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Sum Divisor Cordial Labeling of Ring Sum of a Graph With Star Graph

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Abstract— A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,...|V|\}$ such that an edge uv is assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise, then number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we have derived sum divisor cordial labeling of ringsum of some graphs with star graph $K_{1,n}$.

Keywords— Sum divisor cordial labeling, ringsum of two graphs AMS Subject classi cation number: 05C78.

I. Introduction

By a graph, we mean a simple, finite, undirected graph. For terms and notations related to graph theory which are not defined here, we refer to Gross and Yellen[5] and for standard terminology and notations related to number theory we refer to Burton[2]. In this paper we discuss sum divisor cordial graph with a certain graph operation namely ring sum of graphs

Remark 1.1. Throughout this paper $|V(\mathrm{G})|$ and f denote the cardinality of vertex set and edge set of graph G respectively.

1.1 Definitions

Varatharajan et al. introduced the concept of divisor cordial labelling of a graph.

Definition 1.1 (Varatharajan et al.[8]). Let $G=(V,E)$ be a simple graph and $f:V(G)\rightarrow\{1,2,\ldots,|V(G)|\}$ be a bijection. For each edge e = uv, assign the label 1 if $f(u)\,|\,f(v)$ or $f(v)\,|\,f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.

A Lourdusamy, F. Patrick and J. Shiama introduced the concept of sum divisor cordial labeling of graphs.

Definition 1.2 (Lourdusamy et al.[6]). Let $G=(V,E)$ be a simple graph and $f:V(G)\rightarrow\{1,2,\ldots,|V(G)|\}$ be a bijection. For each edge edge e = uv, assign the label 1 if $f(\mathrm{u}_1)=1$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

The divisor cordial and sum divisor cordial labeling of various types of graphs are presented in [6, 7, 8, 9].

Definition 1.3 (Gallian[3]). Ring sum of two graphs $G_1=(V_1, E_1)$ and $G_2=(V_2, E_2)$ is denoted by $G_1 \oplus G_2$, where $G_1 \oplus G_2 = (V_1 \cup V_2, (E_1 \cup E_2) - (E_1 \cap E_2))$.

Remark 1.2. Throughout this paper we consider the ring sum of a graph G with star graph $K_{1,n}$ by considering any one vertex of G and the apex vertex of $K_{1,n}$ as a common vertex

II. SUM DIVISOR CORDIAL LABELING OF RINGSUM OF GRAPHS WITH STAR GRAPH

Theorem 2.1. $C_n \oplus K_{1,n}$ is a sum divisor cordial graph for all n.

Proof. Let $V(C_n \oplus K_{1,n}) = V_1 \cup V_2$, where $V_1 = V(C_n) = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = V(K_{1,n}) = \{v = u_1, v_1, v_2, \ldots, v_n\}$. Here $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.

$|V(C_n \oplus K_{1,n})| = |E(C_n \oplus K_{1,n})| = 2n$.

We define labeling $f : V(C_n \oplus K_{1,n}) \rightarrow \{1, 2, 3, \ldots, |V(C_n \oplus K_{1,n})|\}$ as follows.

$f(u_i) = 2i - 1; \quad 1 \leq i \leq n,$

$f(v_j) = 2j; \quad 1 \leq j \leq n,$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_{i+1}$ in C_n,

$f(u_i) \mid f(u_{i+1}),\ 1 \leq i \leq n.$

Also

$f(v)$ does not divides $f(v_j)$ $1 \leq j \leq n$

Hence $e_f(0) = n+1, e_f(1) = n+2$.

Example 2.1. Sum divisor cordial labeling of the graph $C_5 \oplus K_{1,5}$ is shown in Figure 1 as an illustration for Theorem 2.1.

