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Profit Analysis of a Computer System with Software Redundancy Subject to Hardware Inspection

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***Abstract* –** In this research paper, the authors examined reliability modelling of a computer system with software redundancy by introducing the concept of inspection of hardware component. The system fails independently from normal mode. All the repair activities such as hardware repair, software up-gradation, hardware inspection and hardware replacement are carried out by a single server immediately on need basis. The failed hardware component undergoes for repair or replacement after inspection. All random variables are statistically independent. The negative exponential distribution is taken for the failure time of the component while the distributions of repair time, up-gradation time and replacement time are assumed arbitrary with different probability density functions. Semi-Markov process and regenerative point technique are used for obtaining the values various performance measures. The behaviour of some important performance measure has been examined for different parameters and costs. The profit comparison of the present model has also been made with that of the model analyzed by Munday and Malik [2015].

***Keywords* -** Computer System, Software Redundancy, Up-gradation, Inspection, Replacement, Profit Analysis and Stochastic Modelling.

I. INTRODUCTION

Now a day's, the importance of computer systems cannot be denied in the corporate or business world, at the workplace and even in one's personnel life. And, the emerging use of computers in all industrial sectors leads to the need to postulate and design computing systems which could fulfil the desires of the targeted applications at the minimum cost. The computer can be found here and there in every store, restaurants, industries, offices, etc. Basically a computer is made up of two components i.e. hardware and software. Disruption in a computer system may be of the type hardware and software failures. Hardware failure may because of several reasons including wear out, abnormal environment, electrical stress and poor design. The software failure in a computer system may because of improper instructions and programming, latent faults or carelessness or wrong coding. Therefore, the computer systems should be allowed to operate under the supervision of a skilled and knowledgeable person in order to maintain the reliability. The reliability analysis deals with the proper functioning of the components. Barlow and Prochan [1965] defined Mathematical Theory on Reliability that focuses on the ability of a system to perform its intended function". Several techniques have been suggested by the designers and engineers for performance improvement of the systems. Branson and Shah [1971] applied semi-Markov approach for evaluating reliability measures of a system with different failure and repair time distributions. Srinivasan and Gopalan [1973] analyzed a two-unit warm standby system with single repair facility by using the regenerative point technique. Arora [1977] introduced the idea of priority while evaluating the reliability of a system. Adachi and Kodama [1980] defined availability analysis of two-unit warm standby system with an inspection time. Gopalan and Naidu [1982] stressed on the cost-benefit analysis of a one-server system subject to inspection. Goel et al. [1986] carried out reliability analysis of a system with preventive maintenance, inspection and two types of repair. Singh and Singh [1992] analyzed profit evaluation of two-unit cold standby system changing two types of independent repair facilities. Tuteja and Malik (1994) developed a system model with pre-inspection and two types of repairman. The unit wise redundancy technique has been considered as one of these in the development of stochastic models for computer systems. Malik and Anand [2012], Kumar et al. [2012] and Malik and Sureria [2012] analyzed computer systems with cold standby redundancy under different failures and repair policies. Also, Malik and Munday [2014] tried to establish a stochastic model for a computer system by providing hardware redundancy in cold standby. Kumar and Saini [2018] analyzed stochastic modeling and cost-benefit analysis of computing device with fault detection subject to expert repair facility. Recently, Munday and Permila (2020) examined reliability measures of a computer system with software redundancy subject to maximum repair time.

The basic interest of the authors in this paper is to evaluate reliability measures of a computer system with software redundancy in cold standby by introducing the concept of inspection of failed hardware component. The system fails independently from normal mode. All the repair activities such as hardware repair, software up-gradation, hardware inspection and hardware replacement are carried out by a single server immediately on need basis. The failed hardware component undergoes for repair or replacement after inspection. All random variables are statistically independent. The negative exponential distribution is taken for the failure time of the component while the distributions of repair time, up-gradation time and replacement time are assumed arbitrary with different probability density functions. Semi-Markov process and regenerative point technique are used for obtaining the values various performance measures. The behaviour of some important performance measure has been examined for different parameters and costs. The profit comparison of the present model has also been made with that of the model analyzed by Munday and Malik [2015].

