\mu_s}\left\{\frac{I_0(\lambda_n) - I_0(\lambda) - \alpha\eta I_1(\eta)}{\lambda^2 I_0(\lambda)}\right\}\right] \tag{14}$$

and

$$v^*(\eta) = \frac{V_c}{R_0}\left[\eta - \frac{2I_1(\lambda_n)}{\lambda I_0(\lambda)}\right], \tag{15}$$

Where $\lambda = kR_0$ and $k^2 = \frac{4\mu_R\mu_s}{\nu(\mu_R+\mu_s)}$

Case II - Pulsatile motion.

In considering the pulsatile flow of blood through a rigidly circular based tube, we consider an arbitrary pressure gradient in general form as

$$-\frac{\partial p}{\partial t} = \begin{Bmatrix} 0 & if \;\; t < 0 \\ P(t) & if \;\; t \geq 0 \end{Bmatrix} \tag{16}$$

As suggested by McDonald [16] the arbitrary time-dependent pulsatile pressure gradient can be expressed in the following form

$$P(t) = A_n \sin(n\omega t) + B_n \cos(n\omega t), \tag{17}$$

where ω denote the fundamental frequency of oscillation (circular) and n = 0, 1, 2, ... However, the experimentally pulsatile motion of blood data [17, 18] is presented for pressure gradient in the form of a sinusoidal gradient superposed on a constant pressure gradient so that the fourier coefficients A_n and B_n reduce to

$$A_1 = P_s, \;\; B_0 = P_m, \;\; A_j = 0, \;\; B_i = 0, \quad (j = 2,3,4\ldots; i = 1,2,3,\ldots) \tag{18}$$

and, therefore, we can express the sinusoidal pressure gradient as

$$P(t) = P_m(1 + \epsilon\sin\omega t) \tag{19}$$

where P_m is the mean pressure gradient which is constant, P_s is the amplitude of sinusoidal pressure gradient and $\epsilon = \frac{P_s}{P_m}$.

Substituting (19) into equations (5) and (6), the basic equations are given by

$$\frac{\mu_R+\mu_s}{R_0^2}\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial W^*}{\partial\eta}\right) - \rho\frac{\partial W^*}{\partial t} + 2\frac{\mu_R}{R_0}\frac{1}{\eta}\frac{\partial}{\partial\eta}(\eta v^*) = P_m(1 + \epsilon\sin\omega t) \tag{20}$$

and

$$\frac{1}{R_0}\frac{\partial v^*}{\partial\eta} = \frac{\gamma}{2\mu_R R_0^2}\frac{\partial}{\partial\eta}\left\{\frac{1}{\eta}\frac{\partial}{\partial\eta}(\eta W^*)\right\} - 2W^*_{(12)} - \frac{P_j}{2\mu_R}\frac{\partial W^*}{\partial t} \tag{21}$$

The boundary conditions are

$$W^* + \alpha\frac{\partial W^*}{\partial\eta} = 0 \quad at \quad \eta = 1 \tag{22}$$

and

$$\frac{1}{\eta}\frac{d}{d\eta}(\eta v^*) = 0 \quad at \quad \eta = 1 \tag{23}$$

Now If we allow the time to approaching to infinity, then the transient solutions approach the steady state solutions for the pulsatile velocity and cell rotational velocity which are obtained by the consecutive use of Hankel and Laplace transformations in the following way,

$$W(r,t) = \frac{2P_m}{\rho^2 jR_0}\sum\left\{\frac{J_0(r\xi_i) - J_0(R_0\xi_i) - \alpha\xi_i J_0'(R_0\xi_i)}{\xi_i r_1 r_2 J_1(R_0\xi_i)}\right\} \times$$

$$\left\{(4\mu_R + \gamma\xi_i^2) + \frac{\epsilon r_1 r_2}{(r_1^2+\omega^2)(r_2^2+\omega^2)}(\alpha_1 \cos\omega t + \beta_1 \sin\omega t)\right\} \quad (24)$$