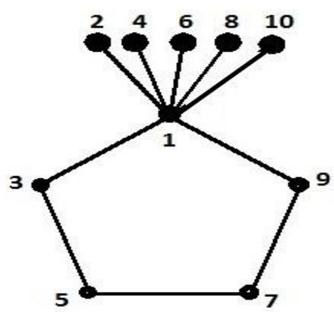


Figure 1

Definition 2.1.[3] A chord of a cycle C_n is an edge joining two non-adjacent vertices

Theorem 2.2. *$G \oplus K_{1,n}$ is sum divisor cordial graph for $n \geq 4$, $n \in \mathbb{N}$, where tt is cycle C_n with one chord and chord forms a triangle with two edges of C_n.*

Proof. Let G be the cycle C_n with one chord, $V(G) = \{u_1, u_2, \ldots, u_n\}$ and $e = u_2u_n$ be the chord of C_n. The vertices $u_1, u_2, \ldots, u_n$ forms a triangle with chord e.

Let $V(K_{1,n}) = \{v, v_1, v_2, \ldots, v_n\}$, where $v = u_1$ is the apex vertex and $v_1, v_2, \ldots, v_n$ are the pendant vertices of $K_{1,n}$.

$|V(G \oplus K_{1,n})| = 2n$ and $|E(G \oplus K_{1,n})| = 2n + 1$.

We define labeling $f : V(G \oplus K_{1,n}) \rightarrow \{1, 2, 3, \ldots, 2n\}$ as follows.

$f(u_i) = 2i - 1; \quad 1 \leq i \leq n,$

$f(v_j) = 2j; \quad 1 \leq j \leq n,$

According to this pattern the vertices are labeled such that for any edge $e = u_iu_{i+1}$ in C_n,

$f(u_i) \mid f(u_{i+1})$ $1 \leq i \leq n$

Also

$f(v)$ does not divides $f(v_j)$ $1 \leq j \leq n$

Hence $e_f(1) = n+1, e_f(0) = n$

Thus $|e_f(0) - e_f(1)| \leq 1$.

So, $G \oplus K_{1,n}$ is a sum divisor cordial graph, where G is the cycle C_n with one chord.

Example 2.2. Sum divisor cordial labeling of ringsum of C_6 with one chord and $K_{1,6}$ is shown in Figure 2 as an illustration for Theorem 2.2.

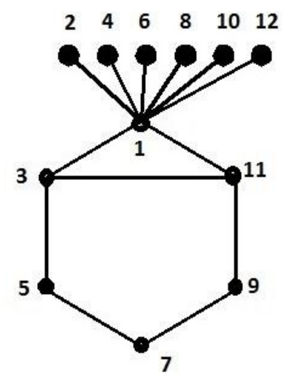


Figure 2

Definition 2.2.[3] Two chords of a cycle C_n are said to be twin chords if they form a triangle with an edge of C_n.

For positive integers n and p with $5 \leq p + 2 \leq n$, $C_{n,p}$ is the graph consisting of a cycle C_n with a pair of twin chords with which the edges of C_n form cycle C_p, C_3 and C_{n+1-p} without chords.

Theorem 2.3. *$C_{n,3} \oplus K_{1,n}$ is a vertex odd divisor cordial graph, for $n \geq 5$, $n \in \mathbb{N}$*

Proof. Let $V(C_{n,3}) = \{u_1, u_2, \ldots, u_n\}$, $e_1 = u_2u_n$ and $e_2 = u_3u_n$ be the chords of C_n.

Let $V(K_{1,n}) = \{v = u_1, v_1, v_2, \ldots, v_n\}$, where v is the apex vertex and $v_1, v_2, \ldots, v_n$ are pendant vertices of $K_{1,n}$.

$|V(C_{n,3} \oplus K_{1,n})| = 2n$ and $|E(C_{n,3} \oplus K_{1,n})| = 2n + 2$.

We define labeling $f : V(C_{n,3} \oplus\in K_{1,n}) \rightarrow \{1, 2, 3, . . . , 2n\}$ as follows.

$$\begin{aligned} f(u_1) &= 1 \\ f(u_2) &= 3 \\ f(u_3) &= 2n \\ f(u_i) &= 2i - 3; \quad 4 \le i \le n. \\ f(v_j) &= 2j; \quad 1 \le j \le n - 1. \\ f(v_n) &= 2n - 1. \end{aligned}$$

According to this labeling, the vertices are labeled such that $e_f(1) = \text{n}+1 = e_f(0)$.