II. NOTATIONS

E: Set of regenerative states
$\bar{E}$: Set of non-regenerative states
O: Computer system is operative
Scs: Software is in cold standby
a/b: Probability that the system has hardware / software failure
α_0/β_0: Probability that the system undergoes for hardware repair /hardware replacement after inspection
λ_1/λ_2 : Hardware/Software failure rate
HFUr /HFWr: The hardware is failed and under repair/waiting for repair
SFUg/SFWUg : The software is failed and under/waiting for up-gradation
HFURp /HFWRp : The hardware is failed and under replacement/waiting for eplacement
HFUi /HFWi : The hardware is failed and under/waiting for inspection
HFUR/HFWR : The hardware is failed and continuously under repair / waiting for repair from previous state
SFUG/SFWUG : The software is failed and continuously under up-gradation /waiting for up- gradation from previous state
HFURP/HFWRP : The hardware is failed and continuously under replacement / waiting for replacement from previous state
HFUI/HFWI : The hardware is failed and continuously under/waiting for inspection from previous state
g(t)/G(t) : pdf/cdf of hardware repair time
f(t)/F(t) : pdf/cdf of software up-gradation time
r(t)/R(t) :pdf/cdf of hardware replacement time
$q_{ij}(t)/Q_{ij}(t)$: pdf / cdf of first passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in (0, t]
$q_{ij.k}(t)/Q_{ij.k}(t)$: pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state $S_k$once in (0, t]
$M_i(t)$: Probability that the system up initially in state $S_i \epsilon E$ is up at time t without visiting to any regenerative state
$W_i(t)$: Probability that the server is busy in the state S_i up to time‘t’ without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t)\,dt = \sum_j m_{ij} ,$$

where T denotes the time to systemfailure.
m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that

$\mu_i = \sum_j m_{ij}$

$$and\ m_{ij} = \int_0^\infty t dQ_{ij}\,(t) = -q_{ij}^{*'}(0)$$

Ⓢ/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/** : Symbol for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)
P1 : Profit of the Model as shown in Munday and Malik (2015)
P : Profit of the present model

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}\,(t)dt$

$p_{01} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}$, $p_{02} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}$, $p_{13} = \frac{\alpha_0}{\alpha_0+\beta_0}$,

$p_{14} = \frac{\beta_0}{\alpha_0+\beta_0}$, $p_{20} = f^*(a\lambda_1 + b\lambda_2)$,

$p_{25} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$,

$p_{26} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$, $p_{30} = g^*(0)$,

$p_{40} = r^*(0)$ $p_{52} = p_{61} = f^*(0)$

For $g(t) = \alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ and $r(t) = \beta e^{-\beta t}$, we have

$p_{21.6} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$

$p_{22.5} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\{1 - f^*(a\lambda_1 + b\lambda_2)\}$

But, $f^*(0) = g^*(0) = r^*(0) = 1$,

$p + q = 1\ and\ \alpha_0 + \beta_0 = 1$

It can be easily verified that

$$p_{01} + p_{02} = p_{13} + p_{14} = p_{20} + p_{25} + p_{26} = p_{30} = p_{40} = p_{52} = p_{61} = p_{20} + p_{21.6} + p_{22.5} = 1$$

The mean sojourn times (μ_i) is the state S_i are

$\mu_0 = \frac{1}{a\lambda_1+b\lambda_2}$ $\mu_1 = \frac{1}{\alpha_0+\beta_0}$

$\mu_2 = \frac{1}{a\lambda_1+b\lambda_2+\theta}$, $\mu_2' = \frac{(a\lambda_1+b\lambda_2)(\theta+1)+\theta^3}{\theta^2(a\lambda_1+b\lambda_2+\theta)^2}$

Also
$m_{01}+m_{02}=\mu_0,\ m_{13}+m_{14}=\mu_1,\ m_{20}+m_{25}+m_{26}=\mu_2,\ m_{30}=\mu_3,\ m_{40}=\mu_4$
and $m_{20}+m_{21.6}+m_{22.5}=\mu_2^{'}$

IV. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$,

$$\phi_0(t)=Q_{02}(t)\,\&\,\phi_2(t)+Q_{01}(t)$$

$$\phi_2(t)=Q_{20}(t)\,\&\,\phi_0(t)+Q_{25}(t)+Q_{26}(t) \quad (1)$$

Taking LST of above relations (1) and solving for $\phi_0^{**}(s)$
We have

$$R^*(s)=\frac{1-\phi_0^{**}(s)}{s}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of the above equation.
The mean time to system failure (MTSF) is given by

$$MTSF=\lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s}=\frac{N_1}{D_1} \quad (2)$$

