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Some Techniques of Ancient Indian Vedic Mathematics for Elliptic Curve Cryptography over the Ring A_4

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Abstract— In this present approach, some efficient computing techniques of Ancient Indian Vedic Mathematics for elliptic curve cryptography (ECC) over the Ring A_4 has been studied, in which, it has been observed that the applications of AIVM Techniques or Sutras decrease the number of multiplications and squares which occur in point doubling and point addition in ECC over the Ring A_4. This paper described the use of AIVM Sutras, Urdhva-Tiryagbhyam for multiplication and Dvandva-Yoga for the square of any number in the ECC over the Ring A_4. The results proved that AIVM based scheme shows better performance in speed, processing time and power consumption of multipliers compared to conventional method. The effect of some AIVM techniques over ECC was investigated and the obtained results are explained in the form of tables and graphs.

Keywords— Cryptography, Dvandva-Yoga, Elliptic Curve, Finite Field, Point Addition, Point Doubling, Ring A_4, Scalar Multiplication, Urdhva-Tiryagbhyam, Vedic Mathematics.

I. INTRODUCTION

Elliptic curve cryptography (ECC) is an approach which is based on public key cryptography. Along the system of a public key cryptography every client or the electronic tools joined in the Network Communication naturally have a couple of keys first one is a public key and other is a private key and a class of numeric operations correlated with keys for doing the various types of cryptographic operation The private key known only by the special client and on the other hand the public key is communicated to all clients which are joints in the communication network [12, 17]. In the core work, we shall discuss the elliptic curves over the ring $A_4 = F_{2^d}[\varepsilon]$ where $\varepsilon^4 = 0$ [1, 5] and we have used some Ancient Indian Vedic Mathematics (AIVM) techniques [9] for the completion of cryptographic operations on these curves. The major congestion that arrangement of the efficiency in ECC operations first is the addition and other is doubling in the schemes which are based on the ECC. In conduct, these operations are found to be very complex and the most time-consuming operations for ECC. It is very well known that the energy efficiency and effective computation methods are the important factors in identifying the performance of any of ECC based algorithm. The performance of an ECC based algorithm can be improved if we are able to reduce its execution time which is basically depending upon the point addition and doubling operations. It is widely known that there exist many simple and time-saving techniques in form of sixteen Sutras and fourteen sub Sutras of AIVM to compute complex Mathematical manipulations such as multiplication and divisions [9]. In the current approach, we have used Urdhva-Tiryagbhyam multiplication and Dvandva-Yoga technique to speed up the above-mentioned complex computations occurring in the ECC based ring A_4 proposed in. The results show that AIVM based ECC over the ring A_4 can give better performance compared to the conventional methods [4, 5, 6, 7, 8, 9, 11, 13, 14, 15].

II. RELATED WORK

In the last three decade across the World Vedic Mathematics branch of Mathematics is using so much in the research area. Ancient Indian Vedic Mathematics (AIVM) is based on sixteen Sutras and fourteen sub Sutras. AIVM has a different and unique technique

