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Re-contextualization of Discretized Fuzzy Cyber-Risk Functional Arguments with Fuzzy Polynomials

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***Abstract*— Re-contextualization is basically a transformative process of reframing a system structure to assume that of another system. It extracts texts, symbols, signs, artefacts, or meanings from its original structure to assume another structure in context. The concept is applied in abstract and quantitative fuzziness to resolve and re-interpret discrete fuzzy function by assuming fuzzy polynomial structure. This study is premised upon the fuzziness in relationships exhibited by cyber-security vulnerabilities and threat constructs as fuzzy functions. The evaluation of discrete fuzzy functions based on arbitrary equidistant computations render the solution indeterminate. However, re-contextualizing the cyber-risk function into a fuzzy polynomial simplifies the solution and depicts its functional nature for real world applications. Numerical examples are used to illustrate the concept by the Ranking method.**

***Keywords*— Cyber-Risk; Re-contextualization; Fuzzy Polynomial; Discrete Fuzzy Functions; Ranking Method;**

I. Introduction

Re-contextualization is basically a process of transformation whereby a structure of a system is reframed to assume a structure of another system. The process may extract text, symbols, signs, artefacts, or meanings from its original context or structure in order to introduce it into another context or structure [1] [2]. In this study, the concept is applied to abstract and quantitative fuzziness in an attempt to solve mathematical equations, and re-interpret the solutions to the equations.

Typically, it is not uncommon for engineers and scientists to use functions to develop the relationships amongst constructs used in modeling problems or solutions of interest [3]. Perfilieva [4] posits that a "fuzzy function is a special fuzzy relation with a generalized property of uniqueness". This notion is alluded to by Zadeh [5] in his extension principle, which espouses that a classical function can be fuzzified. Accordingly, a fuzzy function is interpreted as a fuzzy result or fuzzy functional values formulated by a "fuzzy bunch parameters" [6].

We herewith adapt the approach used by [7] in representing the vagueness or subjectivity in relationships exhibited by cyber-security vulnerabilities and threats constructs as fuzzy functions.

The cyber-risk function is formulated based on the perceived uncertainties and it uses fuzzy linguistic variables to represent real world business situational multifaceted decision-making to model risk assessment. The issue is how to estimate the cumulative impact on business, should these vulnerabilities be exploited? These uncertain attributes, which are representations of fuzziness, are suitably described by fuzzy functions [1]. For example, *Confidentiality* impact is estimated by taking the possibilities of the occurrence of breaches due to human errors, transmission errors, data storage, data disposal, etc. leading to or due to unauthorized disclosure.

The essence of this study is premised upon the cyber-risk defined mathematically as a fuzzy relational function [7]:

$$CyberRisk_{threat} \cong \tilde{F}(Threat_{asset}, Vu\ln erability_{threat}, AssetValue)$$

[1-1] where

- Asset Value is the summation of contributing values from the CIA (Confidentiality, Integrity & Availability) utilities as evaluated in respect of the urgency for restoring a compromised asset and/or its criticality to the business;

[1] A function is a mapping, a transformation, an operation or a relation between a set of inputs, usually called arguments, and each element of a given set of outputs, usually called the value(s). Functions are said to be paramount in investigative research [39]. Literature supports a variety of methods for the representation of functions. The commonest ways are either by a formula or an algorithm, which may describe the relationship existing amongst the outcome and the arguments.

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- Vulnerability is a weakness within and around the system that can be exploited by threat agents; and
- Threat is the possibility of exploiting the weakness, given the conditions of motivation or intent, capability, opportunity and attractiveness of the asset.

The equation [1-1] depicts how the fuzzy cyber-risk function can be evaluated in enterprise systems or in fuzzy multi-attribute decision-making (MADM). From equation [1-1], the parameters Threat, Vulnerability and Asset Value can either be convoluted if numerical possibility measures are available or the linguistic terms or semantic labels are "convoluted" using approximate reasoning (intuitionistic) with rules [8]. The value of an asset determines its attractiveness, whereas the vulnerability with the asset serves as an opportunity for possible threat exploitation leading to the extent of cyber-risk [9].

