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Development and Analysis of Generalized Queuing Model

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Abstract— In the present work a generalized queuing model has been developed to investigate the various queuing characteristics in steady state. The model consists of two global servers having three servers each which are connected in tri-cum biserial way. The comprehensive governing equations has been given in mathematical formulation which has been used to find the various output parameters i.e., queue lengths, variances, joint probabilities, traffic intensities, average waiting time for customers. The present model is named a generalized queuing model because several models available in the literature can be developed as the special cases.

Keywords— Queue length, Average waiting time, Poisson Law, Moment generating function, Probability.

I. INTRODUCTION

Queuing (waiting line) is pretty common in various real time situations e.g., in a shopping complex, in banks, at mobile phone exchange, at railway station, etc. Extensive investigations have been carried out which dealt with the development of various queuing models to facilitate the customer for better decision in practical problems. In this context, Jacksons [1] took the first step to investigate the various characteristics of phase type service based queue system. Maggu [2] considered the time-dependent probability generating function to investigate the various characteristics of biserial based phase type service queuing model. Arya [3] studied the system of two servers connected in biserial way with multiple service channel. Singh, Man [4] focused to investigate the Steady-state characteristics of serial queuing processes. Hassin and Haviv [5] provided an expressions for equilibrium behaviour in Queuing Systems for better decision making. Gupta *et al* [6] explored the various queuing model parameters consist of biserial and parallel channels connected with a common server. Singh *et al* [7, 8] examined the transient behaviour of a queuing network with parallel biserial queues. Authors further extended their work to investigate the steady state characteristics of a queue models with two sub systems connected in biserial way. Paoumy [9] considered various activities such as Balking, Reneging and Heterogeneous servers while studying the queuing model behaviour. Agrawal and Singh [10, 11, 12, 13] performed comprehensive investigation to find the various queuing model parameters of several recently developed tri-cum bi-serial network based queuing models.

II. PRACTICAL ENACTMENT OF THE MODEL

The developed queuing model can be useful in many problems i.e., if GSr_1 and GSr_2 represent the global server 1 and global server 2 respectively which consist of servers $Sr_\alpha, Sr_\beta, Sr_\gamma$ and Sr_u, Sr_v, Sr_w connected in tri cum biserial way as shown in figure 1. Suppose global servers GSr_1 and GSr_2 show the two floors of a commercial shopping complex which are dedicated to male and female sections. In each section, there are three sub sections such as clothing, footwear and cosmetic which are represented by the servers $Sr_\alpha, Sr_\beta, Sr_\gamma$ and Sr_u, Sr_v, Sr_w in global servers GSr_1 and GSr_2 respectively. The customer first filtered at entry level where male customer will go to GSr_1 and female customer will go to GSr_2. Further suppose a male customer who entered in GSr_1 can avail the facility at server Sr_α which is clothing section then he can go to Sr_β and Sr_γ which depends on his will and requirements. After availing all the facilities, he can exit from the server GSr_1 and move to the server Sr_d which represent the billing section. The same activities is possible while considering the global server GSr_2 which is dedicated to the female customers.

III. MATHEMATICAL DESCRIPTION OF THE MODEL

Let us assume that there are two global servers GSr_1 and GSr_2. Each global server consist of three servers named $Sr_\alpha, Sr_\beta, Sr_\gamma$ and Sr_u, Sr_v, Sr_w which are connected in tri cum biserial way as shown in figure 1. It is evident from the figure that customer entered in any of the global server can avail the facilities available at each server i.e., if customer entered in GSr_1 then he/she can avail the facility Sr_α or Sr_β or Sr_γ and then exit from GSr_1 and move to Sr_d .

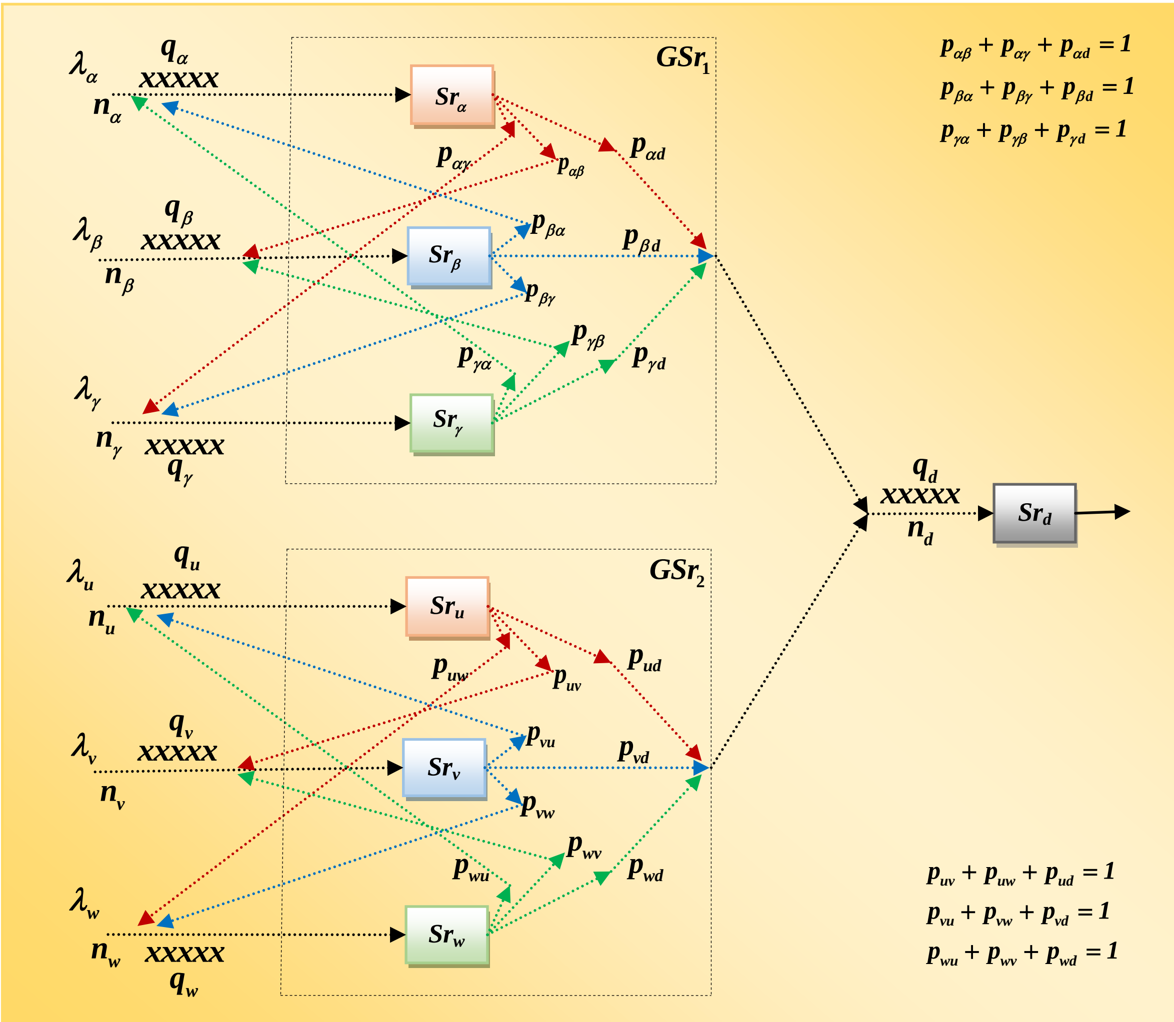


Figure 1. Generalized queuing network

The various combinations of the customer's movement at global servers GSr_1 and GSr_2 are as follows.

$$Sr_\alpha \to Sr_d\,,\quad Sr_\beta \to Sr_d\,,\quad Sr_\gamma \to Sr_d$$

$$Sr_\alpha \to Sr_\beta \to Sr_d\,,\quad Sr_\alpha \to Sr_\gamma \to Sr_d\,,\quad Sr_\beta \to Sr_\alpha \to Sr_d$$

$$Sr_\beta \to Sr_\gamma \to Sr_d\,,\quad Sr_\gamma \to Sr_\alpha \to Sr_d\,,\quad Sr_\gamma \to Sr_\beta \to Sr_d$$

$$Sr_\alpha \to Sr_\beta \to Sr_\gamma \to Sr_d\,,\quad Sr_\alpha \to Sr_\gamma \to Sr_\beta \to Sr_d\,,\quad Sr_\beta \to Sr_\alpha \to Sr_\gamma \to Sr_d$$

$$Sr_\beta \to Sr_\gamma \to Sr_\alpha \to Sr_d\,,\quad Sr_\gamma \to Sr_\alpha \to Sr_\beta \to Sr_d\,,\quad Sr_\gamma \to Sr_\beta \to Sr_\alpha \to Sr_d$$

$$Sr_u \to Sr_d\,,\quad Sr_v \to Sr_d\,,\quad Sr_w \to Sr_d$$

$$Sr_u \to Sr_v \to Sr_d\,,\quad Sr_u \to Sr_w \to Sr_d\,,\quad Sr_v \to Sr_u \to Sr_d$$

$$Sr_v \to Sr_w \to Sr_d\,,\quad Sr_w \to Sr_u \to Sr_d\,,\quad Sr_w \to Sr_v \to Sr_d$$

$$Sr_u \to Sr_v \to Sr_w \to Sr_d\,,\quad Sr_u \to Sr_w \to Sr_v \to Sr_d\,,\quad Sr_v \to Sr_u \to Sr_w \to Sr_d$$

$$Sr_v \to Sr_w \to Sr_u \to Sr_d\,,\quad Sr_w \to Sr_u \to Sr_v \to Sr_d\,,\quad Sr_w \to Sr_v \to Sr_u \to Sr_d$$

Let $\lambda_\alpha, \lambda_\beta, \lambda_\gamma$ and $n_\alpha, n_\beta, n_\gamma$ show the mean arrival rate and number of customers at servers $Sr_\alpha, Sr_\beta, Sr_\gamma$ respectively whereas $q_\alpha, q_\beta, q_\gamma$ denote the queue length associated with these servers respectively. The customer n_α arriving with mean arrival rate λ_α entered to server Sr_α can avail the facility at $Sr_\alpha, Sr_\beta, Sr_\gamma$ such that the cumulative probability $p_{\alpha\beta}+p_{\alpha\gamma}+p_{\alpha d}=1$. The same criterion will be applicable to those customers who entered in GSr_2. The various probabilities associated with the servers at GSr_1 and GSr_2 are as follows.

$$For\ GSr_1 \quad p_{\alpha\beta}+p_{\alpha\gamma}+p_{\alpha d}=1,\ p_{\beta\alpha}+p_{\beta\gamma}+p_{\beta d}=1,\ p_{\gamma\alpha}+p_{\gamma\beta}+p_{\gamma d}=1.$$

$$For\ GSr_2 \quad p_{uv}+p_{uw}+p_{ud}=1,\ p_{vu}+p_{vw}+p_{vd}=1,\ p_{wu}+p_{wv}+p_{wd}=1.$$