compare to other computational technique for calculations. Kan et al. [2]. purposed in his work in 2012, the Design and implementation of low power multiplier using Vedic multiplication technique involving of small key size in ECC cryptographic system it gives faster implementation and best output. It has been observed that AIVM based ECC gives a better result with uses of small key size than other cryptographic systems. In current decade ECC is using because its gives reliability and security ECC is maximally used in security and networking areas because ECC key size is too small. It is using in many devices which have not much storage memory like Smart cards. In the banking sector, ECC makes possible and more secure Smart cards for credit and debit, also electronic tickets and personal registration cards or identification. In the process of encryption and decryption, it is well-explained transforming of an informational into an affine coordinate on the elliptic curve. Nanda and Behera [3] in 2014 proposed the AIVM Sutras based multiplication algorithm Design and Implementation of Urdhva-Tiryagbhyam with the 8×8 bit Vedic Binary Multiplier, this paper shows that AIVM techniques such as Dvandva Yoga to get square of any number and Urdhva-Tiryagbhyam Sutra for computation of n-digits multiplication gives better results comparatively other techniques. Chillali et al. [1] in 2015 purposed ECC over a chain ring of characteristic three this reference explained ECC over the Ring that makes much secure ECC. Anchaliya and Chiranjeevi [14] in 2015 proposed Dvandva Yoga Sutra for Square of a number, Urdhva-Tiryagbhyam Sutra for multiplication, and Dhvajanka Sutra for the division. This paper shows the implementation in ECC by using Vedic Sutra for multiplication of n-digits number and square of a number in point doubling and point addition. The AIVM techniques improve the speed, and time of processing compare to the implementation of other conventional multiplication. Palata et al. [10] have described an Implementation of an efficient multiplier based on AIVM in 2017. This work explained the Implementation of encryption and decryption algorithms in ECC with AIVM Sutras to improve the performance in cryptographic operations. In the above references paper AIVM based ECC gives the best output in faster time execution for a cryptographic system.

III. MATHEMATICAL BACKGROUND OF ELLIPTIC CURVE CRYPTOGRAPHY AND THE RING A_4

A. *Encryption System [12, 17]:*

The cryptographic system or encryption system can be characterized by the tuple $(C, P, K, E, \text{and } D)$ with these axioms:

- The elements of the set C (Cipher text space) are called Cipher text.
- The elements of the set P (Plaintext space) are called Plaintext.
- The elements of the set K (Key space) are called Keys.
- The set $E = \{E_k : k \in K\}$ having a functions $E_k : P \to C$ and all the elements of E is called encryption functions.
- The set $D = \{D_k : k \in K\}$ having a functions $D_k : C \to P$ and all the elements of D is called decryption functions. For each $e \in K$, there is $d \in K$, such that $D_d(E_e(\text{p})) = p$ for all $p \in P$.

B. *Elliptic Curves [12, 17]:*

Let a finite field (F_p) with the prime modulo p and $a, b \in F_p$ such that $(4a^3 + 27b^2) \bmod p \neq 0$.

The equation $y^2 = (x^3 + ax + b) \bmod p$ *i.e.*

$E(F_p) = \{(x, y) : y^2 = (x^3 + ax + b) \bmod p\} \cup \{O\}$

. Satisfied by the $E(F_p)$ where $E(F_p)$ is an elliptic curve with the subject F_p and its having set of points x and y .

The identity element under addition operation *i.e.*

$P + O = O + P = P \quad$ *for every* $\quad P = (x, y) \in E(F_p)$;

where O is showing the point at infinity.

C. *For a Elliptic curve* $E(F_p)$ *two points addition operation defined as [12, 17]:*

Let P and Q are the point of elliptic curve where $P, Q = (x_1, y_1), (x_2, y_2) \in E(F_p)$ then the sum of these point is $R = (x_3, y_3) \in E(F_p)$, that is explain by the following formulation:

$x_3 = (\lambda^2 - x_1 - x_2) \bmod p$ *and* $y_3 = (\lambda(x_1 - x_3) - y_1) \bmod p$,where

$$if\ P \neq Q\ then\ \lambda = \left(y_2 - y_1 / x_2 - x_1\right) \bmod p$$

$$and\ if\ P = Q\ then\ \lambda = \left(3x_1^2 + a / 2\,y_1\right) \bmod p$$

D. *For a Elliptic curve* $E(F_p)$ *Additive inverse element of a point [12, 17]:*

Let P is a point over $E(F_p)$ and inverse of this point is $-P$ these are difined as and $-P = (x, -y \bmod p) \in E(F_p)$

i.e $(x, y) + (x, -y \bmod p) = O$. It can be seen that E(F_p) forms an abelian group under addition operation defined as above:

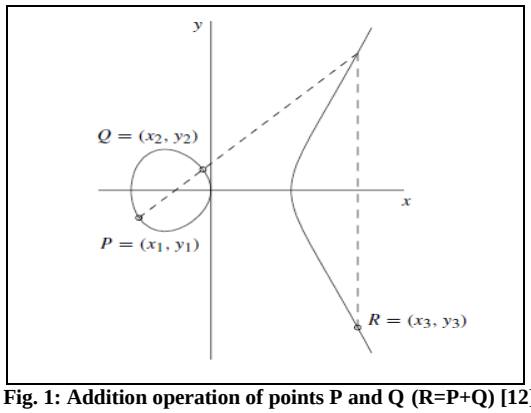

Fig. 1: Addition operation of points P and Q (R=P+Q) [12]

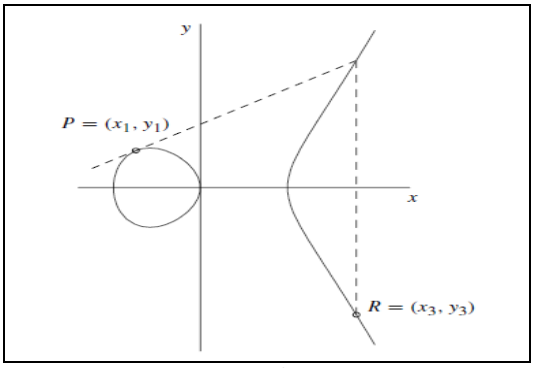

Fig. 2: Doubling operation of point P (R = 2P) [12]

E. The Ring A_4

For a positive integer d , Tadmori et al. [5, 6, 7] considered the quotient ring $A_4 = F_{2^d}[x]/(X^4)$, where F_{2^d} is the finite field of order 2^d . The ring $F_{2^d}[\varepsilon]$ is classified by the ring A_4 ; where $\varepsilon^4 = 0$ i.e. $A_4 = \left\{ x_0 + x_1\varepsilon + x_2\varepsilon^2 + x_3\varepsilon^3 : x_0, x_1, x_2, x_3 \in F_{2^d}^4 \right\}$.

We have the following facts about the ring A_4

- A_4 is a local ring and its maximal ideal is $M = \varepsilon.A_4$.
- The elements which are not invertible in A_4 , are the elements of M .
- A_4 is a vector space over the field F_{2^d} of dimension 4 and $\{1, \varepsilon, \varepsilon^2, \varepsilon^3\}$ is a basis of A_4 .
- If $Y = y_0 + y_1\varepsilon + y_2\varepsilon^2 + y_3\varepsilon^3$ is the inverse of $X = x_0 + x_1\varepsilon + x_2\varepsilon^2 + x_3\varepsilon^3$ then $y_0 = x_0^{-1}$, $y_1 = -x_0^{-1}y_0x_1$, $y_2 = -x_0^{-1}[y_0x_2 + y_1x_1]$, $y_3 = -x_0^{-1}[y_0x_3 + y_1x_2 + y_2x_1]$

IV. ANCIENT INDIAN VEDIC MATHEMATICS

Across India and out of India graph of the research is increasing speedily in the field of AIVM (Ancient Indian Vedic Mathematics) which contain the sixteen Sutras or formulae and fourteen sub Sutras in operations of all branches of Mathematics. In this AIVM section, we will discuss some useful techniques methods first one is Urdhva-Tiryagbhyam and second is Dvandva-Yoga of Ancient Indian Vedic Mathematics which will be used in a later section to improve the performance of ECC based schemes over the Ring A_4 [9].