It is noteworthy that the contributions of the CIA attributes towards the asset value evaluation or computation are such that, the confidentiality implies the asset's confidentiality or privacy or authentication requirements; the integrity implies the asset's trustworthy or accuracy; and the availability implies the asset's requirement of communications security, continuous functionality or accessibility.

Cyber-risk function depicts the ultimate expression of the consequences resulting from a threat agent exploiting an opportune vulnerability in a given asset. The cyber-risk function is used to express the fuzzy relationships existing amongst the functional arguments. It estimates qualitatively the resultant risk value upon assessing the nature and extent of vulnerabilities and the possibility of exploitation and associated severity, should a threat matures for a given cyber-asset [10]. This evaluation is carried out by taking into account all possible (including potential) threats, with the notion that vulnerabilities might be exploited by multiple threat agents.

The implication is that, a higher asset value (which is treated as disutility or negative utility), and a higher vulnerability value (i.e. highly susceptible asset) and/or a higher threat value (i.e. highly motivated threat agent, equipped with necessary capabilities in an opportune moment) will create a high impact or risk to an enterprise [9].

In an attempt to solve equation [1-1], some delimitations or assumptions are imposed as necessary conditions to achieve a practical mathematical solution.

Firstly, equation [1-1] is deemed as having discrete fuzzy functional arguments [11]. Discrete fuzzy functions are evaluated using equidistant approaches, which typically consider the abscissa and ordinate axes separately. In reality, or in practical situations, evaluation using the abscissa discrete measures involves arbitrary discretization which tends to exclude pertinent modal and boundary values [11] [12]. Though the ordinate discrete measures, however, captures the modal and boundary values, for non-linear cyber-security situations, the equidistant discretization measures render the solution inadequate [7] [8].

The equation [1-1] has practical applications in cyber-security threats and vulnerabilities taxonomies, classification, and inferences [7]. The equation is solved using intuitive fuzzy relational approaches, such as fuzzy similarity measures, fuzzy parametric equations, fuzzy reasoning using minimal solutions, etc. By re-contextualizing the discrete fuzzy functions into fuzzy polynomials functions, the problem of finding real solutions is simplified, as exemplified by the numerical illustrations in section 3.2. Even though equation [1-1] has feasible or solvable solutions, the nature of the function is indeterminate; implying that depending on the circumstances the function may be resolved definitively or unresolved. So, the essence of re-contextualizing into fuzzy polynomials is to solve and, more or less, determine the nature of the function.

The paper is organized as follows. In this section, we've given the background to the study and underscored the object of the study. The next section deals with some basic pre-requisites in fuzzy sets. Ensuing sections deals with discretized fuzzy functions, fuzzy polynomials and related works. The key concept of re-contextualization is put in perspective, and followed by fuzzy polynomial solution using the Ranking method. Numerical examples are illustrated and concluded.

II. PRELIMINARIES

Prof. Lotfi A. Zadeh [13] introduced the fuzzy set theory as an approach in treating uncertainties. He posited that whereas probability deals with the uncertainty of

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occurrences due to randomness, possibility deals with the uncertainty of vagueness and he espoused the essence of graded membership functions. Fuzzy sets can practically and quantitatively represent vague concepts. Fuzzy set theory caters for imprecision and vagueness and has wide applications in engineering and sciences. Fuzzy set theory is used in modeling ill-defined and complex nonlinear systems.

The utility of fuzzy set theoretic is seen in the use and application of fuzzy data which is usually expressed subjectively in natural languages with human reasoning. The perceived uncertainties inherent in the metrics used in many systems and applications are appropriately handled and analyzed using fuzzy set theory [13] [14] [15] [16] .