Differential difference equation in steady (transient) state of the model is

$$\begin{aligned}
&\begin{bmatrix} \lambda_\alpha+\lambda_\beta+\lambda_\gamma+\lambda_u+\lambda_v+\lambda_w \\ +\mu_\alpha+\mu_\beta+\mu_\gamma+\mu_u+\mu_v+\mu_w+\mu_d \end{bmatrix} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w,n_d} = \lambda_\alpha P_{n_\alpha-1,n_\beta,n_\gamma,n_u,n_v,n_w,n_d} + \lambda_\beta P_{n_\alpha,n_\beta-1,n_\gamma,n_u,n_v,n_w,n_d} \\
&\quad + \lambda_\gamma P_{n_\alpha,n_\beta,n_\gamma-1,n_u,n_v,n_w,n_d} + \lambda_u P_{n_\alpha,n_\beta,n_\gamma,n_u-1,n_v,n_w,n_d} + \lambda_v P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v-1,n_w,n_d} + \lambda_w P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w-1,n_d} \\
&\quad + \mu_\alpha p_{\alpha\beta} P_{n_\alpha+1,n_\beta-1,n_\gamma,n_u,n_v,n_w,n_d} + \mu_\alpha p_{\alpha\gamma} P_{n_\alpha+1,n_\beta,n_\gamma-1,n_u,n_v,n_w,n_d} + \mu_\alpha p_{\alpha d} P_{n_\alpha+1,n_\beta,n_\gamma,n_u,n_v,n_w,n_d-1} \\
&\quad + \mu_\beta p_{\beta\alpha} P_{n_\alpha-1,n_\beta+1,n_\gamma,n_u,n_v,n_w,n_d} + \mu_\beta p_{\beta\gamma} P_{n_\alpha,n_\beta+1,n_\gamma-1,n_u,n_v,n_w,n_d} + \mu_\beta p_{\beta d} P_{n_\alpha,n_\beta+1,n_\gamma,n_u,n_v,n_w,n_d-1} \\
&\quad + \mu_\gamma p_{\gamma\alpha} P_{n_\alpha-1,n_\beta,n_\gamma+1,n_u,n_v,n_w,n_d} + \mu_\gamma p_{\gamma\beta} P_{n_\alpha,n_\beta-1,n_\gamma+1,n_u,n_v,n_w,n_d} + \mu_\gamma p_{\gamma d} P_{n_\alpha,n_\beta,n_\gamma+1,n_u,n_v,n_w,n_d-1} \qquad (1) \\
&\quad + \mu_u p_{uv} P_{n_\alpha,n_\beta,n_\gamma,n_u+1,n_v-1,n_w,n_d} + \mu_u p_{uw} P_{n_\alpha,n_\beta,n_\gamma,n_u+1,n_v,n_w-1,n_d} + \mu_u p_{ud} P_{n_\alpha,n_\beta,n_\gamma,n_u+1,n_v,n_w,n_d-1} \\
&\quad + \mu_v p_{vu} P_{n_\alpha,n_\beta,n_\gamma,n_u-1,n_v+1,n_w,n_d} + \mu_v p_{vw} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v+1,n_w-1,n_d} + \mu_v p_{vd} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v+1,n_w,n_d-1} \\
&\quad + \mu_w p_{wu} P_{n_\alpha,n_\beta,n_\gamma,n_u-1,n_v,n_w+1,n_d} + \mu_w p_{wv} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v-1,n_w+1,n_d} + \mu_w p_{wd} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w+1,n_d-1} \\
&\quad + \mu_d P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w,n_d+1}
\end{aligned}$$

IV. SOLUTION METHODOLOGY

To solve the governing Equation, Generating function is assumed as

$$F\left(X_1,X_2,X_3,X_4,X_5,X_6,X_7\right)=\sum_{n_\alpha=0}^{\infty}\sum_{n_\beta=0}^{\infty}\sum_{n_\gamma=0}^{\infty}\sum_{n_u=0}^{\infty}\sum_{n_v=0}^{\infty}\sum_{n_w=0}^{\infty}\sum_{n_d=0}^{\infty} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w,n_d} X_1^{n_\alpha} X_2^{n_\beta} X_3^{n_\gamma} X_4^{n_u} X_5^{n_v} X_6^{n_w} X_7^{n_d} \quad (2)$$

such that $|X_1|=|X_2|=|X_3|=|X_4|=|X_5|=|X_6|=|X_7|\le 1$

Also, taking partial generating function as

$$F_{n_\beta,n_\gamma,n_u,n_v,n_w,n_d}\left(X_1\right)=\sum_{n_\alpha=0}^{\infty} P_{n_\alpha,n_\beta,n_\gamma,n_u,n_v,n_w,n_d}.X_1^{n_\alpha} \qquad (3)$$

$$F_{n_\gamma,n_u,n_v,n_w,n_d}\left(X_1,X_2\right)=\sum_{n_\beta=0}^{\infty} F_{n_\beta,n_\gamma,n_u,n_v,n_w,n_d}\left(X_1\right).X_2^{n_\beta} \qquad (4)$$

$$F_{n_u,n_v,n_w,n_d}\left(X_1,X_2,X_3\right)=\sum_{n_\gamma=0}^{\infty} F_{n_\gamma,n_u,n_v,n_w,n_d}\left(X_1,X_2\right).X_3^{n_\gamma} \qquad (5)$$

$$F_{n_v,n_w,n_d}\left(X_1,X_2,X_3,X_4\right)=\sum_{n_u=0}^{\infty} F_{n_u,n_v,n_w,n_d}\left(X_1,X_2,X_3\right).X_4^{n_u} \qquad (6)$$

$$F_{n_w,n_d}\left(X_1,X_2,X_3,X_4,X_5\right)=\sum_{n_v=0}^{\infty}F_{n_v,n_w,n_d}\left(X_1,X_2,X_3,X_4\right).X_5^{\ n_v} \quad (7)$$

$$F_{n_d}\left(X_1,X_2,X_3,X_4,X_5,X_6\right)=\sum_{n_w=0}^{\infty}F_{n_w,n_d}\left(X_1,X_2,X_3,X_4,X_5\right).X_6^{\ n_w} \quad (8)$$

$$F\left(X_1,X_2,X_3,X_4,X_5,X_6,X_7\right)=\sum_{n_d=0}^{\infty}F_{n_d}\left(X_1,X_2,X_3,X_4,X_5,X_6\right).X_7^{\ n_d} \quad (9)$$

Now, on making $n_\alpha, n_\beta, n_\gamma, n_u, n_v, n_w, n_d$ equal to zero with various combinations such as one by one then after considering two of them pairwise, etc. will lead to the development of 128 equations. Now solving equation (1) by using generating function set of equations and the technique given in [2, 7], we can find the probability distribution function in steady (transient) state. We get the subsequent equation

$$\begin{bmatrix}\lambda_\alpha(1-X_1)+\lambda_\beta(1-X_2)+\lambda_\gamma(1-X_3)+\lambda_u(1-X_4)+\lambda_v(1-X_5)+\lambda_w(1-X_6)\\ +\mu_\alpha\left\{1-\frac{p_{\alpha\beta}}{X_1}X_2-\frac{p_{\alpha\gamma}}{X_1}X_3-\frac{p_{\alpha d}}{X_1}X_7\right\}+\mu_\beta\left\{1-\frac{p_{\beta\alpha}}{X_2}X_1-\frac{p_{\beta\gamma}}{X_2}X_3-\frac{p_{\beta d}}{X_2}X_7\right\}\\ +\mu_\gamma\left\{1-\frac{p_{\gamma\alpha}}{X_3}X_1-\frac{p_{\gamma\beta}}{X_3}X_2-\frac{p_{\gamma d}}{X_3}X_7\right\}+\mu_u\left\{1-\frac{p_{uv}}{X_4}X_5-\frac{p_{uw}}{X_4}X_6-\frac{p_{ud}}{X_4}X_7\right\}\\ +\mu_v\left\{1-\frac{p_{vu}}{X_5}X_4-\frac{p_{vw}}{X_5}X_6-\frac{p_{vd}}{X_5}X_7\right\}+\mu_w\left\{1-\frac{p_{wu}}{X_6}X_4-\frac{p_{wv}}{X_6}X_5-\frac{p_{wd}}{X_6}X_7\right\}+\mu_d\left\{1-\frac{1}{X_7}\right\}\end{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6,X_7)$$

$$=\mu_\alpha\left\{1-\frac{p_{\alpha\beta}}{X_1}X_2-\frac{p_{\alpha\gamma}}{X_1}X_3-\frac{p_{\alpha d}}{X_1}X_7\right\}F_0(X_2,X_3,X_4,X_5,X_6,X_7)$$

$$+\mu_\beta\left\{1-\frac{p_{\beta\alpha}}{X_2}X_1-\frac{p_{\beta\gamma}}{X_2}X_3-\frac{p_{\beta d}}{X_2}X_7\right\}F_0(X_1,X_3,X_4,X_5,X_6,X_7)$$

$$+\mu_\gamma\left\{1-\frac{p_{\gamma\alpha}}{X_3}X_1-\frac{p_{\gamma\beta}}{X_3}X_2-\frac{p_{\gamma d}}{X_3}X_7\right\}F_0(X_1,X_2,X_4,X_5,X_6,X_7)$$

$$+\mu_u\left\{1-\frac{p_{uv}}{X_4}X_5-\frac{p_{uw}}{X_4}X_6-\frac{p_{ud}}{X_4}X_7\right\}F_0(X_1,X_2,X_3,X_5,X_6,X_7)$$

$$+\mu_v\left\{1-\frac{p_{vu}}{X_5}X_4-\frac{p_{vw}}{X_5}X_6-\frac{p_{vd}}{X_5}X_7\right\}F_0(X_1,X_2,X_3,X_4,X_6,X_7)$$

$$+\mu_w\left\{1-\frac{p_{wu}}{X_6}X_4-\frac{p_{wv}}{X_6}X_5-\frac{p_{wd}}{X_6}X_7\right\}F_0(X_1,X_2,X_3,X_4,X_5,X_7)$$

$$+\mu_d\left\{1-\frac{1}{X_7}\right\}F_0(X_1,X_2,X_3,X_4,X_5,X_6)$$

Assuming

$$F_0(X_2,X_3,X_4,X_5,X_6,X_7)=F_\alpha,\ F_0(X_1,X_3,X_4,X_5,X_6,X_7)=F_\beta,\ F_0(X_1,X_2,X_4,X_5,X_6,X_7)=F_\gamma$$

