IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Volume-5, Issue-5 E-ISSN: 2347-2693

A Quantum Inspired Evolutionary Computational Technique with Applications to Structural Engineering Design

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Online Available at: www.ijcseonline.org

Received: 02/Apr/2017, Revised: 14/Apr/2017, Accepted: 06/May/2017, Published: 30/May/2017

Abstract**—** A new Quantum Inspired Evolutionary Computational Technique (QIECT) is reported in this work. It is applied to a set of standard test bench problems and a few structural engineering design problems. The algorithm is a hybrid of quantum inspired evolution and real coded Genetic evolutionary simulated annealing strategies. It generates initial parents randomly and improves them using quantum rotation gate. Subsequently, Simulated Annealing (SA) is utilized in Genetic Algorithm (GA) for the selection process for child generation. The convergence of the successive generations is continuous and progresses towards the global optimum. Efficiency and effectiveness of the algorithm are demonstrated by solving a few unconstrained Benchmark Test functions, which are well-known numerical optimization problems. The algorithm is applied on engineering optimization problems like spring design, pressure vessel design and gear train design. The results compare favorably with other state of art algorithms, reported in the literature. The application of proposed heuristic technique in mechanical engineering design is a step towards agility in design.

Keywords**—**Constraint Optimization, Mechanical Engineering Design problems, Quantum Inspired Evolutionary Computational Technique, Unconstrained Optimization

I. INTRODUCTION AND RELATED WORK

Engineering Design is specified as a decision making procedure which leads to the creation of a product that can satisfy particular needs. It involves solving complex objective function with a number of decision variables and number of constraints. In constraint optimization problems main task is to satisfy the constraints in finding the feasible solution. To handle these constraints researchers proposed numerous approaches. One of them is penalty approach as proposed by Deb [1].

The last decade has witnessed remarkable growth in the application of stochastic search techniques for specific well-defined problems from engineering domain. Genetic Algorithm (GA) is one of them, has achieved considerable popularity [2, 3]. Evolutionary algorithms are useful in general function optimization [4, 5]. To find more refine and qualitative solution hybrid methods are adopted and implemented by the researchers [5]. Several researchers have proposed various nature based hybrid methods for solving engineering optimization problems [6, 7, 8, 9, 10]. Hans Raj et al., [11] have proposed a hybrid evolutionary computational technique by combining GA and Simulated Annealing (SA). These nature stimulated evolutionary algorithms succeeded in finding near global optimum for real life problems. Recently researchers have started to integrate quantum mechanics ideas into evolutionary methods [12, 13, 14, 15, 16].

In the present work, QIECT is developed by integrating quantum concepts such as sampling and rotation gate [15] with genetic algorithm and simulated annealing. In QIECT, real variables are used in order to enhance solution accuracy. Q-bit is expressed as the random real number instead of binary bit. The number of Q-bits is equal to the number of variables in the given problem. Rotation gate is applied to improve the initial population. In Evolutionary Computational part, the population is further improved in order to enhance solution accuracy using crossover, mutation, and selection operators. Simultaneously the Simulated Annealing (SA) technique is applied to overcome the problem of getting stuck in local optima [17, 18]. Thus, QIECT has the strong capability to explore the whole search space without getting trapped in local optimum.

In the present work, QIECT is initially used for solving unconstrained benchmark optimization functions and subsequently constrained engineering design problems. This technique provides more rapid and robust convergence for many standard test bench functions. The results of QIECT are compared with results of other state of art algorithms [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] and found comparable in all aspects. The rest of this paper is organized as follows: The basic concepts are reviewed and methodology is discussed in section II. Simulation results and their comparisons are given in section III. Engineering design applications are given in sub-section with conclusions, at the end.

II. METHODOLOGY

A. *Quantum Inspired Evolutionary Computational Technique*

In QIECT real variables are used in order to enhance solution accuracy. Initial population is generated randomly as shown in equation (1). Then, Quantum gate is applied to generate a good improve population from initial population. Phase angle is generated according to the variables as shown in equation (2). It helps in exploring the search space minutely by increasing the diversity of population. Genetic Algorithm and Simulated Annealing (SA) is applied to acquire the best optimized values from population as results. Genetic Algorithm is used to generate children using blend crossover. Afterwards mutation is applied on every parent and children string. Two levels of competition are introduced among the population strings to ensure that the better strings continue in the population. First level of competition is between children. And second level of competition is between the successful child and his parent. "Acceptance Number" concept is introduced so that the algorithm can devotedly explore "better" regions of the search space. The flowchart of the algorithm is given in Figure 1.

1) *Algorithm for Quantum Inspired Evolutionary Computational Technique (QIECT)*

Step1. Initialization: Initialize maximum number of generations, N (population size), dim (dimension)

Step2. Random generation of initial parent population using (1).

$$x_i = x_{i,\min} + (x_{i,\max} - x_{i,\min}) * r_i \tag{1}$$

where x is variable, $1 \le \mathrm{i} \le \mathrm{dim}$

r_i is a random number

Step3. Application of Quantum gate: Quantum gate is applied to improve initial parent population. Evaluate phase angle using eq. (2)

$$Phase\_angle_i^t = \arccos \sqrt{\frac{x_i^t - x_{i,\min}}{x_{i,\max} - x_{i,\min}}} \tag{2}$$

Improve population by updating the phase angle according to eq. (3)

$$\left.\begin{array}{l} If\ \ Phase\_angle_i^t < best\_angle, \\ then\ Phase\_angle_i^{t+1} = Phase\_angle_i^t + \Delta\theta \\ If\ \ Phase\_angle_i^t > best\_angle, \\ then\ Phase\_angle_i^{t+1} = Phase\_angle_i^t - \Delta\theta \\ else\ Phase\_angle_i^t = best\_angle, \\ then\ Phase\_angle_i^{t+1} = Phase\_angle_i^t \end{array}\right\} \tag{3}$$

Where $\Delta\theta$ is randomly generated angle. $1 \le \mathrm{i} \le \mathrm{dim}$, t is generation.