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence the graph under consideration admits sum divisor cordial labeling. Thus $C_{n,3} \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.3. Sum divisor cordial labeling of $C_{7,3} \oplus K_{1,7}$ is shown in Figure 3 as an illustration for Theorem 2.3.

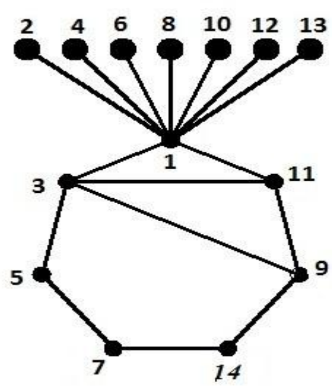


Figure 3

Definition 2.3.[3] The cycle with triangle is a cycle with three chords which by themselves form a triangle. For positive integers p, q, r and $n \ge 6$ with $p + q + r + 3 = n$, $C_n(p, q, r)$ denotes the cycle C_n with triangle whose edges form the edges of cycles C_{p+2}, C_{q+2}, C_{r+2} without chords.

Theorem 2.4. *$C_n(1, 1, n - 5) \oplus K_{1,n}$ is a sum divisor cordial graph, for $n \ge 6$, $n \in \mathbb{N}$.*

Proof. Let $V(C_n(1, 1, n - 5)) = \{u_1, u_2, . . . , u_n\}$, where $e_1 = u_1u_3$, $e_2 = u_3u_{n-1}$ and $e_3 = u_1u_{n-1}$ are chords of C_n which by them selves form triangle.

Let $V(K_{1,n}) = \{v = u_1, v_1, v_2, . . . , v_n\}$, where v is the apex vertex and $v_1, v_2, . . . , v_n$ are the pendant vertices.

$|V(C_n(1, 1, n - 5) \oplus K_{1,n})| = 2n$ and $|E(C_n(1, 1, n - 5) \oplus K_{1,n})| = 2n + 3$.

We define labeling $f : V(C_n(1, 1, n - 5) \oplus K_{1,n}) \rightarrow \{1, 2, 3, . . . , 2n\}$ as follows.

$$\begin{aligned} & f(\mathrm{u}_1) = 1 \\ & f(\mathrm{u}_2) = 2n \\ & f(\mathrm{u}_i) = 2i - 3; 3 \le i \le n, \\ & f(\mathrm{u}_j) = 2j; 1 \le i \le n - 1, \\ & f(v_n) = 2n - 1. \end{aligned}$$

In view of above defined labeling pattern

$$e_f(0) = \text{n}+1, e_f(1) = \text{n}+2$$

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence $C_n(1, 1, n - 5) \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.4. Sum divisor cordial labeling of ringsum of cycle with triangle $C_8(1, 1, 3)$ and $K_{1,8}$ is shown in Figure 4 as an illustration for Theorem 2s.4.

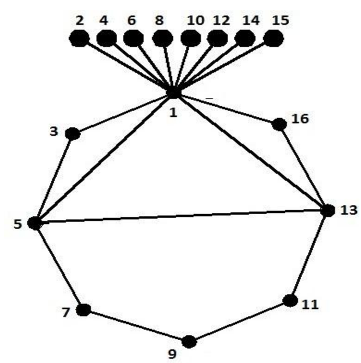


Figure 4

Definition 2.4.[3] *The wheel graph W_n is defined as $C_n + K_1$. The vertices corresponding to C_n are called rim vertices and the vertex corresponding to K_1 is called apex.*

Theorem 2.5. *$W_n \oplus K_{1,n}$ is a sum divisor cordial graph for $n \ge 3$, $n \in \mathbb{N}$.*

Proof. Let V (Wn $\oplus K_{1,n}$) = $V_1 \cup V_2$, where V_1 = V (Wn) = $\{u, u_1, u_2, . . . , u_n\}$, u is apex vertex and $\{u_1, u_2, . . . , un\}$ are rim vertices; $V_2 = V(K_{1,n}) = \{v = u_1, v_1, v_2, . . . , v_n\}$, $v_1, v_2, . . . , v_n$ are pendant vertices and v be the apex vertex.

$|V(W_n \oplus K_{1,n})| = 2n + 1$, $|E(W_n \oplus K_{1,n})| = 3n$.