$$F_0(X_1,X_2,X_3,X_5,X_6,X_7)=F_u,\ F_0(X_1,X_2,X_3,X_4,X_6,X_7)=F_v,\ F_0(X_1,X_2,X_3,X_4,X_5,X_7)=F_w$$

$$F_0(X_1,X_2,X_3,X_4,X_5,X_6)=F_d$$

We get,

$$F(X_1,X_2,X_3,X_4,X_5,X_6,X_7)=\frac{\begin{array}{l}\mu_\alpha\left\{1-\frac{p_{\alpha\beta}}{X_1}X_2-\frac{p_{\alpha\gamma}}{X_1}X_3-\frac{p_{\alpha d}}{X_1}X_7\right\}F_\alpha+\mu_\beta\left\{1-\frac{p_{\beta\alpha}}{X_2}X_1-\frac{p_{\beta\gamma}}{X_2}X_3-\frac{p_{\beta d}}{X_2}X_7\right\}F_\beta\\+\mu_\gamma\left\{1-\frac{p_{\gamma\alpha}}{X_3}X_1-\frac{p_{\gamma\beta}}{X_3}X_2-\frac{p_{\gamma d}}{X_3}X_7\right\}F_\gamma+\mu_u\left\{1-\frac{p_{uv}}{X_4}X_5-\frac{p_{uw}}{X_4}X_6-\frac{p_{ud}}{X_4}X_7\right\}F_u\\+\mu_v\left\{1-\frac{p_{vu}}{X_5}X_4-\frac{p_{vw}}{X_5}X_6-\frac{p_{vd}}{X_5}X_7\right\}F_v+\mu_w\left\{1-\frac{p_{wu}}{X_6}X_4-\frac{p_{wv}}{X_6}X_5-\frac{p_{wd}}{X_6}X_7\right\}F_v\\+\mu_d\left\{1-\frac{1}{X_7}\right\}F_w\end{array}}{\begin{array}{l}\lambda_\alpha(1-X_1)+\lambda_\beta(1-X_2)+\lambda_\gamma(1-X_3)+\lambda_u(1-X_4)+\lambda_v(1-X_5)+\lambda_w(1-X_6)\\+\mu_\alpha\left\{1-\frac{p_{\alpha\beta}}{X_1}X_2-\frac{p_{\alpha\gamma}}{X_1}X_3-\frac{p_{\alpha d}}{X_1}X_7\right\}+\mu_\beta\left\{1-\frac{p_{\beta\alpha}}{X_2}X_1-\frac{p_{\beta\gamma}}{X_2}X_3-\frac{p_{\beta d}}{X_2}X_7\right\}\\+\mu_\gamma\left\{1-\frac{p_{\gamma\alpha}}{X_3}X_1-\frac{p_{\gamma\beta}}{X_3}X_2-\frac{p_{\gamma d}}{X_3}X_7\right\}+\mu_u\left\{1-\frac{p_{uv}}{X_4}X_5-\frac{p_{uw}}{X_4}X_6-\frac{p_{ud}}{X_4}X_7\right\}\\+\mu_v\left\{1-\frac{p_{vu}}{X_5}X_4-\frac{p_{vw}}{X_5}X_6-\frac{p_{vd}}{X_5}X_7\right\}+\mu_w\left\{1-\frac{p_{wu}}{X_6}X_4-\frac{p_{wv}}{X_6}X_5-\frac{p_{wd}}{X_6}X_7\right\}\\+\mu_d\left\{1-\frac{1}{X_7}\right\}\end{array}} \quad (10)$$

As $F(1,1,1,1,1,1,1) = 1$, the entire probability. On considering $X_1 = 1$ as $X_2 \to 1$, $X_3 \to 1$, $X_4 \to 1$, $X_5 \to 1$, $X_6 \to 1$, $X_7 \to 1$, eq (10) $F(X_1,X_2,X_3,X_4,X_5,X_6,X_7)$ is of (0/0) form, which is indeterminate. Therefore, by L-Hospital rule, differentiating eq (10) w.r.t. X_1, we get

$$1=\frac{\mu_\alpha\left(p_{\alpha\beta}+p_{\alpha\gamma}+p_{\alpha d}\right)F_\alpha+\mu_\beta\left(-p_{\beta\alpha}\right)F_\beta+\mu_\gamma\left(-p_{\gamma\alpha}\right)F_\gamma}{-\lambda_\alpha+\mu_\alpha\left(p_{\alpha\beta}+p_{\alpha\gamma}+p_{\alpha d}\right)+\mu_\beta\left(-p_{\beta\alpha}\right)+\mu_\gamma\left(-p_{\gamma\alpha}\right)}$$

where $p_{\alpha\beta}+p_{\alpha\gamma}+p_{\alpha d}=1$

$$\mu_\alpha F_\alpha-\mu_\beta p_{\beta\alpha}F_\beta-\mu_\gamma p_{\gamma\alpha}F_\gamma=-\lambda_\alpha+\mu_\alpha-\mu_\beta p_{\beta\alpha}-\mu_\gamma p_{\gamma\alpha} \quad (11)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_2 by taking $X_2 = 1$ as $X_1 \to 1$, $X_3 \to 1$, $X_4 \to 1$, $X_5 \to 1$, $X_6 \to 1$, $X_7 \to 1$, we get

$$1=\frac{\mu_\alpha\left(-p_{\alpha\beta}\right)F_\alpha+\mu_\beta\left(p_{\beta\alpha}+p_{\beta\gamma}+p_{\beta d}\right)F_\beta+\mu_\gamma\left(-p_{\gamma\beta}\right)F_\gamma}{-\lambda_\beta+\mu_\alpha\left(-p_{\alpha\beta}\right)+\mu_\beta\left(p_{\beta\alpha}+p_{\beta\gamma}+p_{\beta d}\right)+\mu_\gamma\left(-p_{\gamma\beta}\right)}$$

where $p_{\beta\alpha}+p_{\beta\gamma}+p_{\beta d}=1$

$$-\mu_\alpha p_{\alpha\beta}F_\alpha+\mu_\beta F_\beta-\mu_\gamma p_{\gamma\beta}F_\gamma=-\lambda_\beta-\mu_\alpha p_{\alpha\beta}+\mu_\beta-\mu_\gamma p_{\gamma\beta} \quad (12)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_3 by taking $X_3 = 1$ as $X_1 \to 1$, $X_2 \to 1$, $X_4 \to 1$, $X_5 \to 1$, $X_6 \to 1$, $X_7 \to 1$, we get

$$1=\frac{\mu_\alpha\left(-p_{\alpha\gamma}\right)F_\alpha+\mu_\beta\left(-p_{\beta\gamma}\right)F_\beta+\mu_\gamma\left(p_{\gamma\alpha}+p_{\gamma\beta}+p_{\gamma d}\right)F_\gamma}{-\lambda_\gamma+\mu_\alpha\left(-p_{\alpha\gamma}\right)+\mu_\beta\left(-p_{\beta\gamma}\right)+\mu_\gamma\left(p_{\gamma\alpha}+p_{\gamma\beta}+p_{\gamma d}\right)}$$

where $p_{\gamma\alpha}+p_{\gamma\beta}+p_{\gamma d}=1$

$$-\mu_\alpha p_{\alpha\gamma} F_\alpha - \mu_\beta p_{\beta\gamma} F_\beta + \mu_\gamma F_\gamma = -\lambda_\gamma - \mu_\alpha p_{\alpha\gamma} - \mu_\beta p_{\beta\gamma} + \mu_\gamma \quad (13)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_4 by taking $X_4 = 1$ as $X_1 \to 1$, $X_2 \to 1, X_3 \to 1, X_5 \to 1, X_6 \to 1, X_7 \to 1$, we get

$$1 = \frac{\mu_u \left(p_{uv} + p_{uw} + p_{ud}\right) F_u + \mu_v \left(-p_{vu}\right) F_v + \mu_w \left(-p_{wu}\right) F_w}{-\lambda_u + \mu_u \left(p_{uv} + p_{uw} + p_{ud}\right) + \mu_v \left(-p_{vu}\right) + \mu_w \left(-p_{wu}\right)}$$

where $p_{uv} + p_{uw} + p_{ud} = 1$

$$\mu_u F_u - \mu_v p_{vu} F_v - \mu_w p_{wu} F_w = -\lambda_u + \mu_u - \mu_v p_{vu} - \mu_w p_{wu} \quad (14)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_5 by taking $X_5 = 1$ as $X_1 \to 1$, $X_2 \to 1, X_3 \to 1, X_4 \to 1, X_6 \to 1, X_7 \to 1$, we get

$$1 = \frac{\mu_u \left(-p_{uv}\right) F_u + \mu_v \left(p_{vu} + p_{vw} + p_{vd}\right) F_v + \mu_w \left(-p_{wv}\right) F_w}{-\lambda_v + \mu_u \left(-p_{uv}\right) + \mu_v \left(p_{vu} + p_{vw} + p_{vd}\right) + \mu_w \left(-p_{wv}\right)}$$

where $p_{vu} + p_{vw} + p_{vd} = 1$

$$-\mu_u p_{uv} F_u + \mu_v F_v - \mu_w p_{wv} F_w = -\lambda_v - \mu_u p_{uv} + \mu_v - \mu_w p_{wv} \quad (15)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_6 by taking $X_6 = 1$ as $X_1 \to 1$, $X_2 \to 1, X_3 \to 1, X_4 \to 1, X_5 \to 1, X_7 \to 1$, we get

$$1 = \frac{\mu_u \left(-p_{uw}\right) F_u + \mu_v \left(-p_{vw}\right) F_v + \mu_w \left(p_{wu} + p_{wv} + p_{wd}\right) F_w}{-\lambda_w + \mu_u \left(-p_{uw}\right) + \mu_v \left(-p_{vw}\right) + \mu_w \left(p_{wu} + p_{wv} + p_{wd}\right)}$$

where $p_{wu} + p_{wv} + p_{wd} = 1$

$$-\mu_u p_{uw} F_u - \mu_v p_{vw} F_v + \mu_w F_w = -\lambda_w - \mu_u p_{uw} - \mu_v p_{vw} + \mu_w \quad (16)$$

Again differentiating numerator and denominator of eq (10) separately w.r.t. X_7 by taking $X_7 = 1$ as $X_1 \to 1$, $X_2 \to 1, X_3 \to 1, X_4 \to 1, X_5 \to 1, X_6 \to 1$, we get

$$1 = \frac{\mu_\alpha \left(-p_{\alpha d}\right) F_\alpha + \mu_\beta \left(-p_{\beta d}\right) F_\beta + \mu_\gamma \left(-p_{\gamma d}\right) F_\gamma + \mu_u \left(-p_{ud}\right) F_u + \mu_v \left(-p_{vd}\right) F_v + \mu_w \left(-p_{wd}\right) F_w + \mu_d F_d}{\mu_\alpha \left(-p_{\alpha d}\right) + \mu_\beta \left(-p_{\beta d}\right) + \mu_\gamma \left(-p_{\gamma d}\right) + \mu_u \left(-p_{ud}\right) + \mu_v \left(-p_{vd}\right) + \mu_w \left(-p_{wd}\right) + \mu_d}$$

$$\begin{aligned}\mu_\alpha \left(-p_{\alpha d}\right) F_\alpha + \mu_\beta \left(-p_{\beta d}\right) F_\beta + \mu_\gamma \left(-p_{\gamma d}\right) F_\gamma + \mu_u \left(-p_{ud}\right) F_u + \mu_v \left(-p_{vd}\right) F_v + \mu_w \left(-p_{wd}\right) F_w + \mu_d F_d \\ = \mu_\alpha \left(-p_{\alpha d}\right) + \mu_\beta \left(-p_{\beta d}\right) + \mu_\gamma \left(-p_{\gamma d}\right) + \mu_u \left(-p_{ud}\right) + \mu_v \left(-p_{vd}\right) + \mu_w \left(-p_{wd}\right) + \mu_d\end{aligned} \quad (17)$$