Step4. Step3 is repeated for whole population.

Step5. Application of Evolutionary Computational Technique

a. Initialization: Initialize TInit (Initial Temperature) and TFinal (Final Temperature), N parent strings (output from quantum gate), C (Total number of children), Compute m=C/N (Ratio of Children generated per parent), Max_gen (Maximum number of generation).

$T_{Current} \leftarrow T_{Init}$, where $T_{Current}$ is the current temperature.

b. For each parent, generate m children using blend crossover.

c. Application of mutation operator: Apply mutation on each parent and children string

d. 1st level of competition: Select the best child as the parent for the subsequent generations according to the Boltzmann probability criterion.

$Y_{Best\_child} < Y_{Parent}$

OR

$\exp[(Y_{Parent} - Y_{Best\_child}) / T_{Current}] \ge \rho$

Where

Y_{Best_child} is the value of objective function for the best child

Y_{Parent} is the objective function value of its parents

ρ is a random number generated between 0 and 1.

e. Set Count=0

f. Increase Count by 1, if

$Y_{child} < Y_{Parent}$

OR

$\exp[(Y_{Lowest} - Y_{child}) / T_{Current}] \geq \rho$

where

Y_{child} is the objective function value of the child

Y_{Parent} is the objective function value of its parent

Y_{Lowest} is the lowest objective function value ever found

$T_{Current}$ is the temperature co-efficient

ρ is a random number generated between 0 and 1.

g. Step 'f ' is repeated for each child.

h. Step 'e-f ' is repeated for each family.

i. The children which satisfy the above criteria (Step f) are called the 'accepted children'. Count the 'accepted children' for each and every family separately. Acceptance number of the family is equal to the count represented as "A" as given in figure 1(b).

j. Sum of the acceptance numbers is calculated, of all the families denoted as "S" as shown in figure 1(b) with example.

k. For each family, calculate the number of children to be generated in the future generation according to the formula

$$m = (C \times A) / S$$

l. The temperature is decrease according to cooling schedule given below,

$$\beta = \frac{T_{init} - T_{Final}}{T_{init} * T_{Final} * (\max\_T - 1)}$$

Current temperature reduces as given below

$$\frac{T_{Current}}{(1 + \beta * T_{Current})},$$

m. Increase generation by 1

Step6. Step (a to m) is repeated until a maximum number of generation has been reached / no further improvement is observed.

III. Results and Discussion

A. *Performance of QIECT on Benchmark Functions*

The Mathematical Benchmark test functions used in this study are specified in Table 1. Detailed description can be found in [29]. In the present work, empirical experiments are carried out for each function. For all the functions and for engineering applications population size is considered as 100. Offspring size is considered as 1000.

To build performance analysis of QIECT a chain of experiments are carried out. QIECT is run 30 times for each problem to statistically evaluate its performance on benchmark problems. Output graphs of QIECT for benchmark functions are shown in Figure 2. In these graphs, best fitness values are plotted for each function. Results of statistical parameters and Average computation time for all the functions evaluated by QIECT are given in Table 2.

