IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-6, Issue-7, July 2018 E-ISSN: 2347-2693

Simulation of Stochastic Geometric Brownian Motion of Stock Market – Using R Programming

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Available online at: www.ijcseonline.org

Accepted: 13/Jul/2018, Published: 31/July/2018

***Abstract*—** In the Prediction of total stock index, many are faced with some parameters as they are uncertain in future and they can undergo changes, and this uncertainty has a few risks, and for a true analysis, the calculations should be performed under risk conditions. The empirical tests suggest that the stochastic differential equation of GBM model can be used to predict the direction of stock price movement. In terms of predicting the stock price values, the empirical findings suggest that the GBM model performs well in stock market.

***Keywords*—**Prediction, Stock index, Geometric Brownian Motion (GBM), Stochastic differential equation, Stock Market.

I. INTRODUCTION

There is an abundance of literature surrounding the pricing of securities in corporate finance, however there is still a lot of debate as to which method is the most reliable. Financial managers and investors are interested in simulating the price of stock, options, and derivatives in order to make important investment and financing decisions. Simulating the price of a stock means generating price paths that a stock may follow in the future. Many are talk about simulating stock prices because future stock prices are uncertain called stochastic, but we believe that they follow, at least approximately, a set of rules that we can derive from historical data and our knowledge of stock prices. A simulation will be realistic only if the underlying model is realistic. The model must reflect our understanding of stock prices and conform to historical data. In this paper the author deal with the problem of prediction of stock price movement (increase or decrease) that has been there over years. Several methods have been proposed and have predicted stock price movement with variable degrees of accuracy such that until today, prediction of stock price movement is continuously being attempted. A manifold of factors such as economic, political, social and psychological factors interact in a complex way influencing stock price movement. It is no doubt that prediction of stock price movement is quite challenging. This paper is an attempt to predict stock price movement using continuous time models. The author believe that continuous models are suitable to capture the unpredictable dynamics of stock prices to a certain extent. One of most important concept in building such financial model is to understand the geometric Brownian motion, which is a special case of Brownian Motion Process.

The rest of this paper is organized as follows. Section 2 gives some related concepts and examples on the fractional differential equations and Hurst index and then establishes the Geometric Brownian Motion Stochastic differential equation in the stock market. In Section 3, based on the proposed stochastic differential equation, we give the corresponding Ito formula. And we conduct the empirical analysis of fractional order formula of stock price process by using the Geometric Brownian Motion (GBM) simulation method, and we also make comparison analysis of expected value formula under Random Walk method with the Stochastic differential equation based on Geometric Brownian motion. The conclusions drawn from this study are presented in Section 5.

II. RELATED WORK

A. Stock Price Distributions with Stochastic Volatility:

They expose about the stock price distributions that arise when prices follow a diffusion process with a stochastically varying volatility parameter. The authorises analytic techniques to derive an explicit closed-form solution for the case where volatility is driven by an arithmetic ornstein–hollenbeck process. The author then applies our results to two related problems in the finance literature: (i) options pricing in a world of stochastic volatility, and (ii) the relationship between stochastic volatility and the nature of "fat tails" in stock price distributions [1].

B. Fractional Brownian motion, random walks and binary market models:

They prove a Donsker type approximation theorem for the fractional Brownian motion in the case h>1/2.h>1/2. Using this approximation, the author constructs an elementary market model that converges weakly to the fractional analogue of the black–Scholes model. They show that there exist arbitrage opportunities in this model. One such opportunity is constructed explicitly [2].

A common hypothesis about the behaviour of limited liability asset prices in perfect markets is the random walk of returns or in its continuous-time form the geometric Brownian motion hypothesis, which implies that asset prices are stationary and log-normally distributed [3].

C. Geometric Brownian Motion:

The geometric Brownian motion (GBM) process is frequently invoked as a model for such diverse quantities as stock prices, natural resource prices and the growth in demand for products or services. They discussed a process for checking whether a given time series follows the GBM process. Methods to remove seasonal variation from such a time series are also analysed. Of four industries studied, the historical time series for usage of established services meet the criteria for a GBM; However, the data for growth of emergent services do not [4]. let X1(t), ⋯, Xn(t) ben geometric Brownian motions, possibly correlated. They study the optimal stopping problem: found a stopping time $\tau_* < \infty$ such that the sup being taken all over all finite stopping times τ, and ex denotes the expectation when (X1(0), ⋯, Xn(0)) =X=(X1,⋯,Xn) . for n=2 this problem was solved by McDonald and Siegel, but they did not state the precise conditions for their result. They gave a new proof of their solution for n=2 using variational inequalities and the author solve the N -Dimensional case when the parameters satisfy certain that is, Additional Conditions [5].