On solving (11), (12), (13), (14), (15), (16) & (17), we get

$$F_\alpha = 1 - \frac{\lambda_\alpha \left(1 - p_{\gamma\beta} p_{\beta\gamma}\right) + \lambda_\beta \left\{p_{\beta\alpha} (1 - p_{\gamma\beta} p_{\beta\gamma}) + p_{\beta\gamma} \left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)\right\} + \lambda_\gamma \left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)}{\mu_\alpha \left\{\left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)\left(1 - p_{\gamma\beta} p_{\beta\gamma}\right) - \left(p_{\alpha\gamma} + p_{\alpha\beta} p_{\beta\gamma}\right)\left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)\right\}}$$

$$F_\beta = 1 - \frac{\lambda_\alpha \left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right) + \lambda_\beta \left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right) + \lambda_\gamma \left\{p_{\gamma\alpha} \left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right) + p_{\gamma\beta} \left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right)\right\}}{\mu_\beta \left\{\left(1 - p_{\beta\gamma} p_{\gamma\beta}\right)\left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right) - \left(p_{\beta\alpha} + p_{\beta\gamma} p_{\gamma\alpha}\right)\left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right)\right\}}$$

$$F_\gamma = 1 - \frac{\lambda_\alpha \left\{p_{\alpha\beta} \left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right) + p_{\alpha\gamma} \left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)\right\} + \lambda_\beta \left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right) + \lambda_\gamma \left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)}{\mu_\gamma \left\{\left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right)\left(1 - p_{\alpha\beta} p_{\beta\alpha}\right) - \left(p_{\gamma\beta} + p_{\alpha\beta} p_{\gamma\alpha}\right)\left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right)\right\}}$$

$$F_u = 1 - \frac{\lambda_u \left(1 - p_{wv} p_{vw}\right) + \lambda_b \left\{p_{vu}(1 - p_{wv} p_{vw}) + p_{vw}\left(p_{wu} + p_{wv} p_{vu}\right)\right\} + \lambda_w \left(p_{wu} + p_{wv} p_{vu}\right)}{\mu_u \left\{\left(1 - p_{uv} p_{vu}\right)\left(1 - p_{wv} p_{vw}\right) - \left(p_{uw} + p_{uv} p_{vw}\right)\left(p_{wu} + p_{wv} p_{vu}\right)\right\}}$$

$$F_v = 1 - \frac{\lambda_u \left(p_{uv} + p_{uw} p_{wv}\right) + \lambda_v \left(1 - p_{uw} p_{wu}\right) + \lambda_w \left\{p_{wu}\left(p_{uv} + p_{uw} p_{wv}\right) + p_{wv}\left(1 - p_{uw} p_{wu}\right)\right\}}{\mu_v \left\{\left(1 - p_{vw} p_{wv}\right)\left(1 - p_{uw} p_{wu}\right) - \left(p_{vu} + p_{vw} p_{wu}\right)\left(p_{uv} + p_{uw} p_{wv}\right)\right\}}$$

$$F_w = 1 - \frac{\lambda_u \left\{p_{uv}\left(p_{vw} + p_{vu} p_{uw}\right) + p_{uw}\left(1 - p_{uv} p_{vu}\right)\right\} + \lambda_v \left(p_{vw} + p_{vu} p_{uw}\right) + \lambda_w \left(1 - p_{uv} p_{vu}\right)}{\mu_w \left\{\left(1 - p_{uw} p_{wu}\right)\left(1 - p_{uv} p_{vu}\right) - \left(p_{wv} + p_{uv} p_{wu}\right)\left(p_{vw} + p_{vu} p_{uw}\right)\right\}}$$

$$F_d = 1 - \left[\frac{\mu_\alpha p_{\alpha d}}{\mu_d}(1 - F_\alpha) + \frac{\mu_\beta p_{\beta d}}{\mu_d}(1 - F_\beta) + \frac{\mu_\gamma p_{\gamma d}}{\mu_d}(1 - F_\gamma) + \frac{\mu_u p_{ud}}{\mu_d}(1 - F_u) + \frac{\mu_v p_{vd}}{\mu_d}(1 - F_v) + \frac{\mu_w p_{wd}}{\mu_d}(1 - F_w)\right]$$

The solution (Joint Probability) of the model in steady state is written as

$$P_{n_\alpha, n_\beta, n_\gamma, n_u, n_v, n_w, n_d} = \left(1 - F_\alpha\right)^{n_\alpha}\left(1 - F_\beta\right)^{n_\beta}\left(1 - F_\gamma\right)^{n_\gamma}\left(1 - F_u\right)^{n_u}\left(1 - F_v\right)^{n_v}\left(1 - F_w\right)^{n_w}\left(1 - F_d\right)^{n_d} F_\alpha F_\beta F_\gamma F_u F_v F_w F_d \qquad (18)$$

$$P_{n_\alpha, n_\beta, n_\gamma, n_u, n_v, n_w, n_d} = \rho_\alpha^{\;n_\alpha}\, \rho_\beta^{\;n_\beta}\, \rho_\gamma^{\;n_\gamma}\, \rho_u^{\;n_u}\, \rho_v^{\;n_v}\, \rho_w^{\;n_w}\, \rho_d^{\;n_d}\, \left(1 - \rho_\alpha\right)\left(1 - \rho_\beta\right)\left(1 - \rho_\gamma\right)\left(1 - \rho_u\right)\left(1 - \rho_v\right)\left(1 - \rho_w\right)\left(1 - \rho_d\right)$$

Where $\rho_\alpha = 1 - F_\alpha$, $\rho_\beta = 1 - F_\beta$, $\rho_\gamma = 1 - F_\gamma$, $\rho_u = 1 - F_u$, $\rho_v = 1 - F_v$, $\rho_w = 1 - F_w$, $\rho_d = 1 - F_d$

$$\rho_\alpha = \frac{\lambda_\alpha \left(1 - p_{\gamma\beta} p_{\beta\gamma}\right) + \lambda_\beta \left\{p_{\beta\alpha}(1 - p_{\gamma\beta} p_{\beta\gamma}) + p_{\beta\gamma}\left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)\right\} + \lambda_\gamma \left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)}{\mu_\alpha \left\{\left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)\left(1 - p_{\gamma\beta} p_{\beta\gamma}\right) - \left(p_{\alpha\gamma} + p_{\alpha\beta} p_{\beta\gamma}\right)\left(p_{\gamma\alpha} + p_{\gamma\beta} p_{\beta\alpha}\right)\right\}}$$

$$\rho_\beta = \frac{\lambda_\alpha \left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right) + \lambda_\beta \left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right) + \lambda_\gamma \left\{p_{\gamma\alpha}\left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right) + p_{\gamma\beta}\left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right)\right\}}{\mu_\beta \left\{\left(1 - p_{\beta\gamma} p_{\gamma\beta}\right)\left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right) - \left(p_{\beta\alpha} + p_{\beta\gamma} p_{\gamma\alpha}\right)\left(p_{\alpha\beta} + p_{\alpha\gamma} p_{\gamma\beta}\right)\right\}}$$

$$\rho_\gamma = \frac{\lambda_\alpha \left\{p_{\alpha\beta}\left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right) + p_{\alpha\gamma}\left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)\right\} + \lambda_\beta \left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right) + \lambda_\gamma \left(1 - p_{\alpha\beta} p_{\beta\alpha}\right)}{\mu_\gamma \left\{\left(1 - p_{\alpha\gamma} p_{\gamma\alpha}\right)\left(1 - p_{\alpha\beta} p_{\beta\alpha}\right) - \left(p_{\gamma\beta} + p_{\alpha\beta} p_{\gamma\alpha}\right)\left(p_{\beta\gamma} + p_{\beta\alpha} p_{\alpha\gamma}\right)\right\}}$$

$$\rho_u = \frac{\lambda_u \left(1 - p_{wv} p_{vw}\right) + \lambda_b \left\{p_{vu}(1 - p_{wv} p_{vw}) + p_{vw}\left(p_{wu} + p_{wv} p_{vu}\right)\right\} + \lambda_w \left(p_{wu} + p_{wv} p_{vu}\right)}{\mu_u \left\{\left(1 - p_{uv} p_{vu}\right)\left(1 - p_{wv} p_{vw}\right) - \left(p_{uw} + p_{uv} p_{vw}\right)\left(p_{wu} + p_{wv} p_{vu}\right)\right\}}$$

$$\rho_v = \frac{\lambda_u \left(p_{uv} + p_{uw} p_{wv}\right) + \lambda_v \left(1 - p_{uw} p_{wu}\right) + \lambda_w \left\{p_{wu}\left(p_{uv} + p_{uw} p_{wv}\right) + p_{wv}\left(1 - p_{uw} p_{wu}\right)\right\}}{\mu_v \left\{\left(1 - p_{vw} p_{wv}\right)\left(1 - p_{uw} p_{wu}\right) - \left(p_{vu} + p_{vw} p_{wu}\right)\left(p_{uv} + p_{uw} p_{wv}\right)\right\}}$$

$$\rho_w = \frac{\lambda_u \left\{p_{uv}\left(p_{vw} + p_{vu} p_{uw}\right) + p_{uw}\left(1 - p_{uv} p_{vu}\right)\right\} + \lambda_v \left(p_{vw} + p_{vu} p_{uw}\right) + \lambda_w \left(1 - p_{uv} p_{vu}\right)}{\mu_w \left\{\left(1 - p_{uw} p_{wu}\right)\left(1 - p_{uv} p_{vu}\right) - \left(p_{wv} + p_{uv} p_{wu}\right)\left(p_{vw} + p_{vu} p_{uw}\right)\right\}}$$

$$\rho_d = \frac{\mu_\alpha p_{\alpha d}}{\mu_d}(\rho_\alpha) + \frac{\mu_\beta p_{\beta d}}{\mu_d}(\rho_\beta) + \frac{\mu_\gamma p_{\gamma d}}{\mu_d}(\rho_\gamma) + \frac{\mu_u p_{ud}}{\mu_d}(\rho_u) + \frac{\mu_v p_{vd}}{\mu_d}(\rho_v) + \frac{\mu_w p_{wd}}{\mu_d}(\rho_w)$$

The solution of this model in steady state exists if $\rho_\alpha,\ \rho_\beta,\ \rho_\gamma,\ \rho_u,\ \rho_v,\ \rho_w,\ \rho_d < 1$ (19)