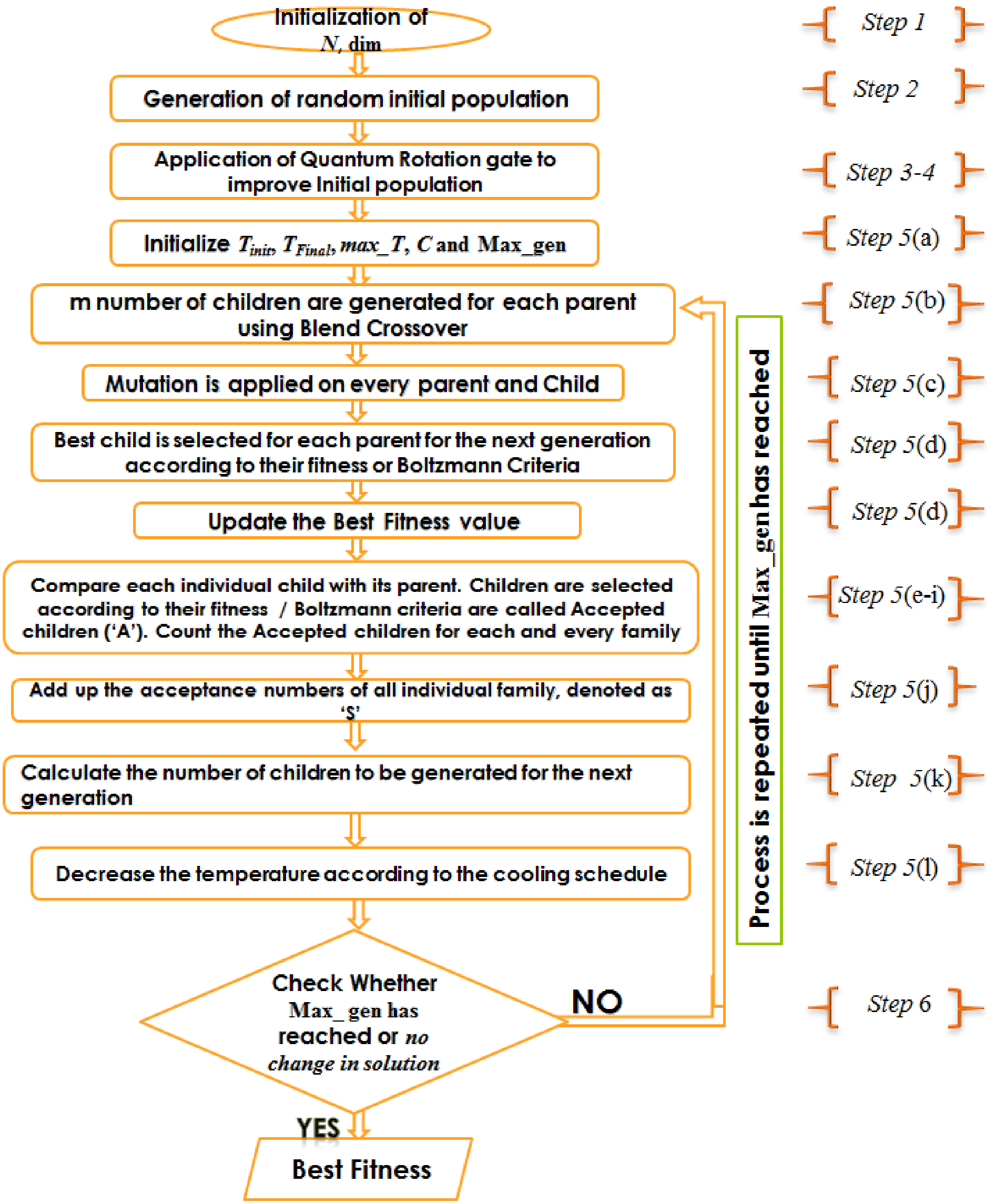

Figure 1 Flowchart of QIECT

Table 1: Benchmark Functions used in section 3

Name	Formula	Dimensions	Range
F1: Sphere	$f(x)=\sum_{i=1}^{n} x_i^2$	10	$x_i \in [-5.12, 5.12]$
F2: Rastrigin	$f(x)=-10n+\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)]$	10	$x_i \in [-5.12, 5.12]$
F3: Griewank	$f(x)=\frac{1}{4000}\sum_{i=1}^{n} x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	10	$x_i \in [-600, 600]$
F4: Ackley	$f(x)=-a\exp(-b.\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i))+a+\exp(1)$	10	$x_i \in [-32.768, 32.768]$
F5: Rozenbrock	$f(x)=\sum_{i=1}^{n-1}[100(x_{i+1}-x_i^2)^2+(1-x_i)^2]$	10	$x_i \in [-2.048, 2.048]$
F6: Six Hump Camel Back	$f(x_1,x_2)=(4-2.1x_1^2+\frac{x_1^4}{3})x_1^2+x_1x_2+(-4+4x_2^2)x_2^2$	2	$x_1 \in [-3,3]$ $x_2 \in [-2,2]$
F7: Schwefel 1-2	$f(x)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_j\right)^2$	10	$x_i \in [-100, 100]$
F8: Goldstein Price	$f(x_1,x_2)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2].$ $[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_1+27x_2^2)$	2	$x_i \in [-2,2]$
F9: Easom	$f(x_{1,}x_2)=-\cos(x_1)\cos(x_2)\exp(-(x_1-\pi)^2-(x_2-\pi)^2)$	2	$x_1 \in [-100,100]$ $x_2 \in [-100,100]$
F10: Weighted Sphere	$f(x)=\sum_{i=1}^{n} ix_i^2$	10	$x_i \in [-5.12, 5.12]$

Table 2. Statistical Performance of QIECT on benchmark functions

FUNCTIONS	OBJECTIVE FUNCTION VALUES					AVERAGE TIME (SECOND)
	BEST	AVERAGE	WORST	STANDARD DEVIATION	MEDIAN	
F1	0	0	0	0	0	62.72
F2	0	1.4484E-03	4.4900E-02	0.008064	0	45.56
F3	0	0	0	0	0	40.72
F4	8.8818E-16	3.0507E-15	1.5099E-14	4.99441E-15	8.8818E-16	513.06
F5	9.9396E-29	3.5716E-22	4.1993E-21	9.74143E-22	3.2283E-26	20034.06
F6	-1.0316	-1.0316	-1.03162	0	-1.03162	16.10
F7	0	0	0	0	0	51.082
F8	3	3	3	0	3	2.43
F9	-1	-1	-1	0	-1	29.44
F10	0	0	0	0	0	49.99

To prove the efficiency of current algorithm, it is compared with reported results of various other reputed algorithms. All the results which are taken for comparison are of the same dimensions as mentioned in Table 1.

Table 3 compares the optimal values of benchmark functions evaluated by QIECT with other algorithms. All the experimental results of QIECT in terms of mean fitness and standard deviation values are summarized in Table 4. It can be noticed that performance outcome of QIECT is favorable in comparison to other state of art algorithms [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28].

