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Sum of The Degrees of Dominating Set And Complementary Dominating Set Using Eulidean Division Algorithm of Divisor 5 For Interval Graph G

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Abstract— Interval graphs, their importance over the years can be seen in the increasing number of researchers trying to explore the field. The concept of the domination is a rapidly developing area in Graph Theory. In this paper, we tried to present some relations on the sum of degree of the vertices in dominating set and complementary dominating set using Euclidean division algorithm of divisor 5 for interval graph G.

Keywords—Interval graph, Domination number, complementary dominating set, complementary domination number.

I. INTRODUCTION

Interval graphs have drawn the attention of many researchers for over 25years.They have extensively been studied and revealed their practical relevance for modeling problems arising in the real world. Let I = $\{ I_1, I_2, \dots, I_n\}$ be an interval family where each I_i is an interval on the real line and I_i= [a_i , b_i] for i=1,2,3,…,n. Here a_i is called the left end point and b_i is called right end point of I_i. Without loss of generality, we assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Let G= (V, E) is called an interval graph if there is a one to one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals have non-intersection in I. We denote this interval graph by G[I].

II. PRELIMINARIES

All graphs considered in this paper are finite, undirected, with no loop or multi edge. A graph is said to be a simple graph if it has no loops and no parallel edges. The number of edges incident with a vertex V is called degree of V and is denoted by d(v). If d(v) = 1 then the vertex v is said to be a pendent vertex. If d(v) = 0 then the vertex v is said to be an isolated vertex. Let G = (V, E) be a graph, a set $D \subseteq V$ is a dominating set of G if every vertex in V\D is adjacent to some vertex in D. A dominating set[2, 3, 5, 9] with minimum cardinality among all the dominating sets of a graph G is said to be Minimum dominating set. Given two integers 'a' and 'b' (≠ 0), there exists unique q and r such that a = b q + r, $0 \le r < b$ where a is called a dividend, b is called a divisor, q is called a quotient and r is called a reminder.

III. PREREQUISITE RESULTS

RESULT1

Let I = $\{ I_1, I_2, I_3, I_4, \cdots\cdots\cdots, I_n \}$, $\forall n \ge 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect the next two intervals only and let G(V,E) be an interval graph corresponding to an interval family I if there is one-to-one correspondence between the vertex set V and interval family I where $V = \{v_1, v_2, v_3, \dots\dots\dots, v_n\}$ then

(i) $d(v_1) = d(v_n) = 2$ (ii) $d(v_2) = d(v_{n-1}) = 3$ and (iii) $d(v_3) = d(v_4) =$ ×××××××××= $d(v_{n-2}) = 4$.

RESULT2

Let I= $\{ I_1, I_2, I_3, I_4, \dots\dots\dots\dots, I_n \}$, $\forall n \ge 5$ be any finite interval family such that every interval I_i , i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 0, then the Minimum dominating set $D = \{ V_{5m-2} \;/\; m \in N, 1 \le m \le q \}$ and the domination number $\gamma(G) = q$ and then the following results are true.

$(i)\sum_{v\in D} d(v)=4q$ $(ii)\sum_{v\in D} d(v)=4\gamma$

$(iii)\sum_{v\in D} d(v)=\frac{4n}{5}$

$(iv)\sum_{v\in D^C} d(v)=16q-6$ $(v)\sum_{v\in D^C} d(v)=4\gamma^C-6$

$(vi)\sum_{v\in D^c} d(v)=\frac{16n-30}{5}$.

RESULT3

Let I= $\{I_1,I_2,I_3,I_4,............,I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1,n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 1, then the Minimum dominating set $D=\{V_{5m-2},V_n \ / \ m\in N,1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v)=4q+2$ $(ii)\sum_{v\in D} d(v)=4\gamma-2$

$(iii)\sum_{v\in D} d(v)=\frac{4n+6}{5}$.

$(iv)\sum_{v\in D^c} d(v)=16q-4$ $(v)\sum_{v\in D^c} d(v)=4\gamma^c-4$

$(vi)\sum_{v\in D^c} d(v)=\frac{16n-36}{5}$.

RESULT4

Let I= $\{I_1,I_2,I_3,I_4,............,I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1,n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 2, then two cases will arise

Case(i): The Minimum dominating set $D=\{V_{5m-2},V_{n-1} \ / \ m\in N,1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v)=4q+3$ $(ii)\sum_{v\in D} d(v)=4\gamma-1$

$(iii)\sum_{v\in D} d(v)=\frac{4n+7}{5}$.

$(iv)\sum_{v\in D^c} d(v)=16q-1$ $(v)\sum_{v\in D^c} d(v)=4\gamma^c-5$

$(vi)\sum_{v\in D^c} d(v)=\frac{16n-37}{5}$.

Case(ii): The Minimum dominating set $D=\{V_{5m-2},V_n \ / \ m\in N,1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v)=4q+2$ $(ii)\sum_{v\in D} d(v)=4\gamma-2$

$(iii)\sum_{v\in D} d(v)=\frac{4n+2}{5}$.

$(iv)\sum_{v\in D^c} d(v)=16q$ $(v)\sum_{v\in D^c} d(v)=4\gamma^c-4$

$(vi)\sum_{v\in D^c} d(v)=\frac{16n-32}{5}$.