V. PERFORMANCE MEASURES

(i) Mean queue length (average number of customers)

$$L_Q = L_\alpha + L_\beta + L_\gamma + L_u + L_v + L_w + L_d$$

$$L_Q = \frac{\rho_\alpha}{1-\rho_\alpha} + \frac{\rho_\beta}{1-\rho_\beta} + \frac{\rho_\gamma}{1-\rho_\gamma} + \frac{\rho_u}{1-\rho_u} + \frac{\rho_v}{1-\rho_v} + \frac{\rho_w}{1-\rho_w} + \frac{\rho_d}{1-\rho_d}$$

Where $L_\alpha = \frac{\rho_\alpha}{1-\rho_\alpha}$, $L_\beta = \frac{\rho_\beta}{1-\rho_\beta}$, $L_\gamma = \frac{\rho_\gamma}{1-\rho_\gamma}$, $L_u = \frac{\rho_u}{1-\rho_u}$, $L_v = \frac{\rho_v}{1-\rho_v}$, $L_w = \frac{\rho_w}{1-\rho_w}$, $L_d = \frac{\rho_d}{1-\rho_d}$

(ii) Fluctuation (Variance) in queue length

$$V_{ar} = V_\alpha + V_\beta + V_\gamma + V_u + V_v + V_w + V_d$$

$$V_{ar} = \frac{\rho_\alpha}{(1-\rho_\alpha)^2} + \frac{\rho_\beta}{(1-\rho_\beta)^2} + \frac{\rho_\gamma}{(1-\rho_\gamma)^2} + \frac{\rho_u}{(1-\rho_u)^2} + \frac{\rho_v}{(1-\rho_v)^2} + \frac{\rho_w}{(1-\rho_w)^2} + \frac{\rho_d}{(1-\rho_d)^2}$$

Where

$$V_\alpha = \frac{\rho_\alpha}{(1-\rho_\alpha)^2}, V_\beta = \frac{\rho_\beta}{(1-\rho_\beta)^2}, V_\gamma = \frac{\rho_\gamma}{(1-\rho_\gamma)^2}, V_u = \frac{\rho_u}{(1-\rho_u)^2}, V_v = \frac{\rho_v}{(1-\rho_v)^2}, V_w = \frac{\rho_w}{(1-\rho_w)^2}, V_d = \frac{\rho_d}{(1-\rho_d)^2}$$

(iii) Average waiting time for customer

$$E_{wt} = \frac{L_Q}{\lambda} \quad , \quad where \quad \lambda = \lambda_\alpha + \lambda_\beta + \lambda_\gamma + \lambda_u + \lambda_v + \lambda_w$$

VI. RESULTS AND DISCUSSION

In the present queuing network, two global servers GSr_1 and GSr_2 are connected in parallel. Both the global servers comprising of three servers connected in tri-cum biserial way and both the global servers are further connected with the exit server Sr_d in series. The detailed discussion of the present model has been done in the aforementioned section 3 along with the detailed pictorial representation. In section 4, the development of various mathematical equations have been carried out which have been used to find the various queuing parameters such as queue lengths, variances, Utilization of servers, average waiting time for customers.

Table 1 shows the various input parameters, i.e., $p_{\alpha\beta}$, $p_{\alpha\gamma}$, $p_{\alpha d}$, p_{uv}, p_{uw}, p_{ud}, n_α, n_β, n_u, n_d etc., which have been used during the calculations of various queuing characteristics.

Table 2 shows the variation of traffic intensities, variances and joint probability with mean arrival rate λ_α at server Sr_α from global server 1 (GSr_1). In bracket, various other input parameters which have been used in the calculation of numerical values shown in the Table 2 are given. It is evident from the results that as λ_α increases traffic intensities ρ_α, ρ_β, ρ_γ, ρ_d and variances increases. It is also observed that the values of ρ_u, ρ_v and ρ_w are unchanged as λ_α increases. This is due to the fact that ρ_u, ρ_v and ρ_w are associated with the global server 2 (GSr_2) which are connected in parallel with global server 1 (GSr_1) therefore as λ_α is associated with GSr_1 only, hence these values remain unchanged. This can also be seen clearly from the queuing network shown in the figure that the servers GSr_1 and GSr_2 which are connected in parallel have their input parameters which are independent to each other.

Figure 2 (a, b) show the variation of mean arrival rate λ_α with the queue length (L_q) and average waiting time (E_{wt}) keeping all the input parameters same as considered for Table 2. It can be seen that as the mean arrival rate λ_α increases queue length (L_q) and average waiting time (E_{wt}) increases. Practically it is possible because as the number of customers at a particular server increases queue length and average waiting time increases. The same conclusion can be drawn for Tables 3–7 and Figures 3–7.

Table 8 shows the variation of traffic intensities, variances and joint probability with mean service rate μ_α at server Sr_α from global server 1 (GSr_1). It is clear from the results that as service rate μ_α increases traffic intensity ρ_α at server Sr_α decreases whereas the traffic intensities ρ_β, ρ_γ, ρ_u, ρ_v, ρ_w and ρ_d at other servers remains unaffected. Variance V_{ar} also decreases as μ_α increases.

The mean service rate μ_α are plotted against queue length (L_q) and average waiting time (E_{wt}) for customers in Figure 8. It is clear from the figure that queue length and average waiting time decreases as the mean service rate μ_α

increases. It is true practically and mathematically also because when the service rate increases, the customers at various servers will be served rapidly as the consequences the queue length and average waiting time decreases. The same outcome can be seen from Tables 9-14 and Figures 9-14.

Table 1. Various input parameters considered in computation of results

$p_{\alpha\beta}$	$p_{\alpha\gamma}$	$p_{\alpha d}$	$p_{\beta\alpha}$	$p_{\beta\gamma}$	$p_{\beta d}$	$p_{\gamma\alpha}$	$p_{\gamma\beta}$	$p_{\gamma d}$	n_α	n_β	n_γ	n_d
0.33	0.33	0.34	0.33	0.33	0.34	0.33	0.33	0.34	1	2	2	13
p_{uv}	p_{uw}	p_{ud}	p_{vu}	p_{vw}	p_{vd}	p_{wu}	p_{wv}	p_{wd}	n_u	n_v	n_w	
0.33	0.33	0.34	0.33	0.33	0.34	0.33	0.33	0.34	2	3	3	

Table 2. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_α

(taking $\lambda_\beta=2$, $\lambda_\gamma=3$, $\mu_\alpha=9$, $\mu_\beta=10$, $\mu_\gamma=11$, $\lambda_u=2$, $\lambda_v=3$, $\lambda_w=4$, $\mu_u=12$, $\mu_v=13$, $\mu_w=14$, $\mu_d=18$)

$\lambda_\alpha\downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
1	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
1.2	0.603	0.603	0.616	0.673	0.679	0.684	0.844	66.434	1.31E-07
1.4	0.636	0.617	0.630	0.673	0.679	0.684	0.856	74.312	1.42E-07
1.6	0.669	0.632	0.643	0.673	0.679	0.684	0.867	84.258	1.50E-07
1.8	0.702	0.647	0.656	0.673	0.679	0.684	0.878	97.079	1.55E-07
2	0.735	0.661	0.669	0.673	0.679	0.684	0.889	114.025	1.55E-07
2.2	0.768	0.676	0.683	0.673	0.679	0.684	0.900	137.125	1.49E-07
2.4	0.801	0.690	0.696	0.673	0.679	0.684	0.911	169.866	1.38E-07
2.6	0.833	0.705	0.709	0.673	0.679	0.684	0.922	218.690	1.22E-07
2.8	0.866	0.720	0.723	0.673	0.679	0.684	0.933	296.763	9.96E-08
3	0.899	0.734	0.736	0.673	0.679	0.684	0.944	435.330	7.38E-08

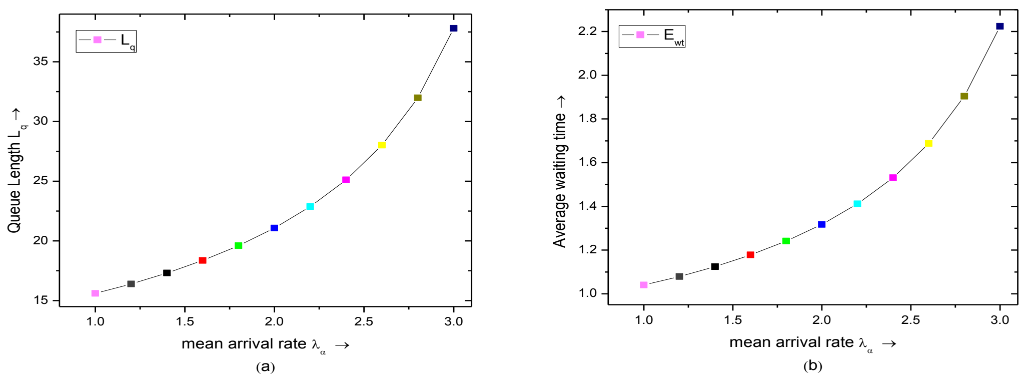


Figure 2 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_α

Table 3. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_β

(taking $\lambda_\alpha=1$, $\lambda_\gamma=3$, $\mu_\alpha=9$, $\mu_\beta=10$, $\mu_\gamma=11$, $\lambda_u=2$, $\lambda_v=3$, $\lambda_w=4$, $\mu_u=12$, $\mu_v=13$, $\mu_w=14$, $\mu_d=18$)

$\lambda_\beta\downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
1	0.489	0.440	0.537	0.673	0.679	0.684	0.778	41.228	4.61E-08
1.2	0.505	0.470	0.550	0.673	0.679	0.684	0.789	43.851	5.80E-08
1.4	0.521	0.499	0.563	0.673	0.679	0.684	0.800	46.922	7.14E-08
1.6	0.538	0.529	0.577	0.673	0.679	0.684	0.811	50.549	8.62E-08
1.8	0.554	0.559	0.590	0.673	0.679	0.684	0.822	54.871	1.02E-07
2	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
2.2	0.586	0.618	0.616	0.673	0.679	0.684	0.844	66.443	1.34E-07
2.4	0.602	0.648	0.630	0.673	0.679	0.684	0.856	74.321	1.49E-07
2.6	0.619	0.677	0.643	0.673	0.679	0.684	0.867	84.244	1.61E-07
2.8	0.635	0.707	0.656	0.673	0.679	0.684	0.878	96.995	1.70E-07
3	0.651	0.736	0.669	0.673	0.679	0.684	0.889	113.776	1.74E-07