Table 3 Best optimal values of benchmark functions, as evaluated by QIECT with other algorithms (NA means not available)

Functions	Exact Values	QIECT	SASP [18]	Hybrid ICA-PSO [19]	BSA [20]
F1	**0**	**0**	NA	1.43E-15	NA
F2	**0**	**0**	2.13E-14	1.24E-12	NA
F3	**0**	**0**	0	NA	NA
F4	**0**	8.**8818E-16**	NA	NA	NA
F5	**0**	**9.94E-29**	NA	6.74E-4	NA
F6	**-1.0316**	**-1.0316**	-1.0316	NA	NA
F7	0	0	NA	NA	NA
F8	**3**	**3**	NA	NA	3
F9	**-1**	**-1**	NA	NA	-1
F10	**0**	**0**	NA	NA	NA

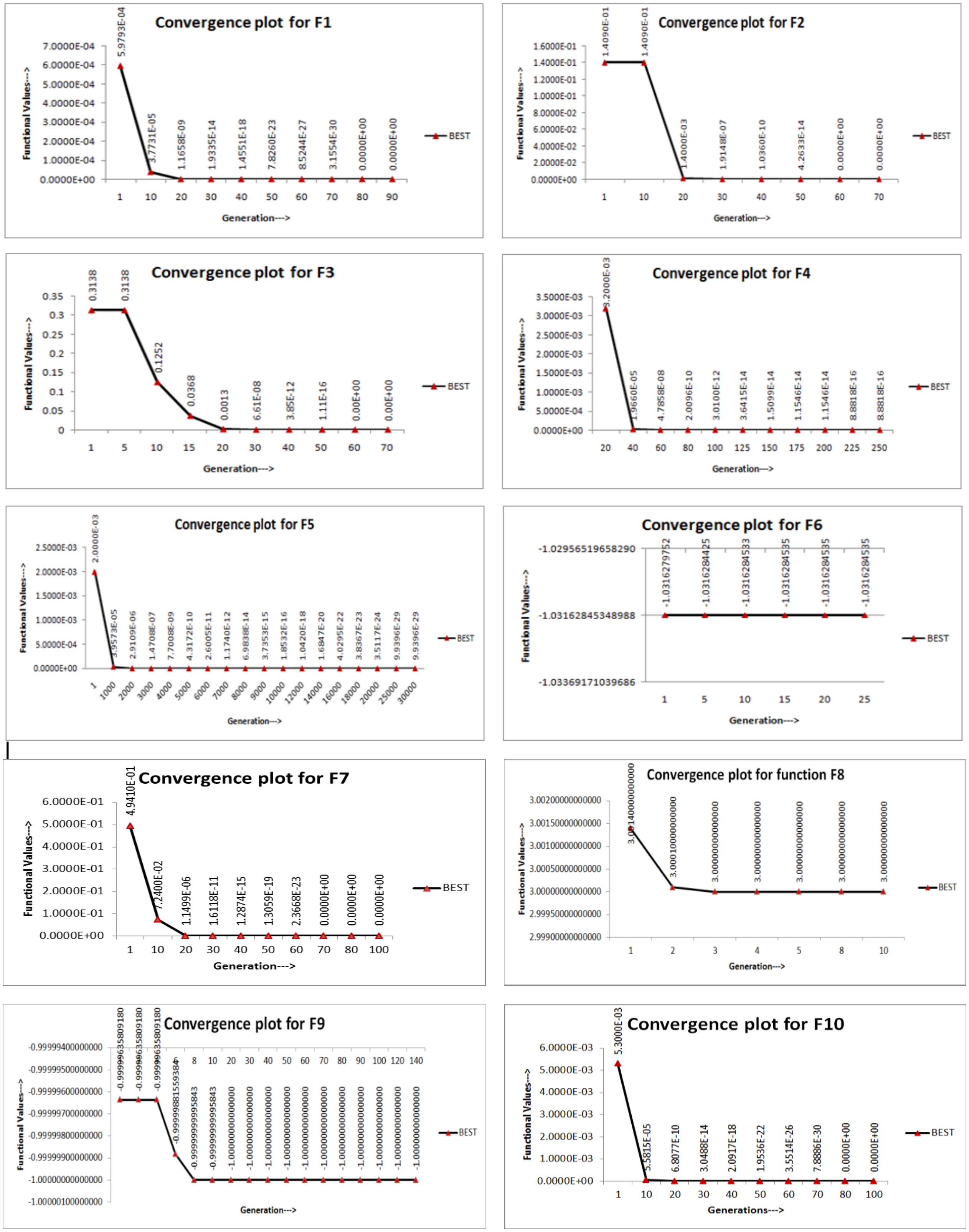

Figure. 2: Convergence plot of optimal values obtained by QIECT for benchmark functions

Table 4 Statistical performances of benchmark functions, as evaluated by QIECT with other state-of-art algorithms ('-' means data is not available)

For F1		
METHODS	**MEAN**	**STD. DEV.**
Hybrid ICA-PSO [19]	3.07E-12	4.34E-12
APSO [22]	0	0
WQPSO [24]	4.71E-106	4.76E-108
KHLD [26]	3.07E-06	2.17E-06
HS [27]	7.69E-05	2.64E-05
QIECT	**0**	**0**

For F2		
METHODS	**MEAN**	**STD. DEV.**
Hybrid ICA-PSO [19]	6.40E-08	1.51E-07
MPSO [21]	1.8407	-
APSO [22]	0.8755	0.8734
IPSO [23]	0.8001	-
WQPSO [24]	1.8857	0.0118
HS [27]	1.38E-02	4.93E-03
QIECT	**1.45E-03**	**0.0081**

For F3		
METHODS	**MEAN**	**STD. DEV.**
MPSO [21]	0.0504	-
IPSO [23]	0.0507	-
WQPSO [24]	1.53E-04	3.37E-04
KHLD [26]	1.00E-06	1.22E-06
HS [27]	4.74E-02	5.99E-02
QIECT	**0**	**0**