RESULT5

Let I= $\{I_1,I_2,I_3,I_4,............,I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1,n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 3, then three cases will arise

Case(i): The Minimum dominating set $D=\{V_{5m-2},V_{n-2} \ / \ m\in N,1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v)=4q+4$ $(ii)\sum_{v\in D} d(v)=4\gamma$

$(iii)\sum_{v\in D} d(v)=\frac{4n+8}{5}$

$(iv)\sum_{v\in D^C} d(v)=16q+2$ $(v)\sum_{v\in D^C} d(v)=4\gamma^C-6$

$(vi)\sum_{v\in D^C} d(v)=\frac{16n-38}{5}$.

Case(ii): The Minimum dominating set $D=\{V_{5m-2},V_{n-1} \ / \ m\in N,1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v) = 4q+3$ $(ii)\sum_{v\in D} d(v) = 4\gamma-1$

$(iii)\sum_{v\in D} d(v) = \frac{4n+3}{5}$

$(iv)\sum_{v\in D^C} d(v) = 16q+3$ $(v)\sum_{v\in D^C} d(v) = 4\gamma^C-5$

$(vi)\sum_{v\in D^C} d(v) = \frac{16n-33}{5}$.

Case(iii): The Minimum dominating set $D=\{V_{5m-2}, V_n \ / \ m\in N, 1\le m\le q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v) = 4q+2$ $(ii)\sum_{v\in D} d(v) = 4\gamma-2$

$(iii)\sum_{v\in D} d(v) = \frac{4n-2}{5}$

$(iv)\sum_{v\in D^C} d(v) = 16q+4$ $(v)\sum_{v\in D^C} d(v) = 4\gamma^C-4$

$(vi)\sum_{v\in D^C} d(v) = \frac{16n-28}{5}$.

RESULT6

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\ge 5$ be any finite interval family such that every interval I_i , i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 4, then two cases will arise

Case(i): The Minimum dominating set $D=\{V_{5m-2}, V_{n-2} \ / \ m\in N, 1\le m\le q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v) = 4q+4$ $(ii)\sum_{v\in D} d(v) = 4\gamma$

$(iii)\sum_{v\in D} d(v) = \frac{4n+4}{5}$.

$(iv)\sum_{v\in D^C} d(v) = 16q+6$ $(v)\sum_{v\in D^C} d(v) = 4\gamma^C-6$

$(vi)\sum_{v\in D^C} d(v) = \frac{16n-34}{5}$.

Case(ii): The Minimum dominating set $D=\{V_{5m-2}, V_{n-1} \ / \ m\in N, 1\le m\le q\}$ and the domination number $\gamma(G) = q+1$ and then the following results are true.

$(i)\sum_{v\in D} d(v) = 4q+3$ $(ii)\sum_{v\in D} d(v) = 4\gamma-1$

$(iii)\sum_{v\in D} d(v) = \frac{4n-1}{5}$.

$(iv)\sum_{v\in D^C} d(v) = 16q+7$ $(v)\sum_{v\in D^C} d(v) = 4\gamma^C-5$

$(vi)\sum_{v\in D^C} d(v) = \frac{16n-29}{5}$.

RESULT7

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$ be any finite interval family and let G be an interval graph corresponding to an interval family I. D be a minimum dominating set and D^c is a compliment dominating set of an interval graph G. And γ and γ^C be the domination numbers of an interval graph G with respect to D and D^C respectively then $\gamma+\gamma^C = n$.

IV. MAIN THEOREMS

THEOREM 1

Let I = $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\ge 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 0, then the Minimum dominating set $D=\{V_{5m-2} \ / \ m\in N, 1\le m\le q\}$ and the domination number $\gamma(G) = q$ and then $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$

PROOF:

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\ge 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={v_1,v_2,v_3,v_4,…………………,v_n}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any integer, n is a number of vertices in G.

Suppose r = 0 then we have n = 5q + 0= 5q $\Rightarrow q = \frac{n}{5}$

And the minimum dominating set $D=\{V_{5m-2} \ / \ m\in N, 1\le m\le q\}$, $\gamma(G)$= q.

That is D = $\{v_3, v_8, v_{13}, ..., v_{n-2}\}$, $\gamma(G) = \frac{n}{5}$

case(i): Theorem will be prove through the quotient :

We know that $\sum_{v\in D} d(v) = 4q$ and $\sum_{v\in D^C} d(v) = 16q - 6.$

And then

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4q + 16q - 6 = 20q - 6 = 4(5q) - 6 = 4n - 6 \quad \because n = 5q$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

case(ii): Theorem will be prove through the domination number :