Where $N_1=\mu_0+p_{02}\mu_2\ and\ D_1=1-p_{02}p_{20}$ (3)

V. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at an instant't' given that the system entered regenerative state $S_i\ at\ t=0$. The recursive relations for $A_i(t)$ are given as:

$A_0(t)=M_0(t)+q_{01}(t)©A_1(t)+q_{02}(t)©\,A_2(t)$
$A_1(t)=q_{13}(t)©A_3(t)+q_{14}(t)©A_4(t)$
$A_2(t)=M_2(t)+q_{20}(t)©A_0(t)+q_{21.6}(t)©\,A_1(t)+q_{22.5}(t)©\,A_2(t)$
$A_3(t)=q_{30}(t)©A_0(t)$
$A_4(t)=q_{40}(t)©A_0(t)$ (4)

Where
$M_0(t)=e^{-(a\lambda_1+b\lambda_2)t}\ and\ M_2(t)=e^{-(a\lambda_1+b\lambda_2)t}\,\overline{F(t)}$
Taking LT of relations (4) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0}s\,A_0^*(s)=\frac{N_2}{D_2} \quad (5)$$

Where
$N_2=(1-p_{22.5})\mu_0+p_{02}\mu_2$ and
$D_2=(1-p_{22.5})\mu_0+p_{02}\mu_2^{'}+\mu_1+\ p_{13}\mu_3+p_{14}\mu_4$ (6)

VI. BUSY PERIOD OF THE SERVER

A. DUE TO HARDWARE REPAIR

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant 't' given that the system entered state $S_i\ at\ t=0$. The recursive relations for $B_i^H(t)$ are as follows:

$B_0^H(t)=q_{01}(t)©B_1^H(t)+\ q_{02}(t)©B_2^H(t)$
$B_1^H(t)=q_{13}(t)©B_3^H(t)+q_{14}(t)©B_4^H(t)$
$B_2^H(t)=q_{20}(t)©B_0^H(t)+q_{21.6}(t)©B_1^H(t)+q_{22.5}(t)©B_2^H(t)$
$B_3^H(t)=W_3^H(t)+q_{30}(t)©B_0^H(t)$
$B_4^H(t)=q_{40}(t)©B_0^H(t)$
where $W_3^H(t)=\ \overline{G(t)}\,dt$ (7)

B. DUE TO SOFTWARE UP-GRADATION

Let $B_i^S(t)$ be the probability that the server is busy due to up-gradation of the software at an instant't' given that the system entered the regenerative state $S_i\ at\ t=0$. We have the following recursive relations for $B_i^S(t)$:

$B_0^S(t)=q_{01}(t)©B_1^S(t)+\ q_{02}(t)©B_2^S(t)$
$B_1^S(t)=q_{13}(t)©B_3^S(t)+q_{14}(t)©B_4^S(t)$
$B_2^S(t)=W_2^S(t)+q_{20}(t)©B_0^S(t)+q_{21.6}(t)©B_1^S(t)+q_{22.5}(t)©B_2^S(t)$
$B_3^S(t)=q_{30}(t)©B_0^S(t)$
$B_4^S(t)=q_{40}(t)©B_0^S(t)$
where
$W_2^S(t)=e^{-(a\lambda_1+b\lambda_2)t}\,\overline{F(t)}+\left(a\lambda_1e^{-(a\lambda_1+b\lambda_2)t}©1\right)\overline{F(t)}+\left(b\lambda_2e^{-(a\lambda_1+b\lambda_2)t}©1\right)\overline{F(t)}$ (8)

C. DUE TO HARDWARE REPLACEMENT

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement of the unit due to hardware failure given that the system entered state $S_i\ at\ t=0$. We have the following recursive relations for $B_i^{Rp}(t)$:

$B_0^{Rp}(t)=q_{01}(t)©B_1^{Rp}(t)+\ q_{02}(t)©B_2^{Rp}(t)$
$B_1^{Rp}(t)=q_{13}(t)©B_3^{Rp}(t)+q_{14}(t)©B_4^{Rp}(t)$
$B_2^{Rp}(t)=q_{20}(t)©B_0^{Rp}(t)+q_{21.6}(t)©B_1^{Rp}(t)+q_{22.5}(t)©B_2^{Rp}(t)$
$B_3^{Rp}(t)=q_{30}(t)©B_0^{Rp}(t)$
$B_4^{Rp}(t)=W_4^{Rp}(t)+q_{40}(t)©B_0^{Rp}(t)$
where $W_4^{Rp}(t)=\ \overline{R(t)}\,dt$ (9)

D. DUE TO HARDWARE INSPECTION

Let $B_i^I(t)$ be the probability that the server is busy in inspection of the unit due to hardware failure given that the system entered state $S_i\ at\ t=0$. We have the following recursive relations for $B_i^I(t)$:

$B_0^I(t)=q_{01}(t)©B_1^I(t)+\ q_{02}(t)©B_2^I(t)$
$B_1^I(t)=W_1^I(t)+q_{13}(t)©B_3^I(t)+q_{14}(t)©B_4^I(t)$
$B_2^I(t)=q_{20}(t)©B_0^I(t)+q_{21.6}(t)©B_1^I(t)+q_{22.5}(t)©B_2^I(t)$
$B_3^I(t)=q_{30}(t)©B_0^I(t)$
$B_4^I(t)=q_{40}(t)©B_0^I(t)$
where $W_1^{Rp}(t)=e^{-(\alpha_0+\beta_0)t}$ (10)

Taking LT of relations (7), (8), (9) and (10), solving for $B_0^{H^*}(t),\ B_0^{S^*}(t),\ B_0^{Rp^*}(t)\ and\ B_0^{I^*}(t)$. The time for which server is busy due to repairs, up-gradations, replacements and inspection respectively are given by

$$B_0^H(t)=\lim_{s\to 0}s\,B_0^{H^*}(t)=\frac{N_3^H}{D_2} \quad (11)$$

$B_0^S(t) = \lim_{s\to 0} s\, B_0^{S^*}(t) = \frac{N_3^S}{D_2}$ (12)

$B_0^{Rp}(t) = \lim_{s\to 0} s\, B_0^{Rp^*}(t) = \frac{N_3^{Rp}}{D_2}$ (13)

$B_0^I(t) = \lim_{s\to 0} s\, B_0^{I^*}(t) = \frac{N_3^I}{D_2}$ (14)

where
$N_3^H = [p_{01}p_{13}(1-p_{22.5}) + p_{02}(1-p_{13}p_{21.6})]\mu_3$,
$N_3^S = p_{02}\mu_2$,
$N_3^{Rp} = [p_{01}(1-p_{22.5}) + p_{02}p_{21.6}]p_{14}\mu_4$, $N_3^I = [p_{01}(1-p_{22.5}) + p_{02}p_{21.6}]\mu_1$
$and\ D_2\ is\ already\ mentioned.$ (15)

VII. EXPECTED NUMBER OF HARDWARE REPAIRS

Let $NHR_i(t)$ be the expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t = 0$. The recursive relations for $NHR_i(t)$ are given as:

$NHR_0(t) = Q_{01}(t)\ \&\ NHR_1(t) + Q_{02}(t)\ \&\ NHR_2(t)$

$NHR_1(t) = Q_{13}(t)\ \&\ [1 + NHR_1(t)] + Q_{14}(t)\ \&\ NHR_4(t)$

$NHR_2(t) = Q_{20}(t)\ \&\ NHR_0(t) + Q_{21.6}(t)\ \&\ NHR_1(t) + Q_{22.5}(t)\ \&\ NHR_2(t)$

$NHR_3(t) = Q_{30}(t)\ \&\ NHR_0(t)$

$NHR_4(t) = Q_{40}(t)\ \&\ NHR_0(t)$ (16)

Taking LST of relations (16) and solving for $NHR_0^{**}(s)$. The expected number of hardware repair is given by
$NHR_0 = \lim_{s\to 0} sNHR_0^{**}(s) = \frac{N_4}{D_2}$ (17)

Where
$N_4 = p_{13}[p_{01}(1-p_{22.5}) + p_{02}p_{21.6}]$ $and\ D_2\ is\ already\ mentioned.$ (18)