Generally, human knowledge is fuzzy; it is usually expressed in linguistic terms such as “young”, “secured”, “vulnerable” – which are fuzzy in nature. The linguistic terms are constructed to form fuzzy sets. Fuzzy logic treats every logical system as a fuzzified system with parameters estimated by degrees of “contribution” [17]. Fuzzy logic is first and foremost multi-valued logic and its grade of membership facilitates the capture and inclusion of most system attributes and constructs.

Definition 2-1: Fuzzy number*:* A fuzzy set $\tilde{\mathbf{A}}$ of the real numbers, $\mathbf{R}$, and with membership function $\tilde{\mathbf{A}}(x):\mathbf{R}\rightarrow[0,1]$ is called a Fuzzy Number, if:

- $\tilde{\mathbf{A}}$ is normal; $\exists$ an element x_0 such that $\tilde{\mathbf{A}}(x_0)=1$;
- $\tilde{\mathbf{A}}$ is fuzzy convex; i.e. $\tilde{\mathbf{A}}(\lambda x_1+(1-\lambda)x_2)\geq\min\{\tilde{\mathbf{A}}(x_1),\tilde{\mathbf{A}}(x_2)\}$ $\forall x_1,x_2\in\mathbf{R},\ \lambda\in[0,1]$;
- $\tilde{\mathbf{A}}(x)$ is upper semi-continuous;
- Support $\tilde{\mathbf{A}}$ is bounded, where support $\tilde{\mathbf{A}}=cl\{x\in\mathbf{R}:\tilde{\mathbf{A}}(x)>0\}$

Definition 2-2: *Membership Function:* Let $\tilde{\mathbf{X}}$ be the set with members or elements x, then a fuzzy set $\tilde{\mathbf{A}}$ defined in $\tilde{\mathbf{X}}$ is given by the duality or the ordered pair $(x,\mu_{\tilde{\mathbf{A}}}(x))$; where $\mu_{\tilde{\mathbf{A}}}(x)$ is the characteristic or membership function for the fuzzy set $\tilde{\mathbf{A}}$. i.e. $\tilde{\mathbf{A}}=\{(x,\mu_{\tilde{\mathbf{A}}}(x))\,|\,x\in X\}$. The membership function $\mu_{\tilde{\mathbf{A}}}(x)$ defines the grade of membership of the members x into the set $\tilde{\mathbf{A}}$. It indicates the degree or extent of compatibility with the fuzzy concept [18]. For a typical fuzzy set $\tilde{\mathbf{A}},\ \mu_{\tilde{\mathbf{A}}}(x):x\rightarrow[0,1],$ such that $\mu_{\tilde{\mathbf{A}}}(x)=0$ implies that the “member” doesn't belong to the set $\tilde{\mathbf{A}}$, whereas $\mu_{\tilde{\mathbf{A}}}(x)=1$ implies full membership. The following are the most common and applicable ones to this study [19] [20]:

i. discrete membership function $\tilde{\mathbf{A}}=\sum_{x_i\in\tilde{\mathbf{X}}}\frac{\mu_{\tilde{\mathbf{A}}}(x_i)}{x_i}$

ii. continuous membership functions $\tilde{\mathbf{A}}=\int_{\tilde{\mathbf{X}}}\frac{\mu_{\tilde{\mathbf{A}}}(x)}{x}$

Definition 2-3: *Domain:* defines the total allowable range of values for a fuzzy linguistic term; usually a set of real numbers, which increase monotonically[2] [18].

Definition 2-4: *Universe of discourse or universe,* $\tilde{\mathbf{X}}$ *:* the universe defines the characteristics and total allowable range of all possible values assigned to the linguistic variables. $\tilde{\mathbf{X}}$ may be discrete or continuous, and may have ordered or non-ordered elements.

Definition 2-5: *Support of membership function (written supp(A)):* The support of a fuzzy set $\tilde{\mathbf{A}}$ is the set containing all the members with membership function values greater than zero (0) and given by $\sup p(\tilde{\mathbf{A}})=\{x\in\tilde{\mathbf{X}}\,|\,\mu_{\tilde{\mathbf{A}}}(x)>0\}$.