We know that $\sum_{v\in D} d(v) = 4\gamma$ and $\sum_{v\in D^C} d(v) = 4\gamma^C - 6.$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4\gamma + 4\gamma^C - 6 = 4(\gamma + \gamma^C) - 6 = 4n - 6 \quad \because \gamma + \gamma^C = n$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

case(iii): Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v\in D} d(v) = \frac{4n}{5}$ and

$$\sum_{v\in D^C} d(v) = \frac{16n - 30}{5}$$

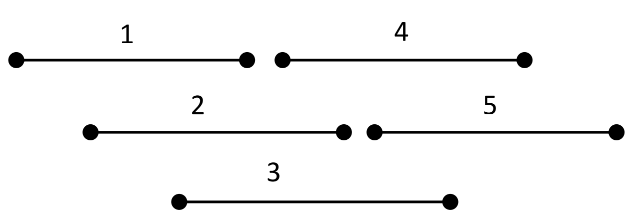
$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n}{5} + \frac{16n - 30}{5} = \frac{4n + 16n - 30}{5}$$

$$= \frac{20n - 30}{5} = 4n - 6 \therefore$$

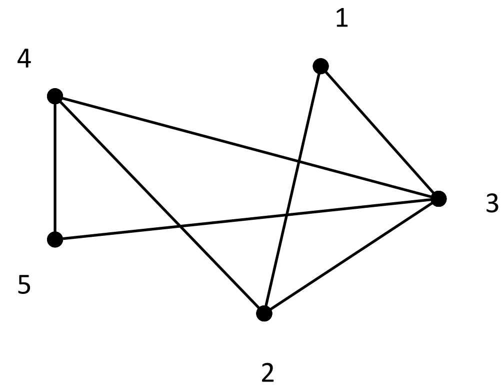
$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

Practical problem

Let I= $\{ I_1, I_2, I_3, I_4, I_5 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRPH G

Here n = 5 = 5 × 1 + 0

This is of the form n = 5q + r here q = 1 and r = 0.

If r = 0, then the Minimum dominating set D = $\{v_3\}$, since theorem 1 and the domination number $\gamma = 1 = q$

And $D^C = \{v_1, v_2, v_4, v_5\}$ and then $\gamma^C = 4$

And also we have $\sum_{v\in D} d(v) = d(v_3) = 4$

$$\sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + d(v_5)$$

$$= 2 + 3 + 3 + 2 = 10$$

$$\therefore \sum_{v\in D^C} d(v) = 10$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4 + 10 = 14 \cdots\cdots\cdots\cdots(1)$$

$$and \ \ 4n - 6 = 4\times 5 - 6 = 20 - 6 = 14 \cdots\cdots\cdots\cdots(2)$$

From (1) and (2)

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

Hence theorem is verified .

THEOREM 2

Let I= $\{ I_1, I_2, I_3, I_4, ..., I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 1, then the Minimum dominating set $D = \{ V_{5m-2}, V_n \ / \ m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q + 1$ and then $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$

PROOF: Let I= $\{ I_1, I_2, I_3, I_4, ..., I_n \}$, $\forall n \ge 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \cdots\cdots\cdots, V_n\}$.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any integer, n is a number of vertices in G.

Suppose r = 1 then we have n = 5q + 1 $\Rightarrow q = \frac{n-1}{5}$

And the minimum dominating set $D = \{ V_{5m-2}, V_n \ / \ m \in N, 1 \le m \le q \}$ and $\gamma(G)$= q + 1.

That is D= $\{v_3, v_8, v_{13}, ..., v_{n-3}, v_n\}$ and $\gamma(G)$= $\frac{n+4}{5}$

case(i): Theorem will be prove through the quotient :

We know that $\sum_{v \in D} d(v) = 4q + 2$ and

$\sum_{v \in D^C} d(v) = 16q - 4$

Now $\sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4q + 2 + 16q - 4$

= 20q – 2= 4(5q) – 2

= 4(n-1) – 2 (since n= 5q + 1)

= 4n – 4 –2 = 4n – 6.

$$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n - 6.$$

case(ii): Theorem will be prove through the domination number :

We know that $\sum_{v \in D} d(v) = 4\gamma - 2$ and

$\sum_{v \in D^C} d(v) = 4\gamma^C - 4$

$$Now \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4\gamma - 2 + 4\gamma^C - 4$$

$$= 4\gamma + 4\gamma^C - 6$$

$$= 4\left(\gamma + \gamma^C\right) - 6$$

$$= 4n - 6 \qquad \because \gamma + \gamma^C = n$$

$$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n - 6.$$

case(iii): Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v \in D} d(v) = \frac{4n+6}{5}$ and

$\sum_{v \in D^C} d(v) = \frac{16n-36}{5}$

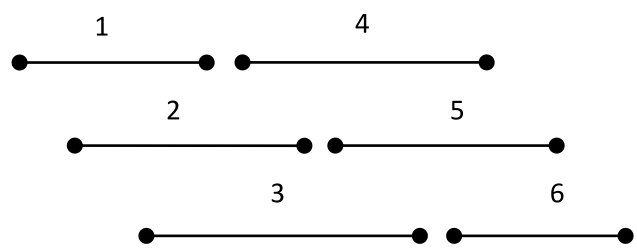
$$Now \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = \frac{4n+6}{5} + \frac{16n-36}{5}$$

$$= \frac{4n+6+16n-36}{5}$$

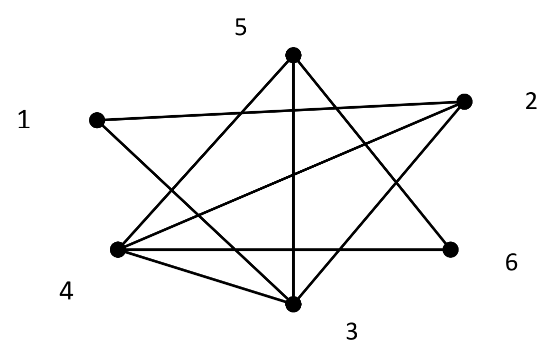
$$= \frac{20n-30}{5} = 4n - 6$$

$$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n - 6.$$

Practical problem: Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 6 = 5 × 1 + 1

This is of the form n = 5q + r and then q = 1, r = 1.