VIII. EXPECTED NUMBER OF SOFTWARE UP-GRADATIONS

Let $NSU_i(t)$ be the expected number of software up-gradations in (0, t] given that the system entered the regenerative state $S_i\ at\ t = 0$. The recursive relations for $NSU_i(t)$ are given as follows:

$NSU_0(t) = Q_{01}(t)\ \&\ NSU_1(t) + Q_{02}(t)\ \&\ [1 + NSU_2(t)]$

$NSU_1(t) = Q_{13}(t)\ \&\ NSU_3(t) + Q_{14}(t)\ \&\ NSU_4(t)$

$NSU_2(t) = Q_{20}(t)\ \&\ NSU_0(t) + Q_{21.6}(t)\ \&\ NSU_1(t) + Q_{22.5}(t)\ \&\ NSU_2(t)$

$NSU_3(t) = Q_{30}(t)\ \&\ NSU_0(t)$

$NSU_4(t) = Q_{40}(t)\ \&\ NSU_0(t)$ (19)

Taking LST of relations (19) and solving for $NSU_0^{**}(s)$. The expected numbers of software up-gradation are given by
$NSU_0(\infty) = \lim_{s\to 0} sNSU_0^{**}(s) = \frac{N_5}{D_2}$ (20)

Where
$N_5 = p_{02}(1-p_{22.5})\ and\ D_2\ is\ already\ mentioned$ (21)

IX. EXPECTED NUMBER OF HARDWARE REPLACEMENT

Let $NHRp_i(t)$ be the expected number of hardware replacement by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t = 0$. The recursive relations for $NHRp_i(t)$ are given as:

$NHRp_0(t) = Q_{01}(t)\ \&\ NHRp_1(t) + Q_{02}(t)\ \&\ NHRp_2(t)$

$NHRp_1(t) = Q_{13}(t)\ \&\ NHRp_3(t) + Q_{14}(t)\ \&\ [1 + NHRp_4(t)]$

$NHRp_2(t) = Q_{20}(t)\ \&\ NHRp_0(t) + Q_{21.6}(t)\ \&\ NHRp_1(t) + Q_{22.5}(t)\ \&\ NHRp_2(t)$

$NHRp_3(t) = Q_{30}(t)\ \&\ NHRp_0(t)$

$NHRp_4(t) = Q_{40}(t)\ \&\ NHRp_0(t)$ (22)

Taking LST of relations (22) and solving for $NHRp_0^{**}(s)$. The expected number of hardware replacement is given by
$NHRp_0 = \lim_{s\to 0} sNHRp_0^{**}(s) = \frac{N_6}{D_2}$ (23)

Where
$N_6 = p_{14}[p_{01}p_{20} + p_{21.6}]$ $and\ D_2\ is\ already\ mentioned.$ (24)

X. EXPECTED NUMBER OF HARDWARE INSPECTION

Let $NHI_i(t)$ be the expected number of hardware inspection by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t = 0$. The recursive relations for $NHI_i(t)$ are given as:

$NHI_0(t) = Q_{01}(t)\ \&\ [1 + NHI_1(t)] + Q_{02}(t)\ \&\ NHI_2(t)$

$NHI_1(t) = Q_{13}(t)\ \&\ NHI_3(t) + Q_{14}(t)\ \&\ NHI_4(t)$

$NHI_2(t) = Q_{20}(t)\ \&\ NHI_0(t) + Q_{21.6}(t)\ \&\ NHI_1(t) + Q_{22.5}(t)\ \&\ NHI_2(t)$

$NHI_3(t) = Q_{30}(t)\ \&\ NHI_0(t)$

$NHI_4(t) = Q_{40}(t)\ \&\ NHI_0(t)$ (25)

Taking LST of relations (25) and solving for $NHI_0^{**}(s)$. The expected number of hardware inspection is given by
$NHI_0 = \lim_{s\to 0} sNHI_0^{**}(s) = \frac{N_7}{D_2}$ (26)

Where
$N_7 = p_{01}(1 - p_{22.5}) and\ D_2\ is\ already\ mentioned.$(27)

XI. COST-BENEFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as:

$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_5B_0^{Rp} - K_6B_0^I - K_3NHR_0 - K_4NSU_0 - K_7NHRp_0 - K_8NHI_0$ (28)