[2] A function f defined on a real interval with arguments x_i, such that $f(x_i)$; i ∈ I, f exhibits the tendency of monotonicity, if:

- $x_1\prec x_2\Rightarrow f(x_1)\prec f(x_2)$, f is said to be increasing monotonically; and
- $x_3\prec x_4\Rightarrow f(x_3)\succ f(x_4)$, f is said to be decreasing monotonically.

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Similarly, a *fuzzy Singleton* is a fuzzy set that has only one element or a single member which has a membership function value $\mu_{\tilde{\mathbf{A}}}(x) = 1$.

Definition 2-6: *Alpha-cut sets or α-cut:* Let $\tilde{\mathbf{A}}$ be a fuzzy set in the universe of discourse, $\tilde{\mathbf{X}}$. Let α be a number belonging to the fuzzy unit interval [0, 1], which is a threshold restriction. Then α-cut of $\tilde{\mathbf{A}}$, denoted by $\tilde{\mathbf{A}}_\alpha$, is a crisp set with all elements of $\tilde{\mathbf{A}}$ with membership function values in $\tilde{\mathbf{A}}$ greater than or equal to α. i.e. $\tilde{\mathbf{A}}_\alpha = \left\{ x : \tilde{\mathbf{A}}(x) \geq \alpha \right\}$ or $\tilde{\mathbf{A}}_\alpha = \left\{ x : \mu_{\tilde{\mathbf{A}}}(x) \geq \alpha \right\}$. The α-cut concept finds applications in engineering and science. It's importance is seen as a facilitator for the execution of fuzzy rules and intersection of multiple fuzzy sets [3] [18].

A strong α-cut is defined as $\tilde{\mathbf{A}}_{\alpha+} = \left\{ x : \tilde{\mathbf{A}}(x) > \alpha \right\}$ or $\tilde{\mathbf{A}}_{\alpha+} = \left\{ x : \mu_{\tilde{\mathbf{A}}}(x) > \alpha \right\}$. If α = 1.0, then the crisp set α-cut is called the CORE set of the fuzzy set $\tilde{\mathbf{A}}$. For ease of statistical inference and interpretation, a nested set of quartile α-cut is defined, such that $nested\ \tilde{\mathbf{A}}_\alpha = \left\{ x : \tilde{\mathbf{A}}(x) \rightarrow \alpha \geq \alpha_i \right\}; \forall i = 1,2,3,4,5$ or α receives the values 1, 0.75, 0.50, 0.25, 0. When α = 0, the α-cut set defines the SUPPORT set of the fuzzy set $\tilde{\mathbf{A}}$. For α values 0.75, 0.50, 0.25, the sets are defined as the upper quartile set, mid-quartile set and the lower quartile set, respectively. When α = 0.5, α-cut set is called the Crossover point; that is $\mu_{\tilde{\mathbf{A}}}(x) = 0.5$.

A. *Discretized Fuzzy Sets*

Typically, a discrete fuzzy function is a relation of the form $\tilde{F}(A \cap B) \square (x_{\tilde{A}}, y_{\tilde{A}}, z_{\tilde{A}}).(x_{\tilde{B}}, y_{\tilde{B}}, z_{\tilde{B}})$, where $\tilde{A}(x_{\tilde{A}}, y_{\tilde{A}}, z_{\tilde{A}})$ and $\tilde{B}(x_{\tilde{B}}, y_{\tilde{B}}, z_{\tilde{B}})$ are fuzzy triangular numbers. It could also be extended using their membership functions, such that $\tilde{F}(\tilde{A} \cap \tilde{B}) \square \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$. In practice, discrete fuzzy functions may be attributable to various reasons, such as discrete fuzzy numbers obtained through surveys with expert opinions elicitation, or discretized membership functions for computational purposes [21].