If r = 1, then the Minimum dominating set D = { v_3, v_6 } and $\gamma = 2$ since theorem 2.

And D^C = {v_1, v_2, v_4, v_5} and then $\gamma^C = 4$

$\sum_{v \in D} d(v) = d(v_3) + d(v_6) = 4 + 2 = 6$

$$\sum_{v\in D^C} d(v)=d(v_1)+d(v_2)+d(v_4)+d(v_5)=2+3+4+3=12$$

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=6+12=18\cdots\cdots\cdots(1)$$

$$and\ \ 4n-6=4\times 6-6=24-6=18\cdots\cdots\cdots(2)$$

From (1) and (2)

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6$$

Hence theorem 2 is verified .

THEOREM 3

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i , i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 2, then two cases will arise

Case(i): The Minimum dominating set $D=\{V_{5m-2}, V_{n-1}\ /\ m\in N, 1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$

Case(ii): The Minimum dominating set $D=\{V_{5m-2}, V_n\ /\ m\in N, 1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$

PROOF:

Let I= $\{I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V=\{v_1, v_2, v_3, v_4, \cdots\cdots\cdots, v_n\}$.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 2 then we have n = 5q + 2 $\Rightarrow q=\frac{n-2}{5}$

Case (i):

The minimum dominating set

$D=\{V_{5m-2}, V_{n-1}\ /\ m\in N, 1\leq m\leq q\}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, \ldots\ldots\ldots\ldots, v_{n-4}, v_{n-1}\}$ and

$\gamma(G)=\frac{n+3}{5}$

Sub case(i):

Theorem will be prove through the quotient :

We know that $\sum_{v\in D} d(v)=4q+3$ and

$$\sum_{v\in D^C} d(v)=16q-1$$

Now $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4q+3+16q-1$

= 20q + 2= 4(5q) + 2

= 4(n – 2) + 2 (since n= 5q + 2)

= 4n – 8 + 2= 4n – 6

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$$

Sub case (ii):

Theorem will be prove through the domination number :

We know that $\sum_{v\in D} d(v)=4\gamma-1$ and

$$\sum_{v\in D^C} d(v)=4\gamma^C-5$$

$$Now \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4\gamma-1+4\gamma^C-5$$

$$=4\gamma+4\gamma^C-6$$

$$=4\left(\gamma+\gamma^C\right)-6$$

$$=4n-6 \qquad \because \gamma+\gamma^C=n$$

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$$

Sub case(iii):

Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v\in D} d(v)=\frac{4n+7}{5}$ and

$$\sum_{v\in D^C} d(v)=\frac{16n-37}{5}$$

$$\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=\frac{4n+7}{5}+\frac{16n-37}{5}$$

$$=\frac{4n+7+16n-37}{5}$$

$$=\frac{20n-30}{5}=4n-6$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Case (ii):

The minimum dominating set

$D=\{V_{5m-2}, V_n \;/\; m\in N, 1\le m\le q\}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, ..., v_{n-4}, v_n\}$ and $\gamma(G)$ = $\frac{n+3}{5}$

Sub case(i):

Theorem will be prove through the quotient :

We know that $\sum_{v\in D} d(v) = 4q+2$ and $\sum_{v\in D^C} d(v) = 16q$

Now $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4q+2+16q$

= 20q + 2= 4(5q) + 2

= 4(n – 2) + 2 (since n= 5q + 2)

= 4n – 8 + 2 = 4n – 6

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case(ii):

Theorem will be prove through the domination number:

We know that $\sum_{v\in D} d(v) = 4\gamma - 2$ and

$$\sum_{v\in D^C} d(v) = 4\gamma^C - 4$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4\gamma - 2 + 4\gamma^C - 4$$

$$= 4\gamma + 4\gamma^C - 6 \qquad \therefore$$

$$= 4(\gamma + \gamma^C) - 6$$

$$= 4n - 6 \qquad \because \gamma + \gamma^C = n$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v\in D} d(v) = \frac{4n+2}{5}$ and

$$\sum_{v\in D^C} d(v) = \frac{16n-32}{5}$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n+2}{5} + \frac{16n-32}{5}$$