D. Differential equations driven by fractional Brownian motion:

A global existence and uniqueness result of the solution for multidimensional, time dependent, stochastic differential equations driven by a fractional Brownian motion with Hurst parameter $h > {1\over 2} $ is proved. It is shown, also, that the solution has finite moments. The result is based on a deterministic existence and uniqueness theorem whose proof uses a contraction principle and a priori estimates [6]. A particle which is caught in a potential hole and which, through the shuttling action of Brownian motion, can escape over a potential barrier yields a suitable model for elucidating the applicability of the transition state method for calculating the rate of chemical reactions [7].

E. Geometric Brownian Motion analysis:

The purpose of this paper is to present a quantitative analyses of oil price's path. They try to argue that, despite its parsimony and simplicity, geometric Brownian motion can perform well as a proxy for the movement of oil prices and for a state variable to evaluate oil deposits. They revealed based on their argument on evidences of very low speed of mean reverting (or long half-life), since unit root tests only can reject its null hypothesis in a sample longer than 100 years. On the other hand, they reject the null hypothesis of unit root with two endogenous breaks, showing that the usual rejection can be attributed to omitted structural breaks. The author concludes that the average half-life of oil price (between 4 and 8 years depending on the model chosen) is long enough to allow a good approximation as a geometric Brownian motion [8].

In this paper they showed, by using dyadic approximations, the existence of a geometric rough path associated with a fractional Brownian motion with Hurst parameter greater than 1/4. Using the integral representation of fractional Brownian motions, they furthermore obtain a skohorod integral representation of the geometric rough path we constructed. By the results, a stochastic integration theory may be established for fractional Brownian motions, and strong solutions and a wong-zakai type limit theorem for stochastic differential equations driven by fractional Brownian motions can be deduced accordingly. The method can actually be applied to a larger class of gaussian processes with covariance functions satisfying a simple decay condition [9].

This paper studied a continuous-time agency model in which the agent controls the drift of the geometric Brownian motion firm size. The changing firm size generates partial incentives, analogous to awarding the agent equity shares according to her continuation payoff. When the agent is as patient as investors, performance-based stock grants implement the optimal contract. Their model generates a leverage effect on the equity returns, and implies that the agency problem is more severe for smaller firms. [10].

III. PROPOSED APPROACH

A. *An introduction to stochastic processes through the use of R:*

Introduction to stochastic processes with R is an accessible and well-balanced presentation of the theory of stochastic processes, with an emphasis on real-world applications of probability theory in the natural and social sciences. The use of simulation, by means of the popular statistical software R, makes theoretical results come alive with practical, hands-on demonstrations. Written by a highly-qualified expert in the field, the author presents numerous examples from a wide array of disciplines, which are used to illustrate concepts and highlight computational and theoretical results.

B. *Use of Brownian Motion in Finance:*

In the middle of this century, work done by M.F.M Osborne showed that the logarithms of common-stock prices, and the value of money, can be regarded as an ensemble of decisions in statistical equilibrium, and that this ensemble of logarithms of prices, each varying with time, has a close analogy with the ensemble of coordinates of a large number of molecules. Using a probability distribution function and the prices of the same random stock choice at random times, he was able to derive a steady state distribution function, which is precisely the probability distribution for a particle in Brownian motion. A similar distribution holds for the value of money, measured approximately by stock market indices. Sufficient, but not necessary conditions to derive this distribution quantitatively are given by the conditions of trading, and the Weber-Fechner law. The Weber-Fechner law states that equal ratios of physical stimulus, for example, sound frequency in vibrations/sec, correspond to equal intervals of subjective sensation, such as pitch. the value of a subjective sensation, like absolute position in physical space, is not measurable, but changes or differences in sensation are, since by experiment they can be equated, and reproduced, thus fulfilling the criteria of measurability.

A consequence of the distribution function is that the expectation values for price itself increases, with increasing time intervals 't', at a rate of 3 to 5 percent per year, with increasing fluctuation, or dispersion, of price. This secular increase has nothing to do with long-term inflation, or the growth of assets in a capitalistic economy, since the expected reciprocal of price, or number of shares purchasable in the future, per dollar, increases with time in an identical fashion.

Thus, it was shown in his paper that prices in the market did vary in a similar fashion to molecules in Brownian motion. In another paper presented around the same period, it was also found that there is definite evidence of periodic in time structure that is, of the prices in Brownian motion corresponding to intervals of a day, week, quarter and year: These being simply the cycles of human attention span.