Where
$K_0 = Revenue\ per\ unit\ up - time\ of\ the\ system$
$K_1 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ repair$
$K_2 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ software\ up - gradation$
$K_3 = Cost\ per\ unit\ repair\ of\ the\ failed\ hardware$
$K_4 = Cost\ per\ unit\ up - gradation\ of\ the\ failed\ software$
$K_5 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ replacement$
$K_6 = Cost\ per\ unit\ time\ for\ which\ server\ is\ busy\ due\ to\ hardware\ inspection$
$K_7 = Cost\ per\ unit\ replacement\ of\ the\ failed\ hardware$
$K_8 = Cost\ per\ unit\ inspection\ of\ the\ failed\ hardware$
$and\ A_0, B_0^H, B_0^S, B_0^{Rp}, B_0^I, NHR_0, NSU_0, NHRp_0, NHI_0\ are\ already\ defined.$

XII. PARTICULAR CASES

Suppose $g(t) = \alpha e^{-\alpha t},\ f(t) = \theta e^{-\theta t}\ and\ r(t) = \beta e^{-\beta t}$

We can obtain the following results:

$MTSF(T_0) = \frac{N_1}{D_1}$, $Availability(A_0) = \frac{N_2}{D_2}$

$Busy\ Period\ due\ to\ hardware\ failure$
$(B_0^H) = \frac{N_3^H}{D_2}$

$Busy\ Period\ due\ to\ software\ failure$
$(B_0^S) = \frac{N_3^S}{D_2}$

$Busy\ Period\ due\ to\ hardware\ replacement$
$(B_0^{Rp}) = \frac{N_3^{Rp}}{D_2}$

$Busy\ Period\ due\ to\ hardware\ inspection$
$(B_0^I) = \frac{N_3^I}{D_2}$

$Expected\ number\ of\ repair\ at\ hardware\ failure$
$(NHR_0) = \frac{N_4}{D_2}$

$Expected\ number\ of\ up - gradation\ at\ software\ failure\ (NSU_0) = \frac{N_5}{D_2}$

$Expected\ number\ of\ repair\ at\ hardware\ replacement\ (NHRp_0) = \frac{N_6}{D_2}$

$Expected\ number\ of\ repair\ at\ hardware\ inspection\ (NHI_0) = \frac{N_7}{D_2}$