$$= \frac{4n+2+16n-32}{5}$$

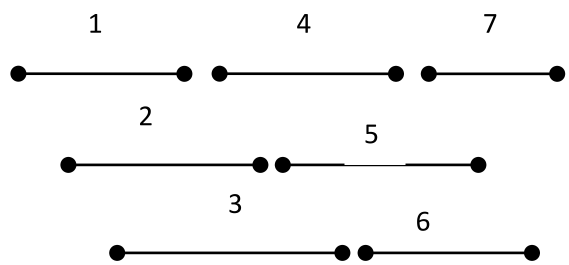
$$= \frac{20n-30}{5}$$

$$= 4n-6$$

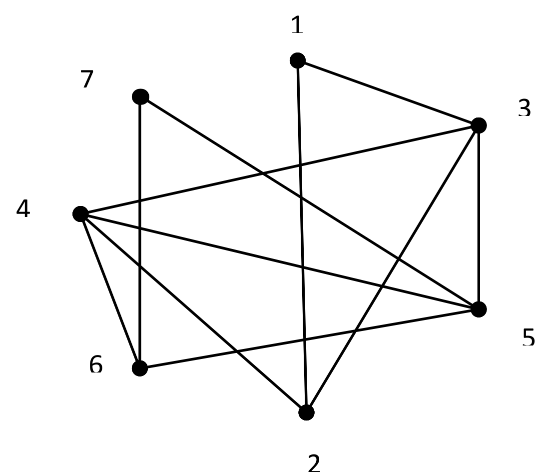
$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Practical problem:

Let I= $\{I_1, I_2, I_3, I_4, I_5, I_6, I_7\}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 7 = 5 × 1 + 2

This is of the form n = 5q + r and then q = 1, r = 2.

If r =2, then two cases will arise

Case(i): The Minimum dominating set D = $\{v_3, v_6\}$ and $\gamma = 2$ since theorem 3.

And D^C = $\{v_1, v_2, v_4, v_5, v_7\}$ and then $\gamma^C = 5$

$$\sum_{v\in D} d(v) = d(v_3) + d(v_6) = 4+3 = 7$$

$$\sum_{v\in D^C} d(v)=d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_7)=2+3+4+4+2=15$$

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=7+15=22\cdots\cdots\cdots(1)$$

$$and\ 4n-6=4\times7-6=28-6=22\cdots\cdots\cdots(2)$$

From (1) and (2)

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6$$

Case(ii):The Minimum dominating set D = {v_3 ,v_7 } and $\gamma = 2$ since theorem 3.

And D^C = {v_1, v_2, v_4, v_5, v_6} and then $\gamma^C = 5$

$$\sum_{v\in D} d(v)=d(v_3)+d(v_7)=4+2=6$$

$$\sum_{v\in D^C} d(v)=d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)=2+3+4+4+3=16$$

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=6+16=22\cdots\cdots\cdots(1)$$

$$and\ 4n-6=4\times7-6=28-6=22\cdots\cdots\cdots(2)$$

From (1) and (2)

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6$$

Hence theorem 3 is verified.

THEOREM 4

Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i , i ≠ $\{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 3, then three cases will arise

Case(i): The Minimum dominating set $D=\{ V_{5m-2}, V_{n-2}\ /\ m\in N, 1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$

Case(ii): The Minimum dominating set $D=\{ V_{5m-2}, V_{n-1}\ /\ m\in N, 1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$

Case(iii): The Minimum dominating set $D=\{ V_{5m-2}, V_{n}\ /\ m\in N, 1\leq m\leq q\}$ and the domination number $\gamma(G)=q+1$ and then $\sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$

PROOF:

Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots\ldots, I_n\}$, $\forall n\geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersects the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={v_1,v_2,v_3,v_4,…………………,v_n}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any integer, n is a number of vertices in G.

Suppose r = 3 then we have n = 5q + 3 $\Rightarrow q=\frac{n-3}{5}$

Case (i):

The minimum dominating set

$D=\{ V_{5m-2}, V_{n-2}\ /\ m\in N, 1\leq m\leq q\}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, \ldots\ldots\ldots\ldots, v_{n-5}, v_{n-2}\}$ and $\gamma(G)=\frac{n+2}{5}$

Sub case(i): Theorem will be prove through the quotient :

$$We\ know\ that\ \sum_{v\in D} d(v)=4q+4\ and\ \sum_{v\in D^C} d(v)=16q+2$$

$$\text{Now}\quad \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4q+4+16q+2$$

= 20q + 6= 4(5q) + 6

= 4(n – 3) + 6 (since n= 5q + 3)

= 4n – 12 + 6 = 4n - 6

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$$

Sub case (ii): Theorem will be prove through the domination number: We know that $\sum_{v\in D} d(v)=4\gamma$ and $\sum_{v\in D^C} d(v)=4\gamma^C-6$

$$Now \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4\gamma+4\gamma^C-6$$

$$=4\gamma+4\gamma^C-6$$

$$=4\left(\gamma+\gamma^C\right)-6$$

$$=4n-6 \qquad (\because \gamma+\gamma^C=n)$$

$$\therefore \sum_{v\in D} d(v)+\sum_{v\in D^C} d(v)=4n-6.$$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

And also we have $\sum_{v\in D} d(v) = \frac{4n+8}{5}$ and

$\sum_{v\in D^C} d(v) = \frac{16n-38}{5}$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n+8}{5} + \frac{16n-38}{5}$$
$$= \frac{4n+8+16n-38}{5}$$
$$= \frac{20n-30}{5}$$
$$= 4n-6$$
$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Case (ii):
The minimum dominating set
$D = \{ V_{5m-2}, V_{n-1} \ / \ m \in N, 1 \le m \le q\}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, ..., v_{n-5}, v_{n-1}\}$ and $\gamma(G)$= $\frac{n+2}{5}$