A. Urdhva-Tiryagbhyam

Urdhva-Tiryagbhyam multiplication technique is used for general multiplication [9]. This Sutra directly explains a way in the form of vertically and crosswise which applying on the digits. This Sutra is very useful in all AIVM Sutras, and it has applied in many applications in all kind of branches of mathematical science. In this work, we are explaining this method for two or three digit numbers. This Sutra is so beneficial because by the use of AIVM technique we get the reduction in multiplications of multi-bits down to one-bits. A Comparison between Vedic multiplier and many others multipliers like Booth and Array multiplier, results show that AIVM based multiplier do not take much time and these multipliers save power, key size, and bits comparatively other multipliers. Naturally, by the use of standard or classical method in multiplication we can get results but the number of operation in the classical method is too high almost m^2 operations, for m-bit digit integers so it looks complicated. Another multiplication method is the

Karatsuba method its require number of operation is $m^{\log 3}$ for two integers of m-bit. Karatsuba technique (method) gives slow output for a small input in any operations comparatively classical methods of multiplication due to the repetition of overhanging operations. To manipulate this type of issue best Sutra of AIVM Urdhva-Tiryagbhyam technique can be applied and it gives best results.

- Multiplication of two digits numbers

$(px+q)(rx+s) = prx^2 + (ps+qr)x + qs$.

- Multiplication of three digits numbers

$$\left(px^2 + qx + r\right)\left(sx^2 + tx + u\right) = psx^4 + \left(pe+qs\right)x^3 + \left(pu+qt+rs\right)x^2 + \left(qu+tr\right)x + ru$$

where x is the base of the number system

Example 1: *Evaluate* 122×134.

1×1	1×3 + 2×1	2×3 + 1×4 + 2×1	2×4 + 2×3	2×4
1	5	12	14	8
1	5 + 1	2 + 1	4	8

= 16348

B. *Dvandva-Yoga*

For squaring by the Dvandva-Yoga, any binary or decimal number, a purposeful architectonics can rise up its performance and best output than other architecture's multiplier. Using Dvandva-Yoga (D_Y) algorithm and rule for squaring of binary or decimal numbers from the AIVM Sutras is explained as [9, 14]:

- To calculate Dvandva-Yoga (D_Y) of a number which contains single digit Dvandva-Yoga expressed that it is the square of that number, Dvandva-Yoga of p_1 is p^2
- To calculate Dvandva-Yoga (D_Y) of a number which contains two digits, Dvandva Yoga expressed that, it's double the multiplication of both digits of that number, Dvandva-Yoga of p_1q_1 is $2 * p_1 * q_1$.
- To calculate Dvandva-Yoga (D_Y) of numbers which contain three digits, Dvandva-Yoga expressed that, it's got double the product of first and third number and gives the square of that number which is placed in the middle, Dvandva-Yoga of $p_1q_1r_1$ is $2 * p_1 * r_1 + q_1^2$.

Example 3: *Evaluate this five digits number square* $(12345)^2$ *using Dvandva Yoga technique.*

STEP1: $D_Y(5) = 5^2 = 25$

STEP 2: $D_Y(45) = 2(4\times5) = 40$

STEP 3: $D_Y(345) = 2(3\times5) + 4^2 = 46$

STEP 4: $D_Y(2345) = 2(2\times5) + 2(3\times4) = 44$

STEP 5: $D_Y(12345) = 2(1\times5) + 2(2\times4) + 3^2 = 35$

STEP6: $D_Y(1234) = 2\times(1\times4) + 2(2\times3) = 20$

STEP 7: $D_Y(123) = 2(1\times3) + 2^2 = 10$

STEP 8: $D_Y(12) = 2(1\times2) = 4$

STEP 9: $D_Y(1) = 1^2 = 1$

Answer is 152399025.