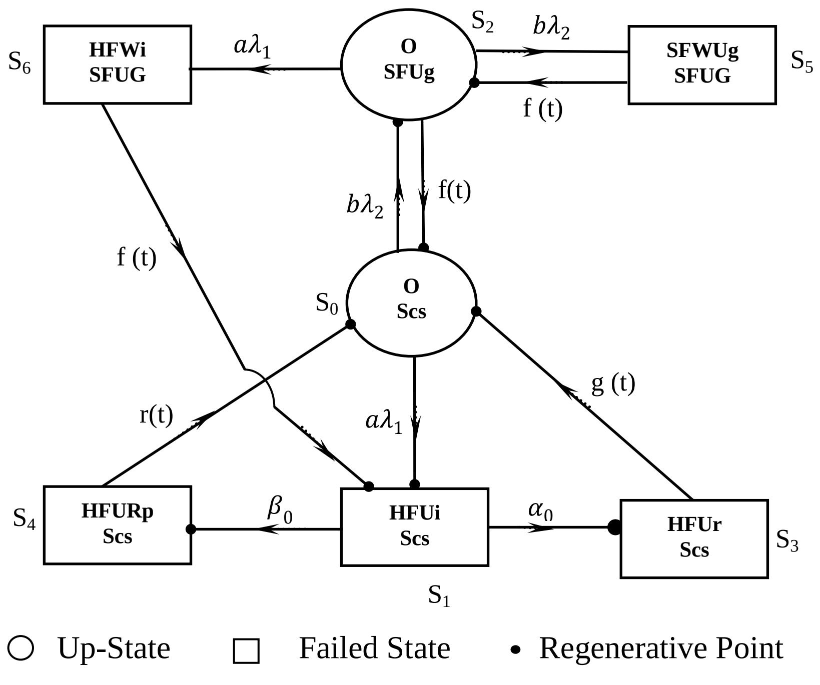

Figure 1: State Transition Diagram

XIII. CONCLUSION

The behaviour of some important performance measures such as MTSF, availability and profit with respect to hardware failure rate (λ_1) has been observed for arbitrary values of various parameters including $K_0 = 15000$, $K_1 = 1000$, $K_2 = 700$, $K_3 = 1500$, $K_4 = 1200$, $K_5 = 500$, $K_6 = 3000$, $K_7 = 400$, $K_8 = 700$ with a=0.6 & b=0.4 and $\alpha_0 = 0.7$and $\beta_0 = 0.3$ as shown respectively in tables 1, 2 and 3. It is observed that these measures go on decreasing with the increase of hardware and software failure rates. But, their values increase with the increase of hardware repair rate (α) and replacement rate (β) and decrease when increase up-gradation rate (θ). On the other hand, if the values of a and b are interchanged i.e. a=0.4 and b=0.6, than MTSF of the system increase while availability and profit declines. And, if the values of α_0 and β_0 are interchanged, than MTSF of the system is constant while availability and profit increases. Also, the system is more profitable when the component goes for repair as compare to replacement. Hence the study reveals that a computer system in which software redundancy is provided in cold standby be more profitable if it has more chances of hardware failure may because of the less hardware repairable cost.

XIV. NUMERICAL PRESENTATION OF RELIABILITY MEASURES

Table 1: MTSF Vs Hardware Failure Rate (λ1)

λ1	λ2=0.001, α=2, θ=5,β=7, a=0.6, b=0.4, α0=0.7, β0=0.3	λ2=0.002	α=3	θ=7	β=9	a=0.4, b=0.6	α0=0.3, β0=0.7
0.01	166.665779	166.6631166	166.665779	166.6660324	166.665779	249.9955048	166.665779
0.02	83.33311168	83.33244687	83.33311168	83.33317489	83.3331117	124.9988771	83.33311168
0.03	55.55545716	55.55516204	55.55545716	55.5554852	55.5554572	83.33283465	55.55545716
0.04	41.66661139	41.66644558	41.66661139	41.66662712	41.6666114	62.49971972	41.66661139
0.05	33.333298	33.333192	33.333298	33.33330805	33.333298	49.99982076	33.333298
0.06	27.77775327	27.77767975	27.77775327	27.77776023	27.7777533	41.66654229	27.77775327
0.07	23.80950582	23.80945187	23.80950582	23.80951093	23.8095058	35.71419441	23.80950582
0.08	20.83331958	20.83327832	20.83331958	20.83332348	20.8333196	31.24993015	20.83331958
0.09	18.51850766	18.51847511	18.51850766	18.51851074	18.5185077	27.77772263	18.51850766
0.1	16.66665788	16.66663154	16.66665788	16.66666037	16.6666579	24.99995537	16.66665788

Table 2: Availability Vs Hardware Failure Rate (λ1)

λ1	λ2=0.001, α=2, θ=5,β=7, a=0.6, b=0.4, α0=0.7, β0=0.3	λ2=0.002	α=3	θ=7	β=9	a=0.4, b=0.6	α0=0.3, β0=0.7
0.01	0.944538404	0.90166544	0.945205	0.872737	0.944593	0.924934029	0.94535474
0.02	0.937233577	0.895090792	0.938506	0.86658	0.937337	0.920281538	0.938792099
0.03	0.930040558	0.888610762	0.931901	0.860508	0.930192	0.915675415	0.932319634
0.04	0.922956801	0.882223325	0.925388	0.854521	0.923155	0.911114971	0.925935498
0.05	0.915979838	0.875926511	0.918965	0.848616	0.916223	0.906599532	0.919637897
0.06	0.909107272	0.86971841	0.91263	0.842791	0.909394	0.902128433	0.913425085
0.07	0.90233678	0.863597161	0.906382	0.837046	0.902666	0.897701028	0.907295363
0.08	0.895666105	0.857560957	0.900219	0.831378	0.896036	0.893316677	0.901247076
0.09	0.889093057	0.851608041	0.894138	0.825786	0.889503	0.888974758	0.895278614
0.1	0.88261551	0.845736703	0.888139	0.820268	0.883064	0.884674657	0.889388407

Table 3: Profit (P) Vs Hardware Failure Rate (λ1)