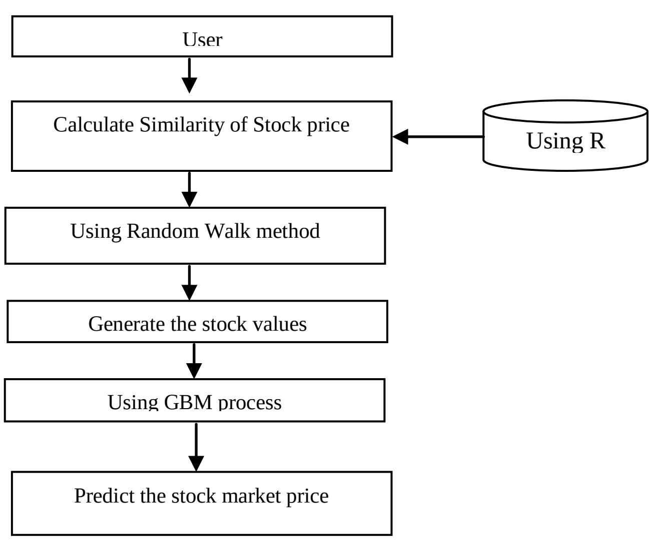


Fig 1 Proposed Architecture

The geometric Brownian motion (GBM) is the most basic processes in financial modelling. Consider a stock price S(t) with dynamics

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

It can be shown (just use Ito`s lemma) that the solution to this stochastic differential equation is

$$S_t = S_0 \exp(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t)$$

where W(t) is a Brownian Motion. A trajectory of this path can be simulated by iteratively sampling a standard normal random variable Y and use,

$$S_{t+h} = S_t \exp(\left(\mu - \frac{\sigma^2}{2}\right)h + \sigma\sqrt{hY})$$

Whether one chooses to simulate the final value S(T) immediately (i.e. using "t=0, h=T), or chooses to simulate through many intermediate steps (as done in the code and seen in the graphic below) does not change the distribution of the final value S(T), but only the granularity of the intermediate steps. In both cases, the value for S(T) will be lognormal distributed.

Thus, a Geometric Brownian motion is nothing else than a transformation of a Brownian motion. For this, we sample the Brownian W(t) (this is "f" in the code, and the red line in the graph). This is being illustrated in the following example, where we simulate a trajectory of a Brownian Motion and then plug the values of W(t) into our stock price S(t).

The exponential lines are the expected value of the GBM's due to

$$E[S_t] = S_0 \exp(\mu t)$$

IV. Results and Conclusion

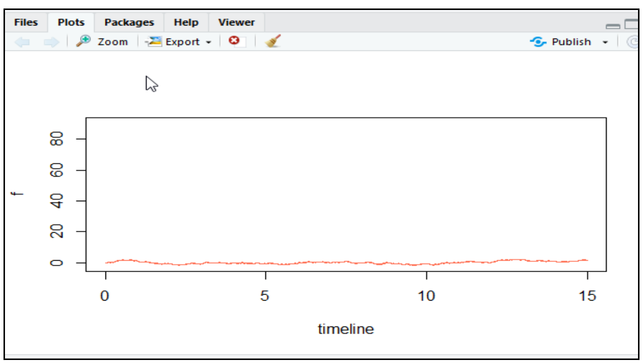
Because the differential equations can capture the effect in the financial system, we established the stochastic differential equation by adding the stochastic process into the differential equation. Based on this stochastic differential equation with Geometric Brownian motion, we apply the fractional stochastic differential equation to the financial market. We constructed the trajectory stock price $S_{t+h} = S_t \exp(\left(\mu - \frac{\sigma^2}{2}\right)h + \sigma\sqrt{hY}$, where t=0, h=T, and derived the stock price process in the cases of and , respectively, and the Stock market pricing formula under the fractional stochastic differential equation. From the

Geometric Brownian motion pricing formula, we find the trend in stock price process.
In addition, we made some comparisons in terms of the pricing option formula and its underlying stock price process between our proposed.
It would be an interesting work if we improve our model by connecting differential equation with fractional Brownian motion in R.

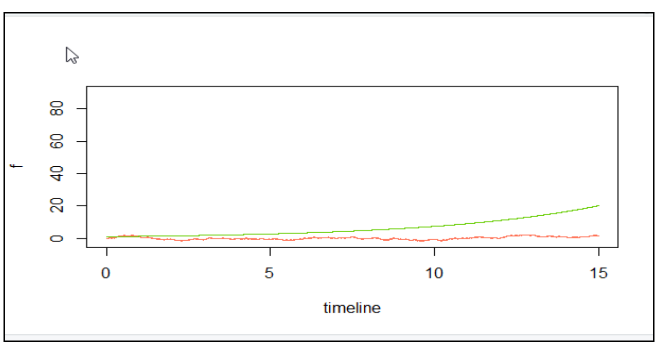
a) Algorithm

Step 1 : Start.
Step 2 : Declare the variables and set values for length and X-axis range.
Step 3 : Set the timeline of X-axis , 0-15.
Step 4 : Set the expected values for S0, r, mu, mu0, sigma, sigma0 to foind the mean and varience.
Step 5 : Assign the expected mean and varience to f, here f is brownian motion.
Step 6 : Simulate the expected values.
Step 7 : Create a Plot using expected values.
Step 8 : Close.