We define labeling $f : V(W_n \oplus K_{1,n}) \to \{1, 2, 3, \ldots, 2n + 1\}$ as follows

$$f(u_i) = \begin{cases} i & ; i \equiv 0 (mod\, 4) \\ i+1 & ; i \equiv 1,2 (mod\, 4) \\ i+2 & ; i \equiv 3 (mod\, 4) \quad (2 \le i \le n) \end{cases}$$

$f(u) = 2$

$f(u_1 = v) = 1$

$f(v_j) = f(u_n) + j;\ 1 \le j \le n.$

Then we have

$$e_f(1) = \left\lceil \frac{3n}{2} \right\rceil, e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$$

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence the graph under consideration admits sum divisor cordial labeling.

Thus $W_n \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.5. Sum divisor cordial labeling of $W_6 \oplus K_{1,6}$ is shown in Figure 5 as an illustration for Theorem *2.5*.

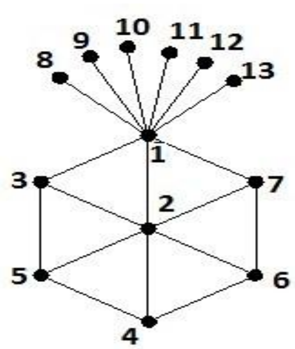


Figure 5

Definition 2.5.[3] The flower graph $fl_n (n \ge 3)$ is obtained from helm H_n by joining each pendant vertex to the central vertex of H_n.

Theorem 2.6. *$fl_n \oplus K_{1,n}$ is a sum divisor cordial graph for $n \ge 3$, $n \in \mathbb{N}$.*

Proof. Let $V(fl_n \oplus K_{1,n}) = V_1 \cup V_2$,

$V_1 = V(fl_n) = \{u, u_1, u_2, \ldots, u_n, w_1, w_2, \ldots, w_n\}$, where u is the apex vertex, $u_1, u_2, \ldots, u_n$ are internal vertices and $w_1, w_2, \ldots, w_n$ are external vertices;

$V_2 = V(K_{1,n}) = \{v = w_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$, where $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.

$|V(fl_n \oplus K_{1,n})| = 3n + 1$, $|E(fl_n \oplus K_{1,n})| = 5n$.

We define labeling $f : V(fl_n \oplus K_{1,n}) \to \{1, 2, 3, \ldots, 3n + 1\}$ as follows.

$f(u) = 1$

$f(u_i) = 2i + 1;\quad 1 \le i \le n$

$f(w_i) = 2i;\quad 1 \le i \le n$

$f(v_j) = f(u_n) + j;\ 1 \le j \le n$

Then we have

$$e_f(1) = \left\lceil \frac{5n}{2} \right\rceil, e_f(0) = \left\lfloor \frac{5n}{2} \right\rfloor$$

Thus $|e_f(0) - e_f(1)| \le 1$..

Hence the graph under consideration admits sum divisor cordial labeling.

Thus $fl_n \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.6. Sum divisor cordial labeling of $fl_4 \oplus K_{1,4}$ is shown in Figure 6 as an illustration for Theorem 2.6.

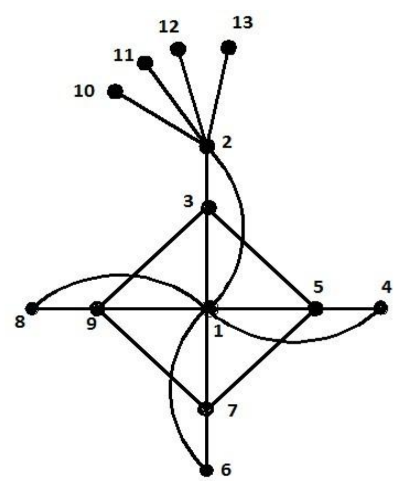


Figure 6

Definition 2.6.[3] The gear graph, denoted by G_n, is obtained from the wheel W_n *by* adding a vertex between every pair of adjacent rim vertices of W_n.