The extension principle facilitates the generalized method for extending crisp mathematical concepts to fuzzy quantities, thus catering for the mapping of crisp functional elements unto fuzzy sets or functional arguments [11]. A cursory look at the fuzzy-based cyber-risk function in equation [1-1] above, reveals that some delimitations may be necessary in order to achieve a practical mathematical solution. Here, the fuzzy arguments are expressed as discrete fuzzy sets [11].

Typical approaches to discretizing fuzzy numbers are either by equidistant of the abscissa or the ordinate axes; i.e. $\Delta x_i = x_{i_{j+1}} - x_{i_j}$ (for the abscissa axis) or $\Delta \mu_i = \mu_{i_{j+1}} - \mu_{i_j}$ (for the ordinate axis).

A key challenge with the abscissa method is that in any real-world parameters (e.g. with practical systems such as assessing the cyber-risk of an enterprise network), the arbitrary discretization process may not guarantee the inclusion of modal and boundary values [11] [12]. Furthermore, in the case of nonlinear systems the equidistant discretization process may be inadequate. However, using the ordinate approach captures the modal and boundary values by evaluating the ordinates $\mu = 0$ and $\mu = 1$.

The notion of discrete fuzzy number has been applied to fuzzy models from politics to engineering to computing. For instance, in determining the preferences, amongst policy alternatives, the abscissa axis represents policy position, whereas the ordinate axis represents extent of politician preference of a given policy [12].

A key axiom in this study, towards the re-contextualization of discrete fuzzy function into fuzzy polynomial function, is borrowed from [12] such that discrete and continuous fuzzy numbers are technically not different. Indeed, "a continuous fuzzy number is simply a discrete fuzzy number with an infinitely high level of granularity" [12, p. 100]. Based on that assumption, we extend the treatise to fuzzy polynomial.

B. *Fuzzy Polynomial*

A polynomial in x is defined as a finite sum of terms of the form $a_i x^n$, where a is a real number and the exponent n

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is a whole number. The term a is called the coefficient and the index n is called the degree of the term. Typically, a polynomial is written in descending order according to degree, and the highest degree term is said to be the leading term with its coefficient, the leading coefficient.

By extension, a polynomial function is defined as a finite sum of terms of the form $f(x)=\sum_{i=1}^{m}(a_i x_i^n)$, $\forall m,n \in I$ (m, n are whole numbers).

Evaluating the polynomial functions implies that, we can estimate the extent of vulnerabilities and cyber-risk, given certain susceptibilities.

A polynomial $P(x) \subset F[x]$ is factorable, or reducible, if there exist polynomials $f(x), g(x) \subset F[x]$ of degree $\geq 1 \ \exists \ P(x) = f(x)g(x)$. If $P(x)$ is not factorizable, it is called a prime or an irreducible polynomial.

Polynomials are used in representing various systems and applications in applied mathematics, engineering and computing technologies. Typical fuzzy polynomials are of the form $\tilde{\mathbf{A}}_1 x + \tilde{\mathbf{A}}_2 x^2 + \dots\dots\dots + \tilde{\mathbf{A}}_n x^n = \tilde{\mathbf{A}}_0$, where $x^i, \tilde{\mathbf{A}}_j \in \tilde{\mathbf{X}}^i \quad \forall i = 1,2,\dots\dots,n$ and $j = 0,1,2,\dots\dots,n$ (if they exist). Note that $\tilde{\mathbf{X}}^i$ is the set of all fuzzy numbers.

Various studies on fuzzy polynomials are noted in the following works:

- Buckley & Eslami [22] for the consideration of fuzzy neural networks based systems and solutions;
- Otadi & Mosleh [23] for the consideration of fuzzy neural network architectures;
- Abbasbandy & Asady [24] for the consideration of nonlinear fuzzy equations using Newton's method;
- Sala & Arino [25] for the consideration of fuzzy polynomial approximation using Taylor-series solutions;
- Abbasbandy & Otadi [26] for the consideration of fuzzy neural networks (FNNs) as equivalent systems of fuzzy polynomials;
- Rahman & Abdullah [27] for the consideration of multi-solutions reviews employing fuzzy polynomials.