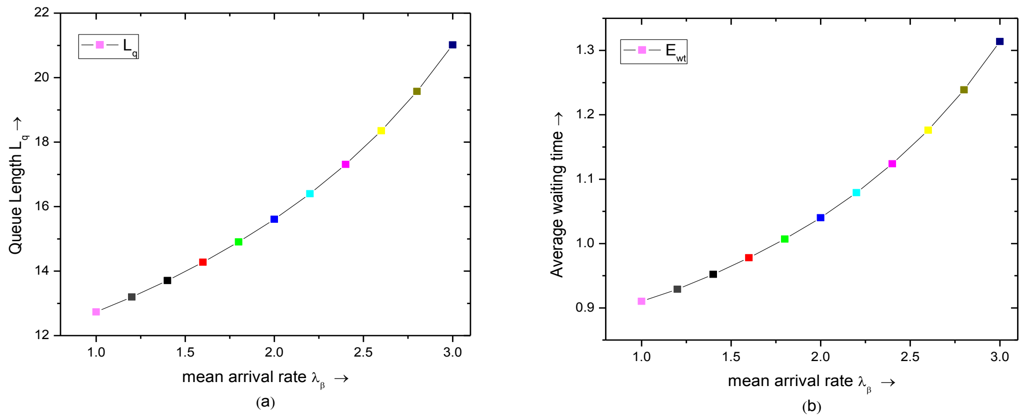


Figure 3 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_β

Table 4. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_γ

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\lambda_\gamma \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
1	0.408	0.442	0.334	0.673	0.679	0.684	0.722	32.397	1.19E-08
1.2	0.424	0.457	0.361	0.673	0.679	0.684	0.733	33.723	1.64E-08
1.4	0.440	0.471	0.388	0.673	0.679	0.684	0.744	35.226	2.21E-08
1.6	0.457	0.486	0.415	0.673	0.679	0.684	0.756	36.940	2.91E-08
1.8	0.473	0.501	0.441	0.673	0.679	0.684	0.767	38.906	3.77E-08
2	0.489	0.515	0.468	0.673	0.679	0.684	0.778	41.173	4.78E-08
2.2	0.505	0.530	0.495	0.673	0.679	0.684	0.789	43.807	5.95E-08
2.4	0.521	0.544	0.522	0.673	0.679	0.684	0.800	46.889	7.26E-08
2.6	0.538	0.559	0.549	0.673	0.679	0.684	0.811	50.527	8.71E-08
2.8	0.554	0.574	0.576	0.673	0.679	0.684	0.822	54.861	1.02E-07
3	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07

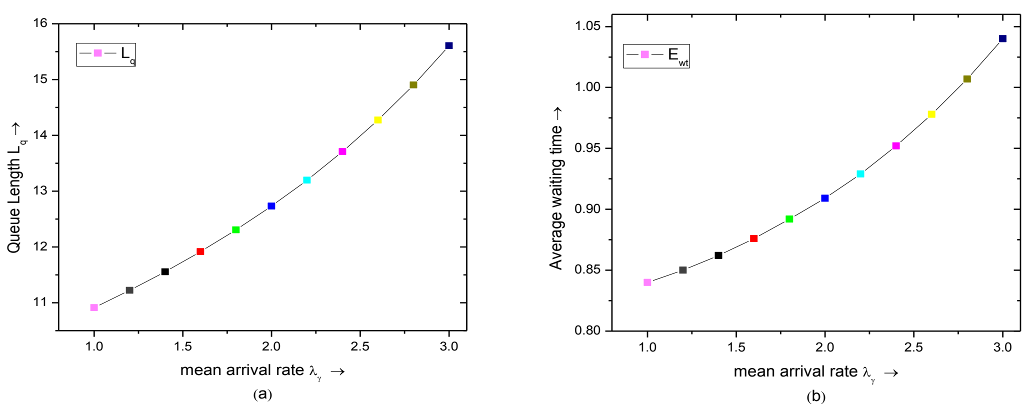


Figure 4 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_γ

Table 5. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_u

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, , $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\lambda_u \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
2	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
2.2	0.570	0.588	0.603	0.697	0.690	0.694	0.844	67.504	1.34E-07
2.4	0.570	0.588	0.603	0.722	0.701	0.705	0.856	76.674	1.48E-07
2.6	0.570	0.588	0.603	0.747	0.712	0.715	0.867	88.206	1.60E-07
2.8	0.570	0.588	0.603	0.771	0.724	0.726	0.878	103.022	1.69E-07

3	0.570	0.588	0.603	0.796	0.735	0.736	0.889	122.554	1.73E-07
3.2	0.570	0.588	0.603	0.821	0.746	0.747	0.900	149.133	1.72E-07
3.4	0.570	0.588	0.603	0.845	0.757	0.757	0.911	186.785	1.64E-07
3.6	0.570	0.588	0.603	0.870	0.769	0.767	0.922	242.998	1.49E-07
3.8	0.570	0.588	0.603	0.895	0.780	0.778	0.933	333.202	1.26E-07
4	0.570	0.588	0.603	0.920	0.791	0.788	0.944	494.245	9.72E-08

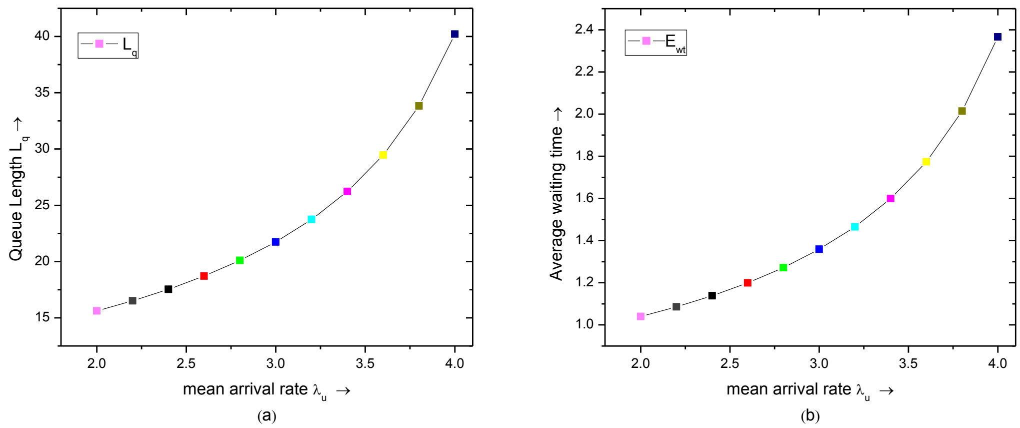


Figure 5 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_u

Table 6. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_v

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, , $\lambda_u = 2$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\lambda_v \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
2	0.570	0.588	0.603	0.612	0.565	0.632	0.778	37.835	4.52E-08
2.2	0.570	0.588	0.603	0.624	0.588	0.642	0.789	40.969	5.66E-08
2.4	0.570	0.588	0.603	0.636	0.610	0.653	0.800	44.618	6.98E-08
2.6	0.570	0.588	0.603	0.648	0.633	0.663	0.811	48.904	8.47E-08
2.8	0.570	0.588	0.603	0.660	0.656	0.674	0.822	53.988	1.01E-07
3	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
3.2	0.570	0.588	0.603	0.685	0.702	0.694	0.844	67.482	1.36E-07
3.4	0.570	0.588	0.603	0.697	0.724	0.705	0.856	76.597	1.53E-07
3.6	0.570	0.588	0.603	0.709	0.747	0.715	0.867	88.016	1.68E-07
3.8	0.570	0.588	0.603	0.721	0.770	0.726	0.878	102.612	1.81E-07
4	0.570	0.588	0.603	0.733	0.793	0.736	0.889	121.721	1.89E-07

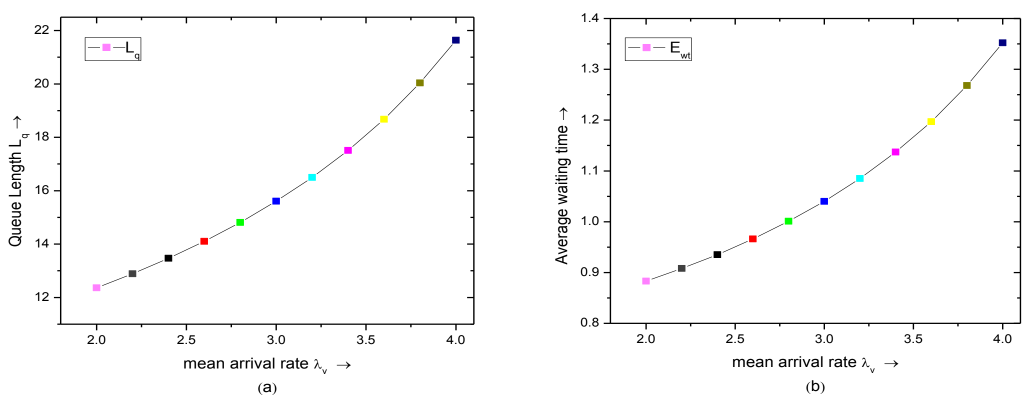


Figure 6 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_v

Table 7. Utilization of servers, Variances and Joint Probabilities for various mean arrival rates λ_w

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, , $\lambda_u = 2$, $\lambda_v = 3$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\lambda_w \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
2	0.570	0.588	0.603	0.551	0.566	0.472	0.722	27.185	1.22E-08
2.2	0.570	0.588	0.603	0.563	0.578	0.493	0.733	28.808	1.64E-08
2.4	0.570	0.588	0.603	0.575	0.589	0.515	0.744	30.640	2.17E-08
2.6	0.570	0.588	0.603	0.587	0.600	0.536	0.756	32.719	2.83E-08
2.8	0.570	0.588	0.603	0.600	0.611	0.557	0.767	35.091	3.64E-08
3	0.570	0.588	0.603	0.612	0.623	0.578	0.778	37.812	4.61E-08
3.2	0.570	0.588	0.603	0.624	0.634	0.599	0.789	40.956	5.74E-08
3.4	0.570	0.588	0.603	0.636	0.645	0.620	0.800	44.614	7.05E-08
3.6	0.570	0.588	0.603	0.648	0.656	0.642	0.811	48.908	8.51E-08
3.8	0.570	0.588	0.603	0.660	0.668	0.663	0.822	53.994	1.01E-07
4	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07

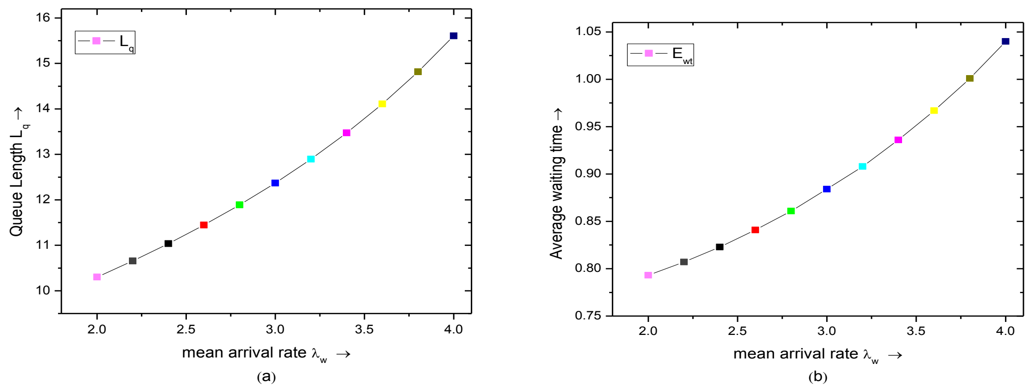


Figure 7 (a, b). Mean queue length and Average waiting time for various mean arrival rates λ_w