Theorem 2.7. *$G_n \oplus K_{1,n}$ is a sum divisor cordial graph for all n.*

Proof Let $V(G_n \oplus K_{1,n}) = V_1 \cup V_2$,

$V_1 = V(G_n) = \{u, u_1, u_2, \ldots, u_{2n}\}$, where u is the apex vertex, $u_1, u_3, \ldots, u_{2n-1}$ are vertices with degree 3 and $u_2, u_4, \ldots, u_{2n}$ are vertices with degree 2;
$V_2 = V(K_{1,n}) = \{v = w_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$, where $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.
$|V(G_n \oplus K_{1,n})| = 3n + 1$, $|E(G_n \oplus K_{1,n})| = 4n$.
We define labeling $f : V(G_n \oplus K_{1,n}) \to \{1, 2, 3, \ldots, 3n + 1\}$ as follows.

Case:1 $n \equiv 1, 3 (mod 4)$

$$f(u) = 2$$
$$f(u_1 = v) = 1$$
$$f(v_j) = f(u_n) + j; \quad 1 \le j \le n.$$
$$f(u_i) = \begin{cases} i+1 & ; i \equiv 1,2 (mod 4) \\ i+2 & ; i \equiv 3 (mod 4) \\ i & ; i \equiv 0 (mod 4) \quad (2 \le i \le n) \end{cases}$$

Case:2 $n \equiv 2, 4 (mod 4)$

$$f(u) = 2$$
$$f(u_1 = v) = 1$$
$$f(v_j) = f(u_n) + j; \quad 1 \le j \le n-1.$$
$$f(v_n) = 2n$$
$$f(u_n) = 3n + 1$$
$$f(u_i) = \begin{cases} i+1 & ; i \equiv 1,2 (mod 4) \\ i+2 & ; i \equiv 3 (mod 4) \\ i & ; i \equiv 0 (mod 4) \quad (2 \le i \le n-1) \end{cases}$$

Then we have $e_f(1) = n - 1, e_f(0) = n$.

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence $G_n \oplus K_{1,n}$ is a sum divisor cordial graph.

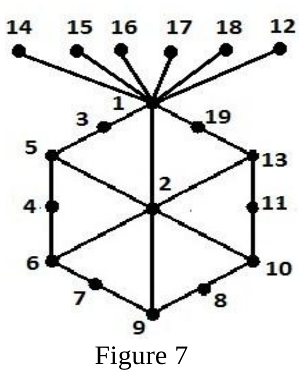


Figure 7

Example 2.7. Sum divisor cordial labeling of $G_6 \oplus K_{1,6}$ is shown in Figure 7 as an illustration for Theorem 2.7.

Theorem 2.8. *$P_n \oplus K_{1,n}$ is a sum divisor cordial graph for all n.*

Proof. Let $V(P_n \oplus K_{1,n}) = V_1 \cup V_2$, where $V_1 = V(Pn) = \{u_1, u_2, \ldots, u_n\}$ and $V_2 = V(K_{1,n}) = \{v = u_1, v_1, v_2, \ldots, v_n\}$. Here $v_1, v_2, \ldots, v_n$ are the pendant vertices and, v is the apex vertex.

$|V(P_n \oplus K_{1,n})| = 2n$, $|E(P_n \oplus K_{1,n})| = 2n - 1$.

We define labeling $f : V(P_n \oplus K_{1,n}) \to \{1, 2, 3, \ldots, 2n\}$ as follows.

$$f(u_i) = 2i; \quad 1 \le i \le n,$$
$$f(v_j) = 2j - 1; 1 \le j \le n.$$

Then we have $e_f(1) = n - 1, e_f(0) = n$.

Thus $|e_f(0) - e_f(1)| \le 1$.

Hence the graph under consideration admits sum divisor cordial labeling.
Thus $P_n \oplus K_{1,n}$ is a sum divisor cordial graph.
Example 2.8. Sum divisor cordial labeling of $P_5 \oplus K_{1,5}$ is shown in Figure 8 as an illustration for Theorem 2.8.