For F4		
METHODS	**MEAN**	**STD. DEV.**
APSO [22]	0.0064	7.48E-4
C-Catfish PSO [25]	8.88E-16	-
KHLD [26]	0.0013	3.83E-04
HS [27]	1.12E-02	2.17E-03
IGAL-ABC [28]	4.44E−15	0
QIECT	**3.05E-15**	**1.30E-15**

For F5		
METHODS	**MEAN**	**STD. DEV.**
Hybrid ICA-PSO [19]	1.75	2.1894
MPSO [21]	1.0194	-
APSO [22]	2.3878	1.1055
WQPSO [24]	10.1650	0.2345
C-Catfish PSO [25]	1.2780	0.6500
KHLD [26]	2.12E-04	2.01E-04
HS [27]	4.18E-02	4.87E-02
QIECT	**3.57E-22**	**9.74E-22**

For F6		
METHODS	**MEAN**	**STD. DEV.**
Hybrid ICA-PSO [19]	-1.0316	0
KHLD [26]	2.99E-05	3.65E-05
HS [27]	-1.03	0
QIECT	**-1.0316**	**0**

For F7		
METHODS	**MEAN**	**STD. DEV.**
MPSO [21]	0	-
QIECT	**0**	**0.1109**

For F8		
METHODS	**MEAN**	**STD. DEV.**
BSA [20]	2.9999	1.10E-15
KHLD [26]	3.006	0.0041
QIECT	**3**	**0**

For F9		
METHODS	**MEAN**	**STD. DEV.**
KHLD [26]	-1	6.92E-09
QIECT	**-1**	**0**

For F10		
METHODS	**MEAN**	**STD. DEV.**
KHLD [26]	1.35E-05	7.31E-05
QIECT	**0**	**0**

B. *Performance of QIECT on Structural Engineering Optimization Problems*

1) *Spring Design*

It is one of the well-researched mechanical design problem. The minimization of the weight of a spring under tension/compression is considered subject to constraints of minimum deflection, shear stress, surge frequency, and limits on the outside diameter and design variables. The design variables x1, x2, and x3 are the wire diameter (d), the mean coil diameter (D), and the number of active coils (P) as illustrated in Figure 3. This problem is described in [30]. Engineering design problems has been solved by several engineers using numerous algorithms [2, 3, 6, 7, 9, 10, 11, 14, 30, 31 32, 33, 34, 35, 36, 37, 38].

$$
\begin{aligned}
\textit{Minimize} \quad f(x) &= (x_3+2)x_2x_1^2 \\
s.t. \quad g_1(x) &= 1-\frac{x_2^3x_3}{71785x_1^4} \le 0 \\
g_2(x) &= \frac{4x_2^2-x_1x_2}{12566(x_2x_1^3-x_1^4)}+\frac{1}{5108x_1^2}-1 \le 0 \\
g_3(x) &= 1-\frac{140.45x_1}{x_1^2x_3} \le 0 \\
g_4(x) &= \frac{x_2+x_1}{1.5}-1 \le 0
\end{aligned}
$$

$$0.05 \le x_1 \le 2\,; \quad 0.25 \le x_2 \le 1.3\,; \quad 2 \le x_3 \le 15$$

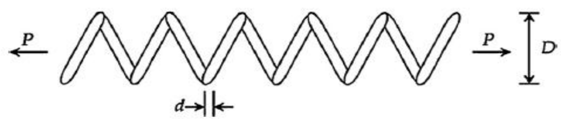

Figure 3 Tension / Compression spring design problem

In solving this problem

$$\text{Penalty} = \sum_{i=1}^{4} g_i \times 10^4$$

is used, where gi is the ith constraints deviation from limits. The algorithm has also successfully obtained optimal value $f(x)$=0.0126652, Corresponding variables [x_1, x_2, x_3]= [0.35670, 0.05169, 11.28999]. Constraints are [g_1, g_2, g_3, g_4]= [-1.26E-08, -2.20E-09, -6.43582, -6.09E-03]. Convergence plot is depicted in Figure 4.

Values of statistical parameters as evaluated by QIECT and other methods are reported in Table 5. It shows that the proposed algorithm gives comparable results for spring design problem. The Table 6 shows the variables and constraints obtained for the spring design problem using QIECT and others reported in literature. It can be clearly observed from both the tables that the results obtained from QIECT algorithm are more accurate and consistent as compared to the other methods reported in literature.