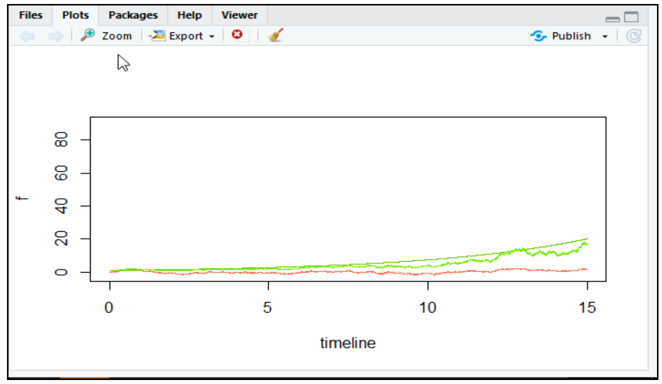
b) Screenshots



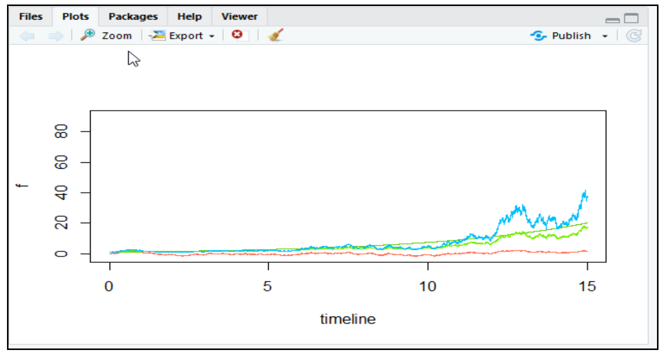
Range of Brownian Motion



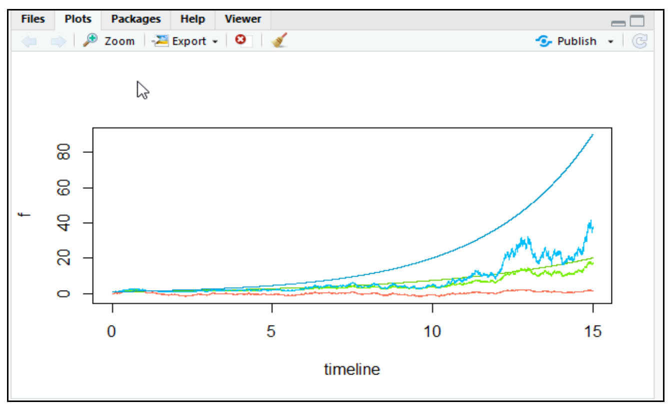
The green line is the expected value



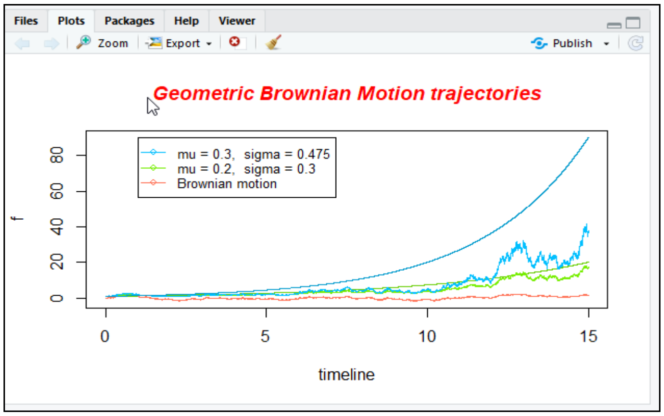
The range of green line is above to its expected value.



The range of blue line, it is a another expected value.



The range of blue line is below to its expected value.



As the result the more likely it is for the GBM to be significantly below its expected value.

An interesting observation is that around time point 12 the green GBM is above its expected value, whereas the blue line is below its expected value, even when they follow the same underlying trajectory of a BM. This may seem odd at first, and in fact the author thought she had made a mistake in the program. Whenever the author simulated a trajectory, in most cases the blue line was below its expected value. should the stock price not be above its expectation value when the BM is positive, and below it is negative?
The answer is no. Even though the increment of a BM from 0 to 15 is normally distributed (with mean zero and variance 15), wrapped in an exponential function, positive deviations of the BM from 0 have an exponential effect on the mean value, whereas negative trajectories are bound from below by 0 and thus have limited downside effect.
The conclusion of this is that the longer the timeline graph goes, the more likely it is for the GBM to be significantly below its expected value.

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