Sub case(i): Theorem will be prove through the quotient :

$$We know that \sum_{v\in D} d(v) = 4q+3 \ and \sum_{v\in D^C} d(v) = 16q+3$$
$$\text{Now} \quad \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4q+3+16q+3$$

= 20q + 6= 4(5q) + 6

= 4(n – 3) + 6 (since n= 5q + 3)

= 4n – 12 + 6 = 4n – 6

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case (ii): Theorem will be prove through the domination number:

We know that $\sum_{v\in D} d(v) = 4\gamma - 1$ and

$\sum_{v\in D^C} d(v) = 4\gamma^C - 5$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4\gamma - 1 + 4\gamma^C - 5$$
$$= 4\gamma + 4\gamma^C - 6$$
$$= 4(\gamma + \gamma^C) - 6$$
$$= 4n - 6 \qquad (\because \gamma + \gamma^C = n)$$
$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

And also we have $\sum_{v\in D} d(v) = \frac{4n+3}{5}$ and

$\sum_{v\in D^C} d(v) = \frac{16n-33}{5}$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n+3}{5} + \frac{16n-33}{5}$$
$$= \frac{4n+3+16n-33}{5}$$
$$= \frac{20n-30}{5}$$
$$= 4n-6$$
$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Case (iii):
The minimum dominating set
$D = \{ V_{5m-2}, V_{n} \ / \ m \in N, 1 \le m \le q\}$ and $\gamma(G)$= q + 1.

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Sub case(i): Theorem will be prove through the quotient :

$$We know that \sum_{v\in D} d(v) = 4q+2 \ and \sum_{v\in D^C} d(v) = 16q+4$$
$$\text{Now} \quad \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4q+2+16q+4$$

= 20q + 6

= 4(5q) + 6

= 4(n – 3) + 6 (since n= 5q + 3)

= 4n – 12 + 6

= 4n – 6

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case (ii): Theorem will be prove through the domination number:

We know that $\sum_{v\in D} d(v) = 4\gamma - 2$ and

$$\sum_{v\in D^C} d(v) = 4\gamma^C - 4$$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4\gamma - 2 + 4\gamma^C - 4$$

$$= 4\gamma + 4\gamma^C - 6$$

$$= 4\left(\gamma + \gamma^C\right) - 6$$

$$= 4n - 6 \qquad (\because \gamma + \gamma^C = n)$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

And also we have $\sum_{v\in D} d(v) = \frac{4n-2}{5}$ and

$$\sum_{v\in D^C} d(v) = \frac{16n-28}{5}$$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n-2}{5} + \frac{16n-28}{5}$$

$$= \frac{4n-2+16n-28}{5}$$

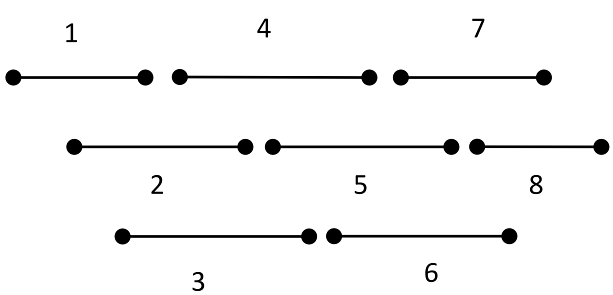
$$= \frac{20n-30}{5}$$

$$= 4n - 6$$

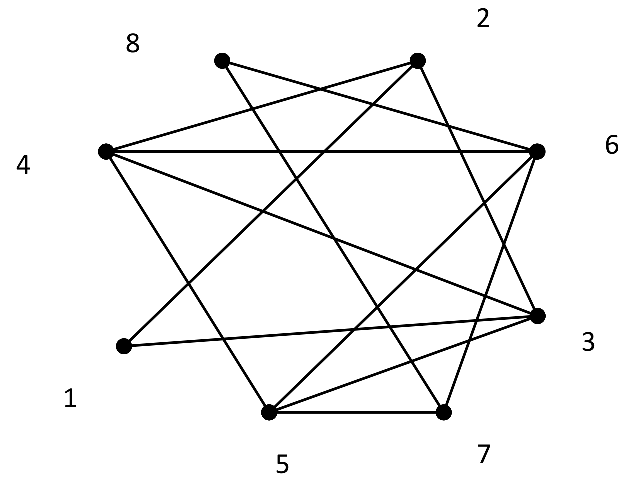
$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6.$$

Practical problem:

Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 8 = 5 × 1 + 3

This is of the form n = 5q + r and then q = 1, r = 3.

If r =3, then three cases will arise.

Case(i): The Minimum dominating set D = $\{v_3, v_6\}$ and $\gamma = 2$ since theorem 4.