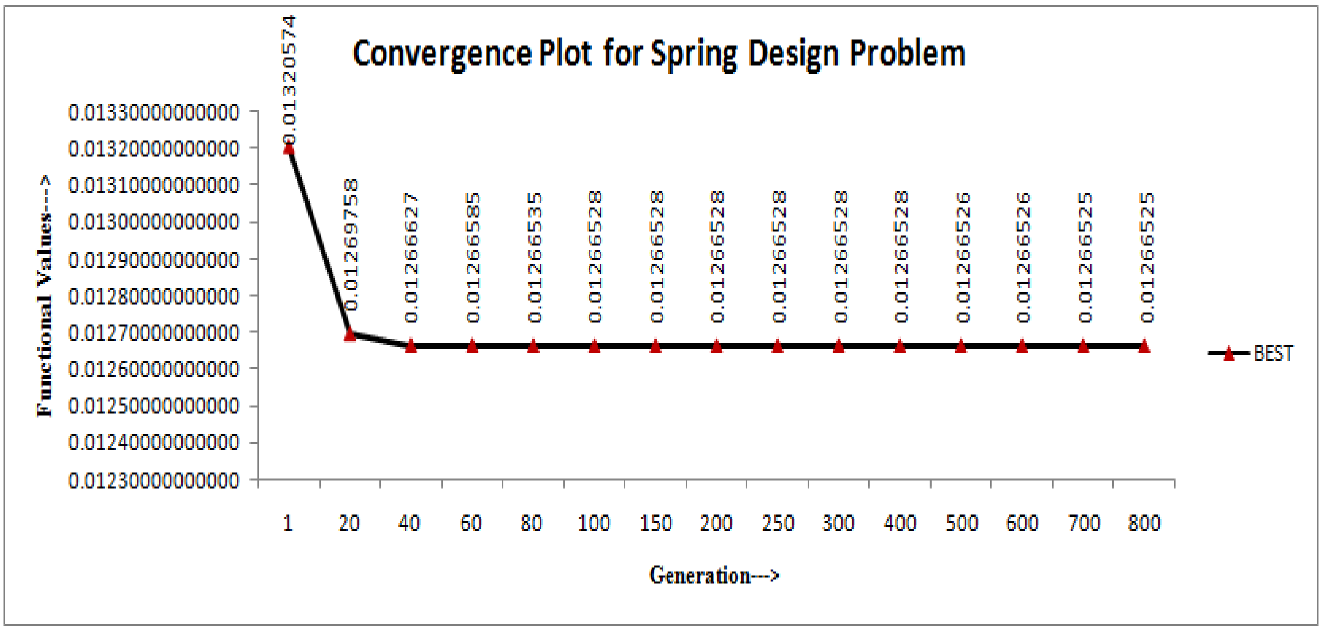

Figure 4. Convergence plot for Spring Design Problem

2) *Pressure Vessel problem*

A compressed air storage tank with a working pressure of 2000psi and a maximum volume of 750ft3 is designed. A cylindrical vessel is shown in Figure 5. The shell is made in two halves that are joined by two longitudinal welds to form a cylinder. The objective is to minimize the total cost, including the cost of material, forming and welding [7]. There are four design variable associated with it namely as thickness of the pressure vessel, Ts = x_1, thickness of the head, T_h = x_2, inner radius of the vessel, R = x_3, and length of the vessel without heads, L=x_4.

In solving this problem,

$$\text{Penalty} = \sum_{i=1}^{4} g_i$$

is used, where g_i is the ith constraints deviation from limits.

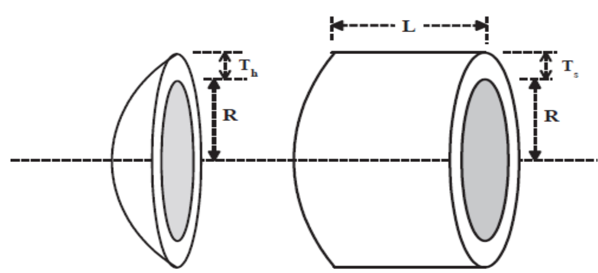

Figure 5 Design of Pressure Vessel Problem

Table 5. Comparison of Statistical results of Constraint optimization problems, as evaluated by QIECT with other methods ('-' means data is not available)

		METHODS										
PROBLEMS	**STATISTICAL PARAMETERS**	**QIECT**	**Coello [2]**	**Coello [3]**	**Akay [6]**	**Garg [7]**	**Kaveh [9]**	**Askarzadeh [10]**	**Hans Raj [11]**	**Coelho [14]**	**He [31]**	**Cagnina [32]**
SPRING DESIGN	**BEST**	**0.0126652**	0.0127048	0.0126810	0.012665	0.0126652	0.0126432	0.0126652	0.0127000	0.0126650	0.0126653	0.0126650
	MEAN	**0.01300**	0.01277	0.01274	0.01271	0.01267	0.01272	0.01267	0.01273	0.01300	0.01270	0.01310
	WORST	**0.01272**	0.01282	0.01297	NA	0.01271	0.01288	0.01267	0.01301	0.01587	-	-
	STD. DEV	**1.93E-5**	3.94E-5	5.90E-5	0.01281	9.43E-6	3.49E-5	1.36E-6	0.0024	0.0006	4.12E-5	4.10E-4
	MEDIAN	**0.01269**	-	-	-	0.01267	-	-	0.01270	0.01271	-	-
PRESSURE VESSEL	**BEST**	**5894.02**	6288.74	6059.94	6059.71	5885.40	6059.73	6059.71	-	6059.72	6059.71	6059.71
	MEAN	**6272.89**	6293.84	6177.25	6245.31	5887.56	6081.78	6342.49	-	6440.37	6289.93	6092.04
	WORST	**7761.79**	6308.15	6469.32	NA	5895.13	6150.13	7332.84	-	7544.49	-	-
	STD. DEV.	**440.560**	7.41000	130.930	205.000	2.75000	67.2400	384.950	-	448.470	305.780	12.1700
	MEDIAN	**6085.06**	-	-	-	5886.15	-	-	-	6257.59	-	-

Table 6. Comparison of Variables, Constraints and objective function value of **Spring Design Problem**, as evaluated by QIECT and other methods