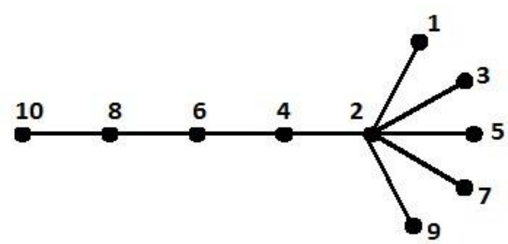


Figure 8

Definition 2.7.[3] The shell Sn(n ≥ 4, n ∈ N) is the graph obtained by taking n − 3 concurrent chords in the cycle Cn.
The vertex at which all the chords are concurrent is called the apex vertex of S_n.

Theorem 2.9. *$S_n \oplus K_{1,n}$ is a sum divisor cordial graph for all $n \in$ N.*

Proof. Let $V(S_n \oplus K_{1,n}) = V_1 \cup V_2$,
$V_1 = V(S_n) = \{u_1, u_2, \ldots, u_n\}$, where u_1 is apex vertex; $V_2 = V(K_{1,n}) = \{v = u_1, v_1, v_2, \ldots, v_n\}$, where $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.
$|V(S_n \oplus K_{1,n})| = 2n + 1$, $|E(S_n \oplus K_{1,n})| = 3n - 1$.
We define labeling $f : V(S_n \oplus K_{1,n}) \to \{1, 2, 3, \ldots, 2n + 1\}$ as follows

$$f(u_i)=\begin{cases} i+1 & ; i\equiv 0,1(mod\,4)\\ i+2 & ; i\equiv 2(mod\,4)\\ i & ; i\equiv 3(mod\,4) \quad (2\le i\le n)\end{cases}$$

$f(u_1 = v) = 1,$

$f(v_j) = f(u_n) + j; \qquad 1 \le j \le n.$

Then we have

Cases of n	Edge conditions
$n \equiv 0, 2(mod\ 4)$	$e_f(0)=\left\lceil \frac{3n-1}{2}\right\rceil, e_f(1)=\left\lfloor \frac{3n-1}{2}\right\rfloor$
$n \equiv 1, 3(mod\ 4)$	$e_f(1)=\frac{3n-1}{2}=e_f(1)$

Thus $|e_f(0)-e_f(1)|\le 1.$

Hence the graph under consideration admits sum divisor cordial labeling.

Thus Sn ⨁ K1,n is a sum divisor cordial graph.

Example 2.9. Sum divisor cordial labeling of S7 ⨁ K1,7 is shown in Figure 9 as an illustration for Theorem 2.9.

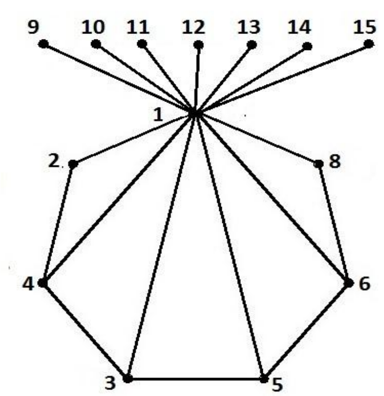


Figure 9

Definition 2.8.[3] The double fan DF_n is obtained by $P_n + 2K_1$.

Theorem 2.10. *$DF_n \oplus K_{1,n}$ is a sum divisor cordial graph for all n.*

Proof. Let V (DFn ⨁ $K_{1,n}$) = $V_1 \cup V_2$,

V_1 = V (DFn) = {$u, w, u_1, u_2, \ldots, u_n$}, where u, w are two apex vertices of DFn;

$V_2 = V(K_{1,n})$ = {$v = w, v_1, v_2, \ldots, v_n$}, where $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.

$|V(DF_n \oplus K_{1,n})| = 2n + 2$, $|E(DF_n \oplus K_{1,n})| = 4n - 1$.

We define labeling $f : V(DF_n \oplus K_{1,n}) \rightarrow \{1, 2, 3, \ldots, 2n + 2\}$ as follows.