Kharrati et al. [28] used a genetic algorithm (GA) fitness function to determine the coefficients of fuzzy polynomial membership functions. It can be deduced, based on similar studies by [28] that each consequent arguments of the cyber-risk fuzzy polynomial in equation [1-1] is a fuzzy polynomial.

Some methods used in evaluating various models of fuzzy polynomials include Newton-Raphson method, Ranking method, modified Adomain Decomposition method, etc.

In a review of fuzzy polynomials, [27] demonstrated that real roots of fuzzy polynomials can be found using various algorithms, if the roots exist. In this study, fuzzified cyber-risk polynomial function is solved by the Ranking method, due to convenience.

Fuzzy polynomial equations were transformed into a system of crisp polynomial equations using the Ranking method [29], based on three variables – value, ambiguity and fuzziness.

Fuzzy logic or systems approach is often used to solve systemic problems without necessarily knowing the mathematical model for which the solution is based.

III. RE-CONTEXTUALIZATION

According to the Merriam-Webster dictionary, to re-contextualize means to place or view (a work of literature or art, for example) in a new or unfamiliar context, especially in order to suggest a different interpretation. So, re-contextualization is the process of extracting text, symbols, signs or meaning from its original context or structure in order to introduce it into another context or structure [1] [2].

In essence, re-contextualize, as applied to reasoning abstractly and quantitatively, implies solving for an arithmetic or mathematical equation, and then interpreting the meaning of the numbers within the context of the problem at stake or to re-interpret the solution to an equation [30].

The crux of the re-contextualization is to attempt to close the gap left in evaluating cyber-risk parameters with discrete fuzzy numbers. Using the principle of re-contextualization, each argument of the cyber-risk fuzzy function is viewed in context of what it represents [31] and "contexts differ in respect of their structure" [32, p. 24].

Similarly, the principle of re-contextualization is defined such that structure which is empirically designated by an activity, may "tend to subordinate" its own structure to that recruited from another activity [32, p. 24]. By extension, the re-contextualization is basically concerned with relationship;

- it's about solving for roots of crisp polynomials versus those of fuzzy polynomials;
- it's about dealing with linear systems versus non-linear fuzzy systems;
- it's about merely estimating a fuzzy relational function, intuitively, versus definitive fuzzy functions or equations of polynomial terms;
- it's about dealing with context-independent accounts versus context-dependent practices.

Bernstein's [33] definition of re-contextualization is described as the "primary context" of producing scientific knowledge, as against the "secondary context" of re-producing the knowledge. Here, it deals with the complexities with which knowledge is transformed into a virtual discourse.

The concept of re-contextualization is premised on the fact that there exists multi-attributes or criteria for the assessment of cyber-risk using fuzzy constructs. The implication is that, these assessment methods are in themselves fuzzy, which invariably are expert opinions elicited through data collection processes. Once "forms of expression and content" [32, p. 136] are re-contextualized, they exhibit [new expression and content], re-produced forms based on the regulated principles of the new activity [34].

In terms of related works, re-contextualization was employed in virtual machine (VM) configuration to contextualize with specific settings for runtime in VM migration in cloud computing environment [35].

In this study, we focus on constructing the fuzzy based cyber-risk function as constituting of [solvable] fuzzy polynomials rather than finding the roots of the polynomials, or the exact form of fuzzy polynomial terms.

A. *Solving Fuzzy Polynomials by Ranking Method*

Possibility theory, rather than the probability theory, is used in recent developments of computational intelligence and similar applications, to make decision amongst multi-criteria or attributes options. Fuzzy numbers are interpreted as resultant decision judgments or alternatives collated from elicited expert opinions. Various methods are used in ordering these fuzzy numbers; in essence, they rank them, thus calling these approaches Ranking methods [36].