$$v(r,t) = \frac{4\mu_R P_m}{\rho^2 j R_0}\sum \frac{J_1(r\xi_i)}{r_1 r_2 J_1(R_0\xi_i)} \times \left\{1 + \frac{\epsilon\, r_1 r_2}{(r_1^2+\omega^2)(r_2^2+\omega^2)}(\alpha_1 \cos\omega t + \beta_1 \sin\omega t)\right\} \quad (25)$$

r_1 and r_2 being the roots of the equation

$$r^2 + \frac{\xi_i^2}{\rho j}\left\{j(\mu_R + \mu_s) + \gamma + \frac{4\mu_R}{\xi_i^2}\right\} r + \frac{\xi_i^2 \gamma}{\rho^2 j}(\mu_R + \mu_s)(\xi_i^2 + k^2) = 0 \quad (26)$$

and

$$\alpha_1 = \rho j\omega_0(r_1 r_2 - \omega_0^2) + \omega_0(r_1 + r_2)(4\mu_R + \gamma\xi_i^2),$$
$$\beta_1 = (r_1 r_2 - \omega_0^2)(4\mu_R + \gamma\xi_i^2) - \rho j\omega_0^2(r_1 + r_2),$$
$$\alpha_2 = \omega_0(r_1 + r_2),$$
$$\beta_2 = r_1 r_2 - \omega_0^2. \quad (27)$$

In the above, J_0 and J_1 are respectively the zero th order and first order Bessel functions and ξ_i (i = 1, 2, 3, 4.....) are chosen to be the positive roots of thetranscendental equation $J_0(R_0\xi_i) = 0$.

IV. RESULTS AND DISCUSSIONS

The velocity variations of $W^*$and cellrotational velocity v^* for varying η or r are shown graphically form for various values of α, the slip parameter and tube radius λ = (kR_0). It is interesting to notethat the slip parameter has no effect on the cell rotational velocity.

For numerical calculations, then numerical values of different parameters are taken as follows: μ_R = 0.98CP, $\mu_s = 1$ CP, $R_0 = 0.15$ cm, $\gamma = 12 \times 10^{-6}$ gm.Cm/sec, c = 0.5, $\omega = 8.4$ Hz, j = 8.5 gm. Cm/s-CP, $\rho = 1.05$ gm./cc., $\alpha = 2$, P_m = 0.25 , P_s = 0.30 .

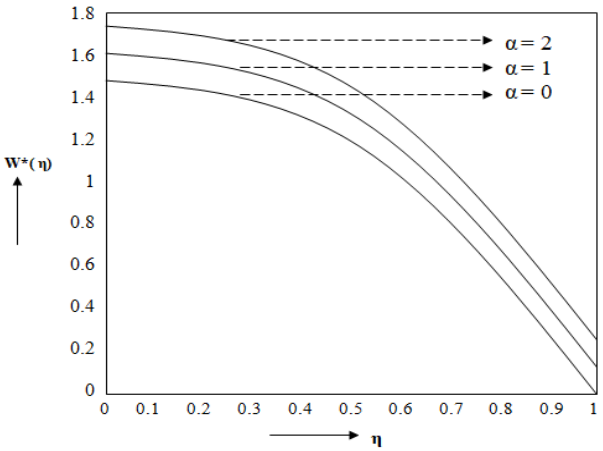

Figure 1 : Variation of blood velocity (steady case)

Fig.1 shows the variation of blood velocity (steady case) for different values of the slip parameter α. It may be observed that the slip parameter decreases the blood velocity for the steady flow.