	Design Variables			**Constraints**				
Methods	x_1	x_2	x_3	g_1	g_2	g_3	g_4	**Objective Function**
Coello [4]	0.35166	0.05148	11.6322	-0. 003336	-0. 000109	-679.1016	-0. 61402	0.0127047
Garg [7]	0.35672	0.05169	11.2888	-2.53E-13	-5.755E-13	−4.053784	-0.72773	0.0126652
Hans Raj [11]	0.41128	0.05386	8.68438	0.000001	6.8601E-6	-387.3829	-0.55281	0.0127484
Coelho [14]	0.35253	0.05152	11.5389	-4.834E-5	-3.5770E-5	4.0455	-0.73064	0.012665
Hans Raj [16]	0.35133	0.05147	11.6221	-	-	-	-	0.01267
Arora [30]	0.39918	0.05340	9.1854	-0.001234	-1.8099E-5	-431.3057	-0.56522	0.1273027
Belegundu [33]	0.3159	0.05	14.25	-0. 001267	-0. 003782	-1001.7839	-0. 65077	0.0127047
Ray [34]	0.32153	0.05042	11.9799	0.141412	-0. 012943	-819.5482	0	0.01306
He and Wang [35]	0.35764	0.05173	11.2445	-0.000845	-1.2600E-5	-4.0513	-0.72709	0.0126747
Wang [36]	0.35969	0.05181	11.1193	-1.620E-4	-4.2000E-5	-4.058572	-0.72566	0.0126682
Eskandar [37]	0.35672	0.05169	11.2890	-1.65E-13	-7.900E-14	-4.053399	-0.72786	0.012665
Long [38]	0.35672	0.05169	11.2890	-2.74E-13	2.045E-14	-4.053785	-0.72773	0.0126652
QIECT	**0.3567**	**0.051688**	**11.2999**	**-1.265E-8**	**-2.2049E-9**	**-6.435824**	**-6.0884E-3**	**0.0126652**

Table 7 Comparison of variables and constraints and best solutions for **Pressure Vessel Problem** found by different methods ('-' means data is not available)

Methods	**Design Variables**				**Constraints**				**Obj. Function**
	x_1	x_2	x_3	x_4	g_1	g_2	g_3	g_4	$f(x)$
Sandgren [39]	1.125	0.625	48.3807	11.7449	-0.1913	-0.1634	-75.875	-128.255	8048.619
Zhang and Wang [40]	1.125	0.625	58.29	43.693	-0.025	-0.0689	6.5496	-196.307	7197.7
Deb and Gene [41]	0.9375	0.5	48.329	112.679	-0.0048	-0.0389	-3652.88	-127.321	6370.7035
Coello [2]	0.8125	0.4375	40.3239	200	-0.0034	-0.0528	-27.1058	-40	6288.7445
Coello [3]	0.8125	0.4375	42.0974	176.6541	-	-	-	-	6059.946
Gandomi [42]	0.8125	0.4375	42.0984	176.6366	-	-	-	-	6059.7143
He et al. [43]	0.8125	0.4375	42.0984	176.6366	-	-	-	-	6059.7143
Lee and Geem [44]	1.125	0.625	58.2789	43.7549	-	-	-	-	7198.433
He and Wang [35]	0.8125	0.4375	42.0913	176.7465	-	-	-	-	6061.0777
Montes [45]	0.8125	0.4375	42.0984	176.6361	-	-	-	-	6059.7017
Montes [46]	0.8125	0.4375	42.0981	176.6405	-	-	-	-	6059.7456
Cagnina et al. [32]	0.8125	0.4375	42.0984	176.6366	-	-	-	-	6059.7143
Kaveh [9]	0.8125	0.4375	42.1036	176.5732	-	-	-	-	6059.0925
Kaveh [47]	0.8125	0.4375	42.0984	176.6378	-	-	-	-	6059.7258
Coelho [14]	0.8125	0.4375	42.0984	176.6372	-8.79E-7	-3.59E-2	-0.2179	-63.3628	6059.7208
Youyun and Hongqin [48]	0.7782	0.3846	40.3196	200	-	-	-	-	5885.3328
Akay and Karaboga [6]	0.8125	0.4375	42.0984	176.6366	-	-	-	-	6059.7143
Garg [7]	0.7782	0.3847	40.3211	199.9802	-0.39E-6	-2.8E-6	-1.1418	-40.0197	5885.4033
QIECT	**0.7826**	**0.4218**	**40.3196**	**200**	**-0.0044**	**-0.0372**	**0**	**-40**	**5894.0229**

The algorithm has also successfully obtained optimal value **f(x) = 5894.0229**, Corresponding **variables [x_1, x_2, x_3, x_4]= [0.7826, 0.4219, 40.3196, 200].** Constraints are **[g_1, g_2, g_3, g_4] = [-0.0045, -0.0372, 0, -40].** Pressure Vessel design problem has been solved by several engineers using numerous algorithms [2, 3, 6, 7, 9, 10, 11, 14, 31, 32, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. Comparison of statistical parameters between QIECT and other methods are given in Table 5. Comparison of objective function value, variables and constraints, as evaluated by QIECT and different methods are given in Table 7. It can be clearly observed from the table that the results obtained from QIECT algorithm are more accurate and consistent as compared to the other methods reported in literature.

1) Gear Train Design

The weight of the gear train is to be minimized subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shafts. The variables x_1, x_2, x_3,…., x_7 are the face width, module of teeth, number of teeth in the pinion, length of the first shaft between bearings, length of the second shaft between bearings and the diameter of first and second shafts as shown in Figure 6. The details for this single objective problem with 11 behavioral constraints are taken from literature [49].