And $D^C = \{v_1, v_2, v_4, v_5, v_7, v_8\}$ and then $\gamma^C = 6$

$$Now \sum_{v\in D} d(v) = d(v_3) + d(v_6) = 4 + 4 = 8$$

$$\sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + d(v_5) + d(v_7) + d(v_8) = 2+3+4+4+3+2 = 18$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 8 + 18 = 26 \cdots\cdots\cdots (1)$$

$$and \quad 4n - 6 = 4\times 8 - 6 = 32 - 6 = 26 \cdots\cdots\cdots (2)$$

From (1) and (2)

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6$$

Case(ii): The Minimum dominating set D = $\{v_3, v_7\}$ and $\gamma = 2$ since theorem 4.

And $D^C = \{v_1, v_2, v_4, v_5, v_6, v_8\}$ and then $\gamma^C = 6$

$$Now \sum_{v\in D} d(v) = d(v_3) + d(v_7) = 4 + 3 = 7$$

$$\sum_{v\in D^C} d(v) = d(v_1) + d(v_2) + d(v_4) + d(v_5) + d(v_6) + d(v_8) = 2+3+4+4+4+2 = 19$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 7 + 19 = 26 \cdots\cdots\cdots (1)$$

$$and \quad 4n - 6 = 4\times 8 - 6 = 32 - 6 = 26 \cdots\cdots\cdots (2)$$

From (1) and (2)

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n - 6$$

Case(iii): The Minimum dominating set D = $\{v_3, v_8\}$ and $\gamma = 2$ since theorem 4.

And $D^C = \{v_1, v_2, v_4, v_5, v_6, v_7\}$ and then $\gamma^C = 6$

$$Now \sum_{v\in D} d(v) = d(v_3) + d(v_8) = 4 + 2 = 6$$

$$\sum_{v\in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7) = 2+3+4+4+4+3 = 20$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 6 + 20 = 26 \cdots\cdots\cdots(1)$$

$$and \quad 4n-6 = 4\times 8 - 6 = 32 - 6 = 26 \cdots\cdots\cdots(2)$$

From (1) and (2)

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6$$

Hence theorem 4 is verified.

THEOREM 5 Let I= $\{I_1, I_2, I_3, I_4, ..., I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n-1, n\}$ intersect next two intervals only and let G be an interval graph corresponding to an interval family I. suppose n = 5q + r, where r = 0, 1, 2, 3, 4 and q is any positive integer. If r = 4, then two cases will arise

Case(i): The Minimum dominating set $D = \{V_{5m-2}, V_{n-2} \;/\; m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$

Case(ii): The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} \;/\; m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q+1$ and then $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$

PROOF: Let I= $\{I_1, I_2, I_3, I_4, ..., I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i, i ≠ $\{n, n-1\}$ intersects the next two intervals only and let G(V, E) be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where V={$v_1, v_2, v_3, v_4, \ldots, v_n$}.

By Euclidean division algorithm, n = 5q + r where r = 0, 1, 2, 3, 4 and q is any positive integer, n is a number of vertices in G.

Suppose r = 4 then we have n = 5q + 4

$$\Rightarrow q = \frac{n-4}{5} \quad \textit{and then two cases will arise}$$

Case(i): The minimum dominating set $D = \{V_{5m-2}, V_{n-2} \;/\; m \in N, 1 \leq m \leq q\}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, ..., v_{n-6}, v_{n-2}\}$ and $\gamma(G) = \frac{n+1}{5}$

Sub case(i): Theorem will be prove through the quotient:

We know that $\sum_{v\in D} d(v) = 4q+4$ and $\sum_{v\in D^C} d(v) = 16q+6$

$$\text{Now} \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4q+4+16q+6$$

= 20q + 10= 4(5q) + 10

= 4(n – 4) + 10 (since n = 5q + 4)

= 4n – 16 + 10 = 4n – 6

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case (ii): Theorem will be prove through the domination number:

We have $\sum_{v\in D} d(v) = 4\gamma$, $\sum_{v\in D^C} d(v) = 4\gamma^C - 6$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4\gamma + 4\gamma^C - 6$$

$$= 4\gamma + 4\gamma^C - 6$$

$$= 4(\gamma + \gamma^C) - 6$$

$$= 4n - 6 \qquad (\because \gamma + \gamma^C = n)$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v\in D} d(v) = \frac{4n+4}{5}$ and $\sum_{v\in D^C} d(v) = \frac{16n-34}{5}$

$$Now \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \frac{4n+4}{5} + \frac{16n-34}{5}$$

$$= \frac{4n+4+16n-34}{5}$$

$$= \frac{20n-30}{5}$$

$$= 4n-6$$

$$\therefore \sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 4n-6.$$

Case(ii): The minimum dominating set $D = \{ V_{5m-2}, V_{n-1} \;/\; m \in N,\, 1 \le m \le q \}$ and $\gamma(G)$= q + 1.