Definition 3-1: A polynomial equation is of the form $\tilde{\mathbf{A}}_1 x + \tilde{\mathbf{A}}_2 x^2 + \dots\dots\dots + \tilde{\mathbf{A}}_n x^n = \tilde{\mathbf{A}}_0$ - [3-1]

Where $x \in \mathbf{X}$, the coefficients $\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2, \dots\dots, \tilde{\mathbf{A}}_n$ and $\tilde{\mathbf{A}}_0$ are fuzzy numbers, is called a fuzzy polynomial.

Let x be a real number solution for the equation [3-1], then by extension principles, denoting the three arguments in the cyber-risk function (c.f. equation [1-1]); we have:

- Vulnerability fuzzy polynomial $V(\tilde{\mathbf{A}}_1 x + \tilde{\mathbf{A}}_2 x^2 + \dots\dots\dots + \tilde{\mathbf{A}}_n x^n) = V(\tilde{\mathbf{A}}_0)$
- Threat fuzzy polynomial $T(\tilde{\mathbf{A}}_1 x + \tilde{\mathbf{A}}_2 x^2 + \dots\dots\dots + \tilde{\mathbf{A}}_n x^n) = T(\tilde{\mathbf{A}}_0)$
- Asset Value fuzzy polynomial $S(\tilde{\mathbf{A}}_1 x + \tilde{\mathbf{A}}_2 x^2 + \dots\dots\dots + \tilde{\mathbf{A}}_n x^n) = S(\tilde{\mathbf{A}}_0)$

Using the Ranking method, we have a set of fuzzy polynomials representing the cyber-risk function [29];

$$\begin{Bmatrix} V(\tilde{\mathbf{A}}_1)x + V(\tilde{\mathbf{A}}_2)x^2 + \dots\dots\dots\dots + V(\tilde{\mathbf{A}}_n)x^n = V(\tilde{\mathbf{A}}_0) \\ T(\tilde{\mathbf{A}}_1)|x| + T(\tilde{\mathbf{A}}_2)|x^2| + \dots\dots\dots\dots + T(\tilde{\mathbf{A}}_n)|x^n| = T(\tilde{\mathbf{A}}_0) \\ S(\tilde{\mathbf{A}}_1)|x| + S(\tilde{\mathbf{A}}_2)|x^2| + \dots\dots\dots\dots + S(\tilde{\mathbf{A}}_n)|x^n| = S(\tilde{\mathbf{A}}_0) \end{Bmatrix}$$

[3-2]

By solving for the real roots of equation [3-2], using the crisp polynomial equations, we reckon that the approach could be a special case for the fuzzy polynomials.

Rouhparvar [29] recommended two-stage approach, where $x \geq 0$ and $x < 0$; to evaluate the roots. Thus, resulting in both positive and negative real roots.

B. *Numerical Examples*

For simplicity, we use fuzzy triangular numbers, though similar treatment could be made of trapezoidal fuzzy numbers. So, for a triangular fuzzy number $\tilde{\mathbf{A}}_i = (u_i, \alpha_i, \beta_i)\ \forall\ i = 0,1,2,\dots\dots,n$. Pursuant to equation [3-2] above, we herewith create a set of fuzzy polynomials using their membership functions [29]:

$$(u_1 + \frac{\beta_1 - \alpha_1}{6})x + \dots\dots + (u_n + \frac{\beta_n - \alpha_n}{6})x^n = (u_0 + \frac{\beta_0 - \alpha_0}{6})$$

$$(\frac{\beta_1 + \alpha_1}{6})|x| + \dots\dots + (\frac{\beta_n + \alpha_n}{6})|x^n| = (\frac{\beta_0 + \alpha_0}{6})$$

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$$(\frac{\beta_1+\alpha_1}{4})|x|+......+(\frac{\beta_n+\alpha_n}{4})|x^n|=(\frac{\beta_0+\alpha_0}{4})$$

Now, consider the following fuzzy polynomial: $(0,1,1)x+(0,2,2)x^2+(1,1,1)x^3=(-1,4,4)$ with exact solution for $x=-1$.