V. ELLIPTIC CURVE CRYPTOSYSTEM USING URDVA-TIRYAGHBHYAM AND DVANDVA-YOGA TECHNIQUES

This section shows some arithmetic fundamental of ECC using AIVM Sutras. The elliptic curve is expressed as:

$$E(a,b) = \left\{ \begin{matrix} (x,y): y^2 = x^3 + ax + b \\ and\ 4a^3 + 27b^2 \neq 0 \end{matrix} \right\} \cup \{O\}.$$

To find the value of λ^2, $3x^2$ *and* $\lambda(x_1 - x_3)$ we can use AIVM Sutra Urdhva-Tiryagbhyam in the operations of ECC first is addition and second is doubling:

For Addition, the sum of two points $P+Q$ is R where $P=(x_1,y_1)$, $Q=(x_2,y_2)$ then R will be (x_3,y_3) where x_3, y_3, λ are

$$\begin{bmatrix} (\lambda^2 - x_1 - x_2), \\ (\lambda(x_1 - x_3) - y_1), \\ (y_2 - y_1)/(x_2 - x_1) \end{bmatrix}$$ Respectively.

For doubling, the point doubling of a point $P(x_1y_1) + P(x_1y_1) = 2P(x_1y_1) = R(x_3y_3)$ where x_3, y_3, λ are

$$\begin{bmatrix} \lambda^2 - 2x), \\ (\lambda(x - x_1) - y), \\ (3x^2 + a)/2y \end{bmatrix}$$ Respectively.

All these values of point addition and doubling can be determine easily by the help of AIVM useful Sutra.

VI. ELLIPTIC CURVES OVER THE RING A_4

Explanation: We characterize an elliptic curve over the ring A_4; distinguished $E_{a,b}(A_4)$ as an arc or curve obsessed by such like Weierstrass equation define as [5, 6, 7]:

$Y^2Z + XYZ = X^3 + aX^2Z + bZ^3$; In this expression *b* is invertible and A_4 containing a *and* b.

$$E_{a,b}(A_4) = \left\{ Y^2Z + XYZ = X^3 + aX^2Z + bZ^3 \;\middle|\; [X:Y:Z] \in P_2(A_4) \right\}.$$

A. *Addition of two points in ecc over the ring a_4 [6]:*

If $P_1 = [X_1 : Y_1 : Z_1]$ *and* $P_2 = [X_2 : Y_2 : Z_2]$
then the Addition of two points over $E_{a,b}(A_4)$
is $P_3 = P_1 + P_2 = [X_3 : Y_3 : Z_3]$ *where*

$$X_3 = X_1Y_2^2Z_1 + X_2Y_1^2Z_2 + X_1^2Y_2Z_2 + X_2^2Y_1Z_1 + aX_1^2X_2Z_2 + aX_1X_2^2Z_1 + bX_1Z_2^2Z_1 + bX_2Z_1^2Z_2.$$

$$Y_3 = X_1^2X_2Y_2 + X_1X_2^2Y_1 + Y_1^2Y_2Z_2 + Y_1Y_2^2Z_1 + X_1^2Y_2Z_2 + X_2^2Y_1Z_1 + aX_1^2Y_2Z_2 + aX_2^2Y_1Z_1 + aX_1^2X_2Z_2 + aX_1X_2^2Z_1 + bY_1Z_2^2Z_1 + bY_2Z_1^2Z_2 + bX_1Z_2^2Z_1 + bX_2Z_1^2Z_2.$$

$$Z_3 = X_1^2X_2Z_2 + X_1X_2^2Z_1 + Y_1^2Z_2^2 + Y_2^2Z_1^2 + X_1Y_1Z_2^2 + X_2Y_2Z_1^2 + aX_1^2Z_2^2 + aX_2^2Z_1^2.$$

B. *Algorithm of addition of two points in ECC over ring a_4*

Addition of $P_1 = (X_1, Y_1, Z_1)$ and $P_2 = (X_2, Y_2, Z_2)$

Input : $P_1 = [X_1 : Y_1 : Z_1]$ *and* $P_2 = [X_2 : Y_2 : Z_2]$, *a, b in ECC over ring* A_4

Output : $P_3 = [X_3 : Y_3 : Z_3] = P_1 + P_2$; *in ECC over ring* A_4.