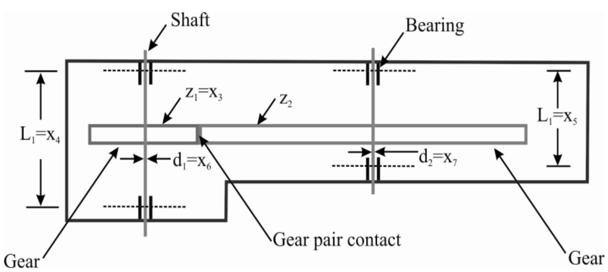

Figure 6. Gear Train design used as third example

The mathematical formulation of this problem can be described as follows:

$$\text{Minimize } f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$

Subject to

$$g_1(x) = 27x_1^{-1}x_2^{-2}x_3^{-1} \le 1$$

$$g_2(x) = 397.5x_1^{-1}x_2^{-2}x_3^{-2} \le 1$$

$$g_3(x) = 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \le 1$$

$$g_4(x) = 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \le 1$$

$$g_5(x) = x_2x_3 \le 40.0$$

$$g_6(x) = 5 \le \frac{x_1}{x_2} \le 12.0$$

$$g_7(x) = (1.5x_6 + 1.9)x_4^{-1} \le 1$$

$$g_8(x) = (1.1x_7 + 1.9)x_5^{-1} \le 1$$

$$g_9(x) = \frac{[(745x_4x_2^{-1}x_3^{-1})^2 + 16.9\times10^6]^{1/2}}{[110.0x_6^3]} \le 1$$

$$g_{10}(x) = \frac{[(745x_5x_2^{-1}x_3^{-1})^2 + 157.5\times10^6]^{1/2}}{[85.0x_7^3]} \le 1$$

$$\textit{where } 2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8,$$
$$17 \le x_3 \le 28, 7.3 \le x_4 \le 8.3, 7.8 \le x_5 \le 8.3,$$
$$2.9 \le x_6 \le 3.9, 5.0 \le x_7 \le 5.5$$

The Table 8 shows the results of statistical parameters, obtained for the gear train design using QIECT algorithm. Comparison of objective function values and variables, as evaluated by QIECT and other methods are given in Table 9. It can be clearly observed from the tables that the results obtained from QIECT algorithm are more accurate and consistent as compared to the values reported in a number of publications [11, 16, 34, 49, 50, 51]. In solving this problem following penalty function is used:

$$\text{Penalty} = \sum_{i=1}^{10} g_i^2$$

Where g_i is the *i*th constraints deviation from limits. The algorithm has also successfully obtained optimal value for gear train problem that is $f(x)$ = **2559.2000**, Corresponding Variables [x_1, x_2, x_3, x_4, x_5, x_6, x_7]**=[3.5680, 0.7000, 28.0000, 8.3000, 3.9000, 5.5000]** and Constraints are [g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9]**=[0.1716, -0.0052, -0.2369, -0.8713, -27.6348, 5, -0.1400, -0.0871, 0.3070, 0.1175].**

II. CONCLUSION

This paper presents a Quantum Inspired Evolutionary Computational Technique (QIECT) that integrates concepts of quantum computing, genetic evolution and simulated annealing. The simulation results of QIECT tested on benchmark functions demonstrate that QIECT compares well with other reputed algorithms in all aspects. It is seen that QIECT has not only a good exploration capability but also has good local exploitation ability. It can easily avoid premature convergence. The benchmark problems illustrate its versatility and effectiveness in solving optimization problems with single objective function. The values of statistical parameters evaluated by QIECT are compared with other methods for spring design, pressure vessel and gear train design problems. The results obtained from QIECT presented earlier, illustrate the accuracy of QIECT as compared to other methods reported in literature. It shows the applicability of QIECT for mechanical engineering design and in particular agile design.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the inspiration and guidance provided by most revered Prof. P. S. Satsangi, Chairman of Advisory Committee on Education, Dayalbagh.

We are also indebted to Prof. V. Maharaj Kumar for the help he gave in completing this work. We duly acknowledge the motivation of Prof. M. P. Gupta IIT Delhi.

Table 8 Comparison of Statistical results of **Gear Train Problem**, as evaluated by QIECT with other methods ('-' means data is not available)

STATISTICAL PARAMETERS	QIECT	Hans Raj [11]	Hans Raj [16]	Ray [34]	Rao [49]	Li [50]	Kunag [51]
BEST	**2559.20**	2724.05	2723.34	2732.9	2985.22	2994.4	2876.2
MEAN	**2716.08**	3026.64	3054.63	-	-	-	-
WORST	**2740.01**	3212.72	3234.75	-	-	-	-
STD. DEV.	**37.7034**	79.8742	75.1000	-	-	-	-
MEDIAN	**2723.58**	2957.68	2996.48	-	-	-	-

Table 9. Comparison of variables and objective function value of **Gear Train problem**, as evaluated by QIECT and other methods

Methods	Design Variables							
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Objective $f(x)$
Rao [49]	3.5	0.7	17	7.3	7.3	3.35	5.29	2985.2
Li [50]	3.5	0.7	17	7.3	7.7153	3.3505	5.2867	2994.4
Kunag [51]	3.6	0.7	17	7.3	7.8	3.4	5	2876.2
Ray [34]	3.5	0.7	17	7.4973	7.8346	2.9018	5.0022	2732.9
Hans Raj [11]	3.5	0.7	17	7.4973	7.8346	2.9018	5.0022	2732.9
Hans Raj [16]	3.5	0.7	17	7.3	7.8	2.9002	5.0002	2723.3
QIECT	3.6	0.7	28	8.3	8.3	3.9	5.5	2559.2

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