$f(u) = 2$

$f(v = w) = 1$

$$f(u_i)=\begin{cases} i+2 & ; i\equiv 0,1(mod\,4)\\ i+1 & ; i\equiv 3(mod\,4)\\ i+3 & ; i\equiv 2(mod\,4) \quad (1\le i\le n)\end{cases}$$

Case:1 n ≡ 0, 1, 3(mod4)

$f(v_j) = f(u_n) + j; \qquad 1 \le j \le n.$

Case:2 $n \equiv 2(mod4)$

$f(v_1) = n + 2$

$f(v_j) = f(u_n) + j; \qquad 2 \le j \le n.$

In view of the above labeling pattern we have

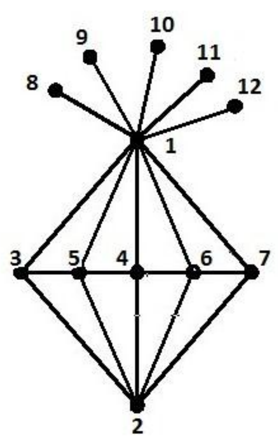
Cases of n	Edge conditions
$n \equiv 0, 2(mod\ 4)$	$e_f(0) = 2n, e_f(1) = 2n - 1$
$n \equiv 1, 3(mod\ 4)$	$e_f(1) = 2n, e_f(0) = 2n - 1$

Thus $|e_f(0)-e_f(1)|\le 1.$

Hence the graph under consideration admits sum divisor cordial labeling.

i.e. $DF_n \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.10. Sum divisor cordial labeling of $DF_5 \oplus K_{1,5}$ is shown in Figure 10 as an illustration



for Theorem 2.10.

Figure 10

Theorem 2.11. *$K_{2,n} \oplus K_{1,n}$ is a sum divisor cordial* graph for all n.

Proof. Let $V(K_{2,n}) = V_1 \cup V_2$, $V_1 = \{u, w\}$, $V_2 = \{u_1, u_2, \ldots, u_n\}$ and

$V(K_{1,n}) = V_3 = \{v = u_1, v_1, v_2, \ldots, v_n\}$, where $v_1, v_2, \ldots, v_n$ are pendant vertices and v is the apex vertex of $K_{1,n}$.

Then $V(K_{2,n} \oplus K_{1,n}) = V_1 \cup V_2 \cup V_3$.

$|V(K_{2,n} \oplus K_{1,n})| = 2n + 2$, $|E(K_{2,n} \oplus K_{1,n})| = 3n$.

We define labeling $f : V(K_{2,n} \oplus K_{1,n}) \rightarrow \{1, 2, 3, \ldots, 2n + 2\}$ *as follows.*

$f(u) = 1.$

$f(w) = 2$

$f(u_i) = i + 2; \quad 1 \le i \le n.$

$f(v_j) = f(u_n) + j; \quad 1 \le j \le n.$

In view of the above labeling pattern we have

Cases of n	Edge conditions
$n \equiv 0, 2 (mod\ 4)$	$e_f(0) = \frac{3n}{2} = e_f(1)$
$n \equiv 1, 3 (mod\ 4)$	$e_f(0) = \left\lceil \frac{3n}{2} \right\rceil, e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$

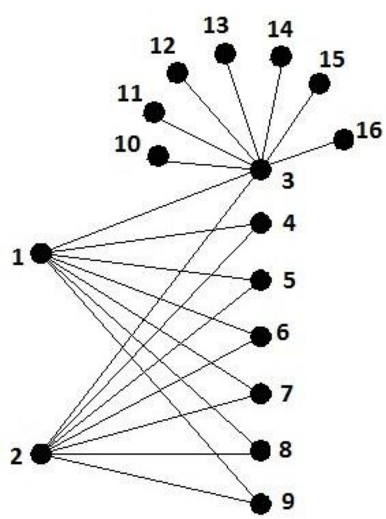
Thus $|e_f(0) - e_f(1)| \le 1.$

Hence the graph under consideration admits sum divisor cordial labeling.

Thus $K_{2,n} \oplus K_{1,n}$ is a sum divisor cordial graph.

Example 2.11. Sum divisor cordial labeling of K2,7 $\oplus$ K1,7 is shown in Figure 11 as an illustration for Theorem 2.11.

Figure 11



III. CONCLUDING REMARKS

The sum divisor cordial labeling is an invariant of divisor cordial labeling by considering codomain as finite set of numbers. It is interesting to see that if two graphs are sum divisor cordial then their ringsum is sum divisor cordial or not. We have investigated eleven sum divisor cordial graphs in context of ringsum of graphs.

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