1. *If* $P_1 = \infty$ *then return* $(X_1 : Y_1 : Z_1)$
2. *If* $P_2 = \infty$ *then return* $(X_2 : Y_2 : Z_2)$
3. $A = X_2 \cdot Y_1$;
4. $B = X_1 \cdot Y_2$;
5. $C = Z_1 \cdot X_2$;
6. $D = Z_2 \cdot X_1$;
7. $E = Z_1 \cdot Y_2$;
8. $F = Z_2 \cdot Y_1$;
9. $G = X_1 \cdot X_2$;
10. $H = Y_1 \cdot Y_2$;
11. $I = Z_1 \cdot Z_2$;
12. $J = D + C$;
13. $K = B \cdot D + A \cdot C$;
14. $L = B \cdot E + A \cdot F$;
15. $X_3 = J[I + aG] + K + L$;
16. $Y_3 = bI[J + M] + K[a + 1] + G[N + aJ] + H[F + E]$;
17. $Z_3 = a[D^2 + C^2] + E[E + C] + F[F + D] + G \cdot J$;
18. *Return* $(X_3 : Y_3 : Z_3)$

Finally, we can calculate the point $P_3\ (X_3, Y_3, Z_3)$ where

$X_3 = J[I + aG] + K + L$,

$Y_3 = bI[J + M] + K[a + 1] + G[N + aJ] + H[F + E]$ *and*

$Z_3 = a[D^2 + C^2] + E[E + C] + F[F + D] + G \cdot J$

here we can use Urdhva-Tiryagbhyam and Dvandva-Yoga technique to evaluate all values of multiplications and squares.

C. *Doubling of a point in ECC over the ring a_4 [6]:*

If $P_1=[X_1:Y_1:Z_1]$ *then the Doubling of a points over* $E_{a,b}(A_4)$ *is* $P_3=2P_1=[X_3:Y_3:Z_3]$ *where*

$$X_3=X_1Y_1Y_2^2+X_2Y_1^2Y_2+X_2^2Y_1^2+X_1X_2^2Y_1+aX_1^2X_2Y_2+aX_1X_2^2Y_1+aX_1^2X_2^2+bX_1Y_1Z_2^2+bX_2Y_2Z_1^2+bX_1^2Z_2^2+bY_1Z_2^2Z_1+bY_2Z_1^2Z_2+bX_1Z_2^2Z_1.$$

$$Y_3=Y_1^2Y_2^2+X_2Y_1^2Y_2+aX_1X_2^2Y_1+a^2X_1^2X_2^2+bX_1^2X_2Z_2+bX_1X_2^2Z_1+bX_1Y_1Z_2^2+bX_1^2Z_2^2+abX_2^2Z_1^2+abX_1^2Z_2^2+bY_1Z_2^2Z_1+bX_1Z_2^2Z_1+abX_1Z_2^2Z_1+abX_2Z_1^2Z_2+b^2Z_1^2Z_2^2.$$

$$Z_3=X_1^2X_2Y_2+X_1X_2^2Y_1+Y_1^2Y_2Z_2+Y_1Y_2^2Z_1+X_1^2X_2^2+X_2Y_1^2Z_2+X_1^2Y_2Z_2+aX_1^2Y_2Z_2+aX_2^2Y_1Z_1+X_1^2X_2Z_2+aX_1X_2^2Z_1+bY_1Z_2^2Z_1+bY_2Z_1^2Z_2+bX_1Z_2^2Z_1.$$