Table 8. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_α

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\beta = 10$, $\mu_\gamma = 11$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\mu_\alpha \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
9	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
9.2	0.558	0.588	0.603	0.673	0.679	0.684	0.833	59.849	1.19E-07
9.4	0.546	0.588	0.603	0.673	0.679	0.684	0.833	59.644	1.20E-07
9.6	0.534	0.588	0.603	0.673	0.679	0.684	0.833	59.464	1.20E-07
9.8	0.524	0.588	0.603	0.673	0.679	0.684	0.833	59.304	1.20E-07
10	0.513	0.588	0.603	0.673	0.679	0.684	0.833	59.162	1.20E-07
10.2	0.503	0.588	0.603	0.673	0.679	0.684	0.833	59.035	1.21E-07
10.4	0.493	0.588	0.603	0.673	0.679	0.684	0.833	58.920	1.20E-07
10.6	0.484	0.588	0.603	0.673	0.679	0.684	0.833	58.816	1.20E-07
10.8	0.475	0.588	0.603	0.673	0.679	0.684	0.833	58.722	1.20E-07
11	0.466	0.588	0.603	0.673	0.679	0.684	0.833	58.637	1.20E-07

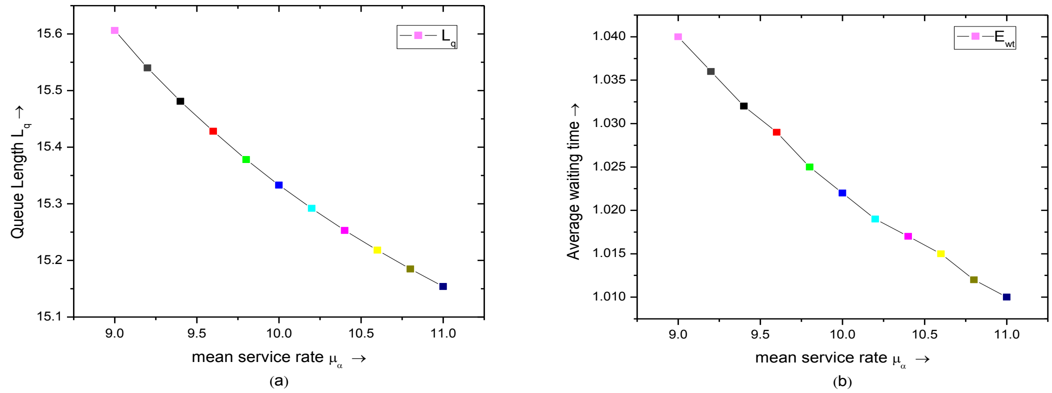


Figure 8 (a, b). Mean queue length and Average waiting time for various mean service rates μ_α

Table 9. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_β

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\gamma = 11$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\mu_\beta \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
9	0.570	0.654	0.603	0.673	0.679	0.684	0.833	62.060	1.23E-07
9.2	0.570	0.639	0.603	0.673	0.679	0.684	0.833	61.530	1.22E-07
9.4	0.570	0.626	0.603	0.673	0.679	0.684	0.833	61.081	1.22E-07
9.6	0.570	0.613	0.603	0.673	0.679	0.684	0.833	60.699	1.21E-07
9.8	0.570	0.600	0.603	0.673	0.679	0.684	0.833	60.369	1.19E-07
10	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
10.2	0.570	0.577	0.603	0.673	0.679	0.684	0.833	59.831	1.17E-07
10.4	0.570	0.566	0.603	0.673	0.679	0.684	0.833	59.610	1.15E-07
10.6	0.570	0.555	0.603	0.673	0.679	0.684	0.833	59.414	1.14E-07
10.8	0.570	0.545	0.603	0.673	0.679	0.684	0.833	59.240	1.12E-07
11	0.570	0.535	0.603	0.673	0.679	0.684	0.833	59.083	1.10E-07

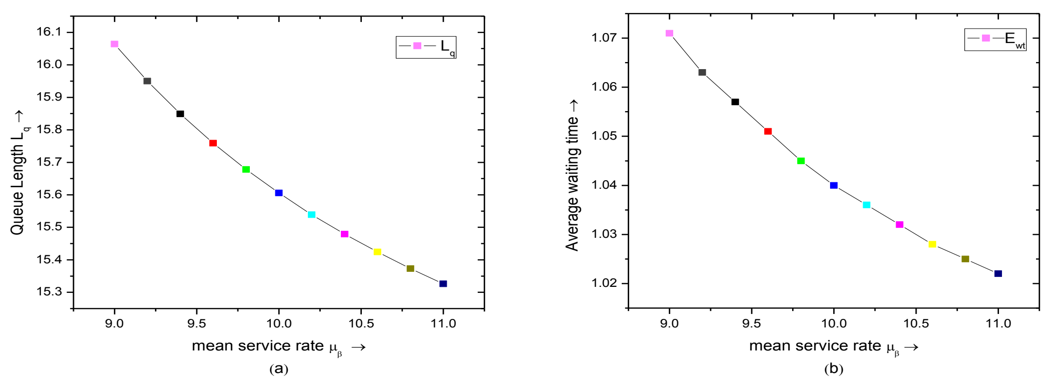


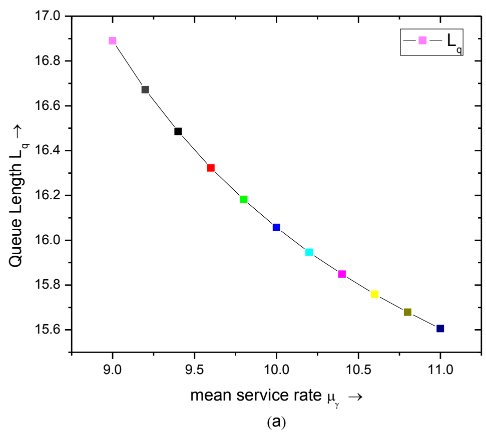
Figure 9 (a, b). Mean queue length and Average waiting time for various mean service rates μ_β

Table 10. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_γ

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\mu_\gamma \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
9	0.570	0.588	0.737	0.673	0.679	0.684	0.833	66.922	1.17E-07
9.2	0.570	0.588	0.721	0.673	0.679	0.684	0.833	65.525	1.19E-07
9.4	0.570	0.588	0.706	0.673	0.679	0.684	0.833	64.406	1.20E-07

9.6	0.570	0.588	0.691	0.673	0.679	0.684	0.833	63.494	1.21E-07
9.8	0.570	0.588	0.677	0.673	0.679	0.684	0.833	62.741	1.21E-07
10	0.570	0.588	0.663	0.673	0.679	0.684	0.833	62.110	1.21E-07
10.2	0.570	0.588	0.650	0.673	0.679	0.684	0.833	61.576	1.21E-07
10.4	0.570	0.588	0.638	0.673	0.679	0.684	0.833	61.119	1.21E-07
10.6	0.570	0.588	0.626	0.673	0.679	0.684	0.833	60.725	1.20E-07
10.8	0.570	0.588	0.614	0.673	0.679	0.684	0.833	60.382	1.19E-07
11	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07



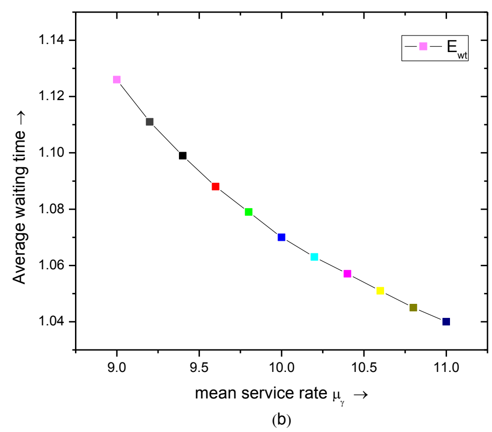


Figure 10 (a, b). Mean queue length and Average waiting time for various mean service rates μ_γ

Table 11. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_u

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_v = 13$, $\mu_w = 14$, $\mu_d = 18$)

$\mu_u \downarrow$	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
12	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
12.2	0.570	0.588	0.603	0.662	0.679	0.684	0.833	59.584	1.18E-07
12.4	0.570	0.588	0.603	0.651	0.679	0.684	0.833	59.148	1.18E-07
12.6	0.570	0.588	0.603	0.641	0.679	0.684	0.833	58.765	1.18E-07
12.8	0.570	0.588	0.603	0.631	0.679	0.684	0.833	58.427	1.17E-07
13	0.570	0.588	0.603	0.621	0.679	0.684	0.833	58.126	1.17E-07
13.2	0.570	0.588	0.603	0.611	0.679	0.684	0.833	57.857	1.16E-07
13.4	0.570	0.588	0.603	0.602	0.679	0.684	0.833	57.615	1.15E-07
13.6	0.570	0.588	0.603	0.594	0.679	0.684	0.833	57.397	1.14E-07
13.8	0.570	0.588	0.603	0.585	0.679	0.684	0.833	57.200	1.13E-07
14	0.570	0.588	0.603	0.577	0.679	0.684	0.833	57.021	1.12E-07

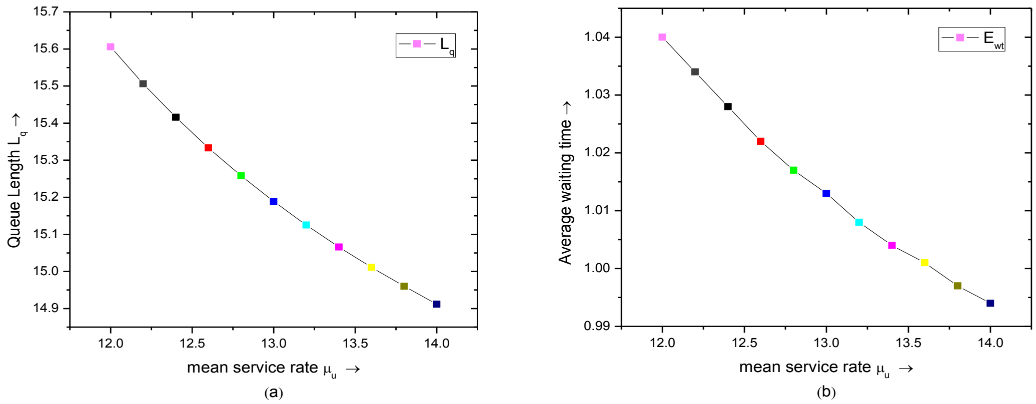


Figure 11 (a, b). Mean queue length and Average waiting time for various mean service rates μ_u

Table 12. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_v

(taking $\lambda_\alpha=1$, $\lambda_\beta=2$, $\lambda_\gamma=3$, $\mu_\alpha=9$, $\mu_\beta=10$, $\mu_\gamma=11$, $\lambda_u=2$, $\lambda_v=3$, $\lambda_w=4$, $\mu_u=12$, $\mu_w=14$, $\mu_d=18$)