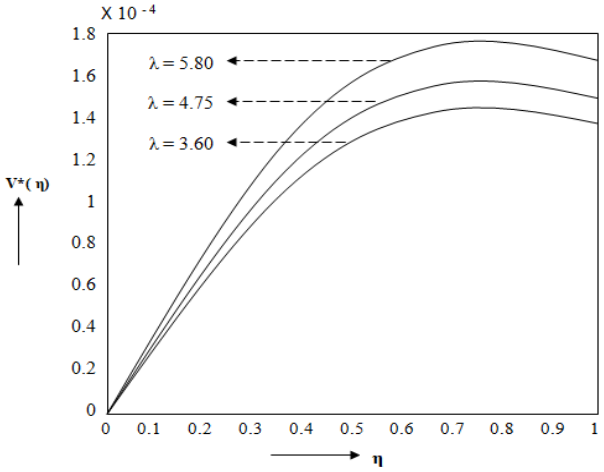

Figure 2 : Variation of cell rotational velocity (steady case)

Fig. 2 shows the variation of cell rotational velocity (steady case) for different values of the parameter λ. It may be observed that the cell rotational velocity increases with increase of λ.

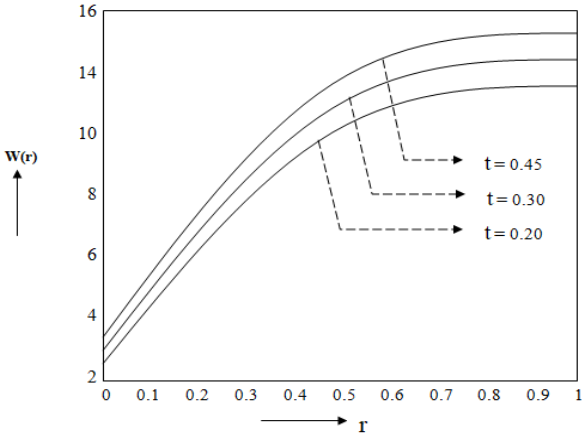

Figure 3: Variation of blood velocity (unsteady case)

Fig. 3 shows the variation of blood velocity (unsteady case) for different values of the time t. It may be observed that the blood motion increases with increasing values of time t.

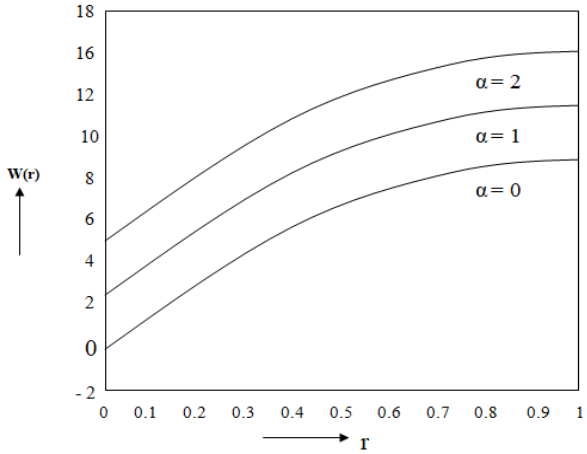

Figure 4: Variation of blood velocity (unsteady case)

Fig.4 shows the variation of blood velocity (unsteady case) for different values of the slip parameter α. The slip parameter increase the blood velocity for unsteady motion.

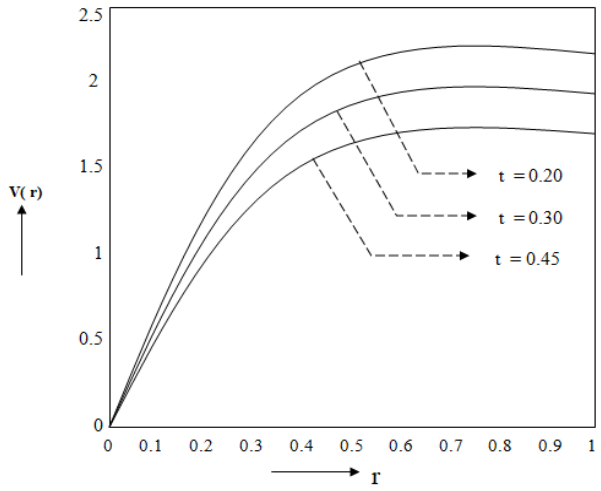

Figure 5: Variation of cell rotational velocity (unsteady case)

Fig.5 shows the variation of cell rotational velocity (unsteady case) for different values of the time t. It may be observed that cell rotational velocity increases decreases with time.

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