λ1	λ2=0.001, α=2, θ=5, β=7, a=0.6, b=0.4, α0=0.7, β0=0.3	λ2=0.002	α=3	θ=7	β=9	a=0.4, b=0.6	α0=0.3, β0=0.7
0.01	14147.99756	13505.15267	14158.7065	13072.52208	14148.83907	13860.3349	14157.8014
0.02	14019.35663	13388.31981	14039.76592	12962.51226	14020.96197	13778.02966	14037.86829
0.03	13892.6951	13273.17906	13922.50433	12854.04474	13895.03926	13696.54946	13919.60656
0.04	13767.96785	13159.69418	13806.88663	12747.08749	13771.02671	13615.88202	13802.98193
0.05	13645.13113	13047.82992	13692.8787	12641.60936	13648.8814	13536.0153	13687.96106

0.06	13524.1425	12937.55205	13580.44736	12537.58002	13528.56169	13456.93752	13574.51151
0.07	13404.96076	12828.82727	13469.56031	12434.96999	13410.02715	13378.6371	13462.60169
0.08	13287.54595	12721.62323	13360.18614	12333.75059	13293.23855	13301.10271	13352.20086
0.09	13171.85927	12615.90847	13252.2943	12233.89387	13178.15781	13224.32323	13243.2791
0.1	13057.86303	12511.65236	13145.85503	12135.37267	13064.74793	13148.28775	13135.80726

Table 4: P1 Vs Hardware Failure Rate (λ1)

P1: Profit of the System Model as discussed in Munday and Malik (2015)

λ1	λ2=0.001, α=2, θ=5, a=0.6, b=0.4	λ2=0.002	α=3	θ=7	a=0.4, b=0.6
0.01	14961.27261	14960.46912	14971.89929	14961.29714	14942.63532
0.02	14923.50045	14922.69785	14944.67531	14923.52542	14886.14565
0.03	14885.87848	14885.07679	14917.52365	14885.90389	14829.99191
0.04	14848.40581	14847.60503	14890.44401	14848.43165	14774.17111
0.05	14811.08156	14810.28169	14863.43611	14811.10782	14718.6803
0.06	14773.90485	14773.10589	14836.49967	14773.93152	14663.51655
0.07	14736.87479	14736.07676	14809.6344	14736.90188	14608.67699
0.08	14699.99053	14699.19342	14782.84002	14700.01803	14554.15877
0.09	14663.25121	14662.45503	14756.11626	14663.2791	14499.95907
0.1	14626.65597	14625.86072	14729.46282	14626.68426	14446.0751

XV. COMPARATIVE STUDY OF PROFITS OF THE SYSTEM MODELS

The profit of the present model has been compared with the model as discussed in Munday and Malik (2015) as shown in Table 5. It is observed that the present model is less profitable as compared to that model. Thus, in a computer system with software redundancy in cold standby, the idea of inspection of hardware failed component is not helpful in increasing the profit of the system if system has more chances of hardware failure than that of software failure (a>b). However, this idea can be helpful in improving the profit of a computer system which has less chances of hardware repair as compared to their replacement.

XVI. NUMERICAL PRESENTATION OF PROFIT DIFFERENCE (P1 – P)

Table 5: (P1–P) VS Hardware Failure Rate (λ1)

λ1	λ2=0.001, α=2, θ=5, β=7, a=0.6, b=0.4, α0=0.7, β0=0.3	λ2=0.002	α=3	θ=7	a=0.4, b=0.6
0.01	813.2750502	1455.316444	813.1927847	1888.775055	1082.300422
0.02	904.1438148	1534.378042	904.9093924	1961.013161	1108.115991
0.03	993.1833791	1611.897721	995.0193213	2031.859146	1133.442454
0.04	1080.437962	1687.910848	1083.55738	2101.344156	1158.289096
0.05	1165.950431	1762.451769	1170.55741	2169.498466	1182.664997
0.06	1249.762351	1835.553844	1256.052315	2236.351504	1206.579035
0.07	1331.914033	1907.249485	1340.074096	2301.931887	1230.039891
0.08	1412.444583	1977.570188	1422.653881	2366.26744	1253.056057
0.09	1491.391943	2046.546563	1503.821954	2429.385232	1275.635838
0.1	1568.792936	2114.208364	1583.607785	2491.311593	1297.787357

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