By implication, we obtain the following using equation [3-2]:

$$V(\tilde{\mathbf{A}}_1)=0;\ V(\tilde{\mathbf{A}}_2)=0;\ V(\tilde{\mathbf{A}}_3)=1;\ V(\tilde{\mathbf{A}}_0)=-1$$

$$T(\tilde{\mathbf{A}}_1)=\frac{1}{3};\ T(\tilde{\mathbf{A}}_2)=\frac{2}{3};\ T(\tilde{\mathbf{A}}_3)=\frac{1}{3};\ T(\tilde{\mathbf{A}}_0)=\frac{4}{3}$$

$$S(\tilde{\mathbf{A}}_1)=\frac{1}{2};\ S(\tilde{\mathbf{A}}_2)=1;\ S(\tilde{\mathbf{A}}_3)=\frac{1}{2};\ S(\tilde{\mathbf{A}}_0)=2$$

So, $\forall\ x=-1;$ equation [3-2] yields:

$$x^3=(-1)^3=-1;$$
$$\frac{1}{3}|x|+\frac{2}{3}|x^2|+\frac{1}{3}|x^3|=\frac{1}{3}+\frac{2}{3}+\frac{1}{3}=\frac{4}{3};$$
$$\frac{1}{2}|x|+|x^2|+\frac{1}{2}|x^3|=\frac{1}{2}+1+\frac{1}{2}=2;$$ QED.

Similarly, we consider the fuzzy polynomial $(2,1,1)x+(1,1,1)x^2+(4,2,1)x^3+(0,1,1)x^4=(7,5,4)$ with exact solution for $x=1$. By computation from equation [3-2], we obtain:

$$V(\tilde{\mathbf{A}}_1)=2;\ V(\tilde{\mathbf{A}}_2)=1;\ V(\tilde{\mathbf{A}}_3)=\frac{23}{6};$$
$$V(\tilde{\mathbf{A}}_4)=0;\ V(\tilde{\mathbf{A}}_0)=\frac{41}{6}$$

$$T(\tilde{\mathbf{A}}_1)=\frac{1}{3};\ T(\tilde{\mathbf{A}}_2)=\frac{1}{3};\ T(\tilde{\mathbf{A}}_3)=\frac{1}{2};$$
$$T(\tilde{\mathbf{A}}_4)=\frac{1}{3};\ T(\tilde{\mathbf{A}}_0)=\frac{3}{2}$$

$$S(\tilde{\mathbf{A}}_1)=\frac{1}{2};\ S(\tilde{\mathbf{A}}_2)=\frac{1}{2};\ S(\tilde{\mathbf{A}}_3)=\frac{3}{4};$$
$$S(\tilde{\mathbf{A}}_4)=\frac{1}{2};\ S(\tilde{\mathbf{A}}_0)=\frac{9}{4}$$

So, $\forall\ x=1;$ equation [3-2] yields:

$$2x+x^2+\frac{23}{6}x^3=2+1+\frac{23}{6}=\frac{41}{6};$$
$$\frac{1}{3}|x|+\frac{1}{3}|x^2|+\frac{1}{2}|x^3|+\frac{1}{3}|x^4|=\frac{1}{3}+\frac{1}{3}+\frac{1}{2}+\frac{1}{3}=\frac{3}{2};$$
$$\frac{1}{2}|x|+\frac{1}{2}|x^2|+\frac{3}{4}|x^3|+\frac{1}{2}|x^4|=\frac{1}{2}+\frac{1}{2}+\frac{3}{4}+\frac{1}{2}=\frac{9}{4};$$

QED.

IV. CONCLUSION

In this study, we endeavored to re-contextualize discretized fuzzy relational function into a set of fuzzy polynomial functions. We demonstrated with the use of Ranking method to solve for the crisp roots of the fuzzy polynomials. Further studies are being considered with the use of fuzzy similarity measures *and Eigenvalue method* to solve fuzzy polynomials.

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