μ_v ↓	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
12	0.570	0.588	0.603	0.673	0.735	0.684	0.833	64.000	1.24E-07
12.2	0.570	0.588	0.603	0.673	0.723	0.684	0.833	62.948	1.23E-07
12.4	0.570	0.588	0.603	0.673	0.712	0.684	0.833	62.060	1.22E-07
12.6	0.570	0.588	0.603	0.673	0.700	0.684	0.833	61.302	1.21E-07
12.8	0.570	0.588	0.603	0.673	0.689	0.684	0.833	60.649	1.20E-07
13	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
13.2	0.570	0.588	0.603	0.673	0.668	0.684	0.833	59.587	1.16E-07
13.4	0.570	0.588	0.603	0.673	0.658	0.684	0.833	59.151	1.15E-07
13.6	0.570	0.588	0.603	0.673	0.649	0.684	0.833	58.766	1.13E-07
13.8	0.570	0.588	0.603	0.673	0.639	0.684	0.833	58.423	1.11E-07
14	0.570	0.588	0.603	0.673	0.630	0.684	0.833	58.116	1.09E-07

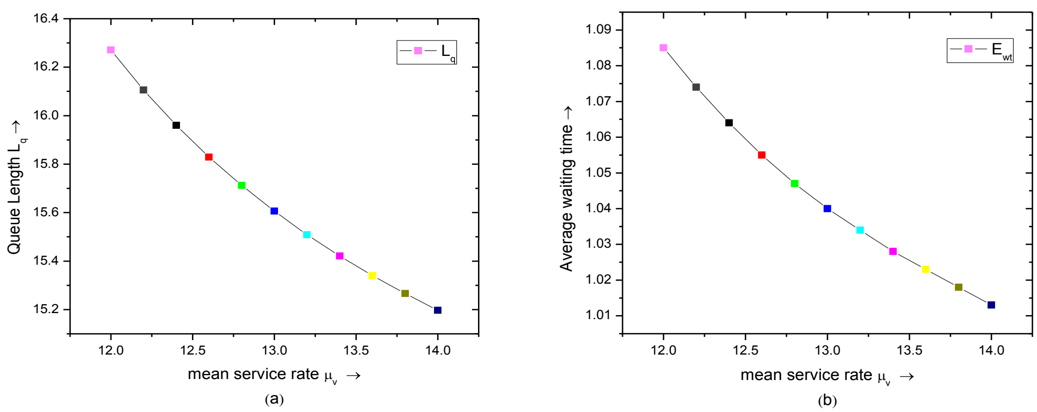


Figure 12 (a, b). Mean queue length and Average waiting time for various mean service rates μ_v

Table 13. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_w

(taking $\lambda_\alpha=1$, $\lambda_\beta=2$, $\lambda_\gamma=3$, $\mu_\alpha=9$, $\mu_\beta=10$, $\mu_\gamma=11$, $\lambda_u=2$, $\lambda_v=3$, $\lambda_w=4$, $\mu_u=12$, $\mu_v=13$, $\mu_d=18$)

μ_w ↓	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
12	0.570	0.588	0.603	0.673	0.679	0.798	0.833	72.781	1.20E-07
12.2	0.570	0.588	0.603	0.673	0.679	0.785	0.833	70.193	1.22E-07
12.4	0.570	0.588	0.603	0.673	0.679	0.772	0.833	68.117	1.23E-07
12.6	0.570	0.588	0.603	0.673	0.679	0.760	0.833	66.423	1.23E-07
12.8	0.570	0.588	0.603	0.673	0.679	0.748	0.833	65.022	1.23E-07

13	0.570	0.588	0.603	0.673	0.679	0.737	0.833	63.849	1.23E-07
13.2	0.570	0.588	0.603	0.673	0.679	0.725	0.833	62.855	1.22E-07
13.4	0.570	0.588	0.603	0.673	0.679	0.715	0.833	62.006	1.22E-07
13.6	0.570	0.588	0.603	0.673	0.679	0.704	0.833	61.275	1.21E-07
13.8	0.570	0.588	0.603	0.673	0.679	0.694	0.833	60.639	1.19E-07
14	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07

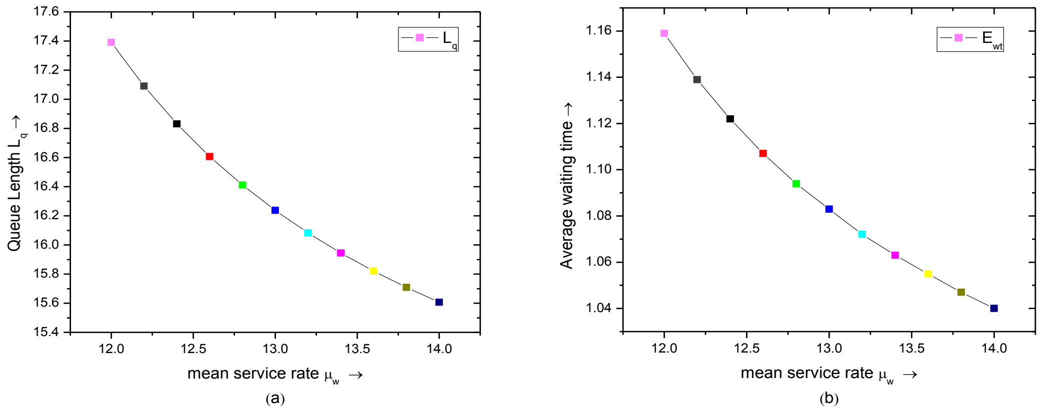


Figure 13 (a, b). Mean queue length and Average waiting time for various mean service rates μ_w

Table 14. Utilization of servers, Variances and Joint Probabilities for various mean service rates μ_d

(taking $\lambda_\alpha = 1$, $\lambda_\beta = 2$, $\lambda_\gamma = 3$, $\mu_\alpha = 9$, $\mu_\beta = 10$, $\mu_\gamma = 11$, $\lambda_u = 2$, $\lambda_v = 3$, $\lambda_w = 4$, $\mu_u = 12$, $\mu_v = 13$, $\mu_w = 14$)

μ_d ↓	ρ_α	ρ_β	ρ_γ	ρ_u	ρ_v	ρ_w	ρ_d	V_{ar}	P
18	0.570	0.588	0.603	0.673	0.679	0.684	0.833	60.082	1.18E-07
18.2	0.570	0.588	0.603	0.673	0.679	0.684	0.824	56.742	1.08E-07
18.4	0.570	0.588	0.603	0.673	0.679	0.684	0.815	53.958	9.84E-08
18.6	0.570	0.588	0.603	0.673	0.679	0.684	0.806	51.610	8.96E-08
18.8	0.570	0.588	0.603	0.673	0.679	0.684	0.798	49.611	8.14E-08
19	0.570	0.588	0.603	0.673	0.679	0.684	0.789	47.895	7.39E-08
19.2	0.570	0.588	0.603	0.673	0.679	0.684	0.781	46.409	6.70E-08
19.4	0.570	0.588	0.603	0.673	0.679	0.684	0.773	45.113	6.07E-08
19.6	0.570	0.588	0.603	0.673	0.679	0.684	0.765	43.976	5.50E-08
19.8	0.570	0.588	0.603	0.673	0.679	0.684	0.758	42.973	4.98E-08
20	0.570	0.588	0.603	0.673	0.679	0.684	0.750	42.082	4.50E-08

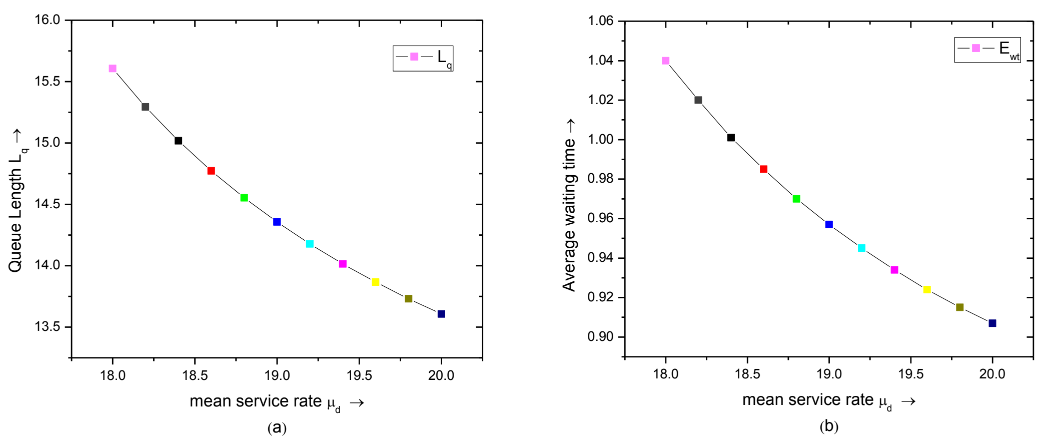


Figure 14 (a, b). Mean queue length and Average waiting time for various mean service rates μ_d

VII. CONCLUSION

In the present study, a complex queuing model has been developed to find the various queuing characteristics such as queue lengths, traffic intensities and average waiting time for customers etc. Various combinations of input parameters have been considered to find the various output parameters. This parametric study can be useful in various practical applications such as shopping complex, banks, railway stations, industries etc.

Some of the important attributes of presently developed queuing model can be summarized as follows

- If we consider only one global server GSr_1 or GSr_2 then the queuing model can be converted to previously developed model which is given by Agrawal and Singh [10].
- If Sr_γ and Sr_w have not considered then the queuing model will deliver the same results as presented by Kumar et al. [8].
- If GSr_2 is completely ignored and in GSr_1 only two servers will be considered then the resulted queuing model will be same as given by Singh et al. [7].

There are several other models available in the literature which can be drawn from the presently developed model therefore the presently developed model is named as generalized queuing model.

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Appendix

Symbol	Notations
Servers	GSr_1, GSr_2, Sr_α, Sr_β, Sr_γ, Sr_u, Sr_v, Sr_w, Sr_d
Joint Probability	$P_{n_\alpha, n_\beta, n_\gamma, n_u, n_v, n_w, n_d}$
Mean arrival rates	λ_α, λ_β, λ_γ, λ_u, λ_v, λ_w
Mean Service Rates	μ_α, μ_β, μ_γ, μ_u, μ_v, μ_w, μ_d

probabilities	$p_{\alpha\beta}$, $p_{\alpha\gamma}$, $p_{\alpha d}$, $p_{\beta\alpha}$, $p_{\beta\gamma}$, $p_{\beta d}$, $p_{\gamma\alpha}$, $p_{\gamma\beta}$, $p_{\gamma d}$, p_{uv}, p_{uw}, p_{ud}, p_{vu}, p_{vw}, p_{vd}, p_{wu}, p_{wv}, p_{wd}
No. of Customers	$n_\alpha, n_\beta, n_\gamma, n_u, n_v, n_w, n_d$
Traffic intensity or utilization of servers	$\rho_\alpha, \rho_\beta, \rho_\gamma, \rho_u, \rho_v, \rho_w, \rho_d$
queues lengths	$L_\alpha, L_\beta, L_\gamma, L_u, L_v, L_w, L_d, L_q$
Variances	$V_\alpha, V_\beta, V_\gamma, V_u, V_v, V_w, V_d, V_{ar}$
Average waiting time for customers	E_{wt}