That is D = $\{v_3, v_8, v_{13}, \ldots\ldots\ldots\ldots, v_{n-6}, v_{n-1}\}$ and $\gamma(G)$= $\frac{n+1}{5}$

Sub case(i): Theorem will be prove through the quotient:

We have $\sum_{v \in D} d(v) = 4q+3$, $\sum_{v \in D^C} d(v) = 16q+7$

Now $\sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4q+3+16q+7$

= 20q + 10

= 4(5q) + 10

= 4(n – 4) + 10 (since n= 5q + 4)

= 4n – 16 + 10

= 4n - 6

$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n-6.$

Sub case (ii): Theorem will be prove through the domination number:

We have $\sum_{v \in D} d(v) = 4\gamma - 1$, $\sum_{v \in D^C} d(v) = 4\gamma^C - 5$

$$Now \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4\gamma - 1 + 4\gamma^C - 5$$

$$= 4\gamma + 4\gamma^C - 6$$

$$= 4(\gamma + \gamma^C) - 6$$

$$= 4n - 6 \qquad (\because \gamma + \gamma^C = n)$$

$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n-6.$

Sub case(iii): Theorem will be prove through the total number of vertices of the interval graph :

We know that $\sum_{v \in D} d(v) = \frac{4n-1}{5}$ and $\sum_{v \in D^C} d(v) = \frac{16n-29}{5}$

$$Now \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = \frac{4n-1}{5} + \frac{16n-29}{5}$$

$$= \frac{4n-1+16n-29}{5}$$

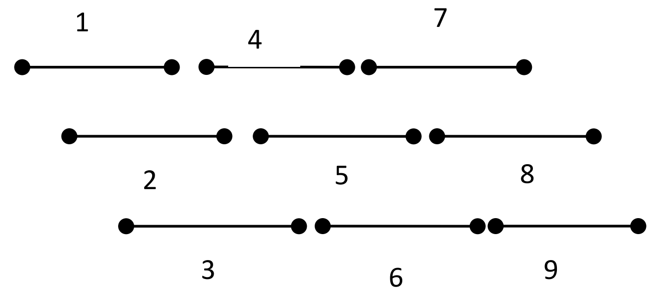
$$= \frac{20n-30}{5}$$

$$= 4n-6$$

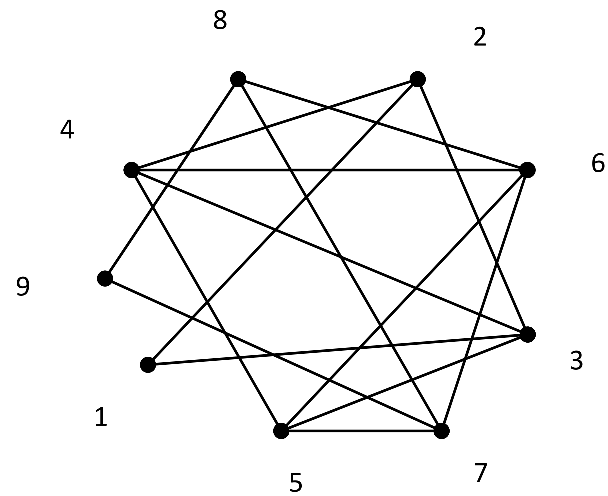
$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 4n-6.$

Practicalproblem:

LetI= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here n = 9 = 5 × 1 + 4

This is of the form n = 5q + r and then q = 1, r = 4.

If r =4, then two cases will arise

Case(i): The Minimum dominating set D = $\{v_3, v_7\}$ and $\gamma = 2$ since theorem 5.

And $D^C = \{v_1, v_2, v_4, v_5, v_6, v_8, v_9\}$ and then $\gamma^C = 7$

$$Now \sum_{v \in D} d(v) = d(v_3) + d(v_7) = 4+4 = 8$$

$$and \sum_{v \in D^C} d(v) = d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_8)+d(v_9) = 2+3+4+4+4+3+2 = 22$$

$$\therefore \sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 8+22 = 30 \cdots\cdots\cdots\cdot (1)$$

$$and\ \ 4n-6=4\times 9-6=36-6=30\cdots\cdots\cdots(2)$$

From (i) and (ii)

$$\sum_{v\in D} d\left(v\right)+\sum_{v\in D^C} d\left(v\right)=4n-6$$

Case(ii): The Minimum dominating set D = {v_3 ,v_8 } and $\gamma = 2$ since theorem 5.

And D^C = {v_1, v_2, v_4, v_5, v_6, v_7, v_9 } and then $\gamma^C = 7$

$$Now\ \ \sum_{v\in D} d(v)=d(v_3)+d(v_8)=4+3=7$$

$$and \sum_{v\in D^C} d(v)=d(v_1)+d(v_2)+d(v_4)+d(v_5)+d(v_6)+d(v_7)+d(v_9)=2+3+4+4+4+4+2=23$$

$$\therefore \sum_{v\in D} d\left(v\right)+\sum_{v\in D^C} d\left(v\right)=7+23=30\cdots\cdots\cdots(1)$$

$$and\ \ 4n-6=4\times 9-6=36-6=30\cdots\cdots\cdots(2)$$

From (i) and (ii)

$$\sum_{v\in D} d\left(v\right)+\sum_{v\in D^C} d\left(v\right)=4n-6$$

Hence theorem 5 is verified.

V. CONCLUSION

In this paper, we focuses on some relations to sum of the degrees of the vertices in dominating set and complementary dominating set by using the Euclidean division algorithm of divisor 5 of the interval graph G corresponding to an interval family I. In future efforts in the paper eventually open up many an avenue in the field of research on interval graphs.

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