D. *Algorithm of doubling of a point in ECC over ring a_4*

Doubling of $P_1=(X_1,Y_1,Z_1)$

Input : $P_1=[X_1:Y_1:Z_1]$, a,b *in ECC over ring* A_4

Output: $P_3=[X_3:Y_3:Z_3]=2P_1$; *in ECC over ring* A_4

1. *If* $P_1=\infty$ *then return* (∞)
2. $A=X_2\cdot Y_1$;
3. $B=X_1\cdot Y_2$;
4. $C=Z_1\cdot X_2$;
5. $D=Z_2\cdot X_1$;
6. $E=Z_1\cdot Y_2$;
7. $F=Z_2\cdot Y_1$;
8. $G=X_1\cdot X_2$;
9. $H=Y_1\cdot Y_2$;
10. $I=Z_1\cdot Z_2$;
11. $J=H+A$;
12. $K=b(F+D)$;
13. $L=A+B$;
14. $M=A+aG$;
15. $N=I+C$;
16. $X_3=G[aL+M]+K[I+D]+bE\cdot N+A\cdot J+H\cdot B$;
17. $Y_3=[D+C][G+aI]+a[C^2+D^2]+K[I+D]+aG\cdot M+H\cdot J+bI^2$;
18. $Z_3=G[L+aC+G+D]+E[H+bI]+aA\cdot C+F\cdot J+I\cdot K$;
19. $Return(X_3:Y_3:Z_3)$

Finally, we can calculate the point $P_3\ (X_3,Y_3,Z_3)$ where

$$X_3=G[aL+M]+K[I+D]+bE\cdot N+A\cdot J+H\cdot B;$$
$$Y_3=[D+C][G+aI]+a[C^2+D^2]+K[I+D]+aG\cdot M+H\cdot J+bI^2;$$
$$Z_3=G[L+aC+G+D]+E[H+bI]+aA\cdot C+F\cdot J+I\cdot K;$$

Here we can use Urdhva-Tiryagbhyam and Dvandva-Yoga technique to evaluate all values of multiplications and squares.

VII. CRYPTOGRAPHIC APPLICATIONS

Let $P\in E_{a,b}(A_4)$ of order τ, we will use the subgroup $<P>$ of $E_{a,b}(A_4)$ to encrypt message, and we denote $G=<P>$.

- **Coding of elements of *G*.**

We will give a code to each element

$Q = m.P \in G,$ where

$m \in \{1, 2...\tau\},$

Let

$$Q = \left[x_0 + x_1\varepsilon + x_2\varepsilon^2 + x_3\varepsilon^3 : y_0 + y_1\varepsilon + y_2\varepsilon^2 + y_3\varepsilon^3 : Z \right]$$

Where, $x_i, y_i \in F_{2^d}$ *for* $i = 0,1,2,3.$ we set:

If Z=1 then

We code Q as it follows:

$$Q = \left[x_0 x_1 x_2 x_3 y_0 y_1 y_2 y_3 : 1 \right]$$

$$Q = \begin{bmatrix} 100010001, 100011111, 100011101, 000011111 \\ 110010001, 100011011, 100010111, 110011111 \\ 101010001, 100010111, 100010101, 100011111 \\ 100110001, 100010011, 100010111, 101011111 \\ 111110001, 111010011, 111110111, \ldots \quad \ldots \quad \ldots \end{bmatrix}$$

Q has containing $2^8 = 256$ *elements.*

VIII. RESULT ANALYSIS AND COMPARISONS

The comparison is based on the number of multiplications and square in point doubling and addition in ECC (Elliptic Curve Cryptography) over the Ring A_4 and VECC (Vedic Mathematics based Elliptic Curve Cryptography) over the Ring A_4. The total arithmetic operations such as multiplication, square are compared in table I and II, which concludes that the number of operations is minimum in table II which is based on VECC.

- **[M (**Number of multiplications**), S (**Number of Squares**), T (**Total number of Arithmetic Operations**)]**

ECC over Ring A_4	M	S	T
Point addition	56	34	90
Point doubling	74	52	126

Table I: Number of operations needed in addition and doubling of points in ECC over Ring A_4.

ECC over Ring A_4	M	S	T
Point addition	25	2	27
Point doubling	23	3	26

Table II: Number of operations needed in addition and doubling of points in VECC over Ring A_4.

The arithmetic operations in ECC over the Ring A_4 for point addition and point are compared in fig.3 which also conclude that the number operations are minimum in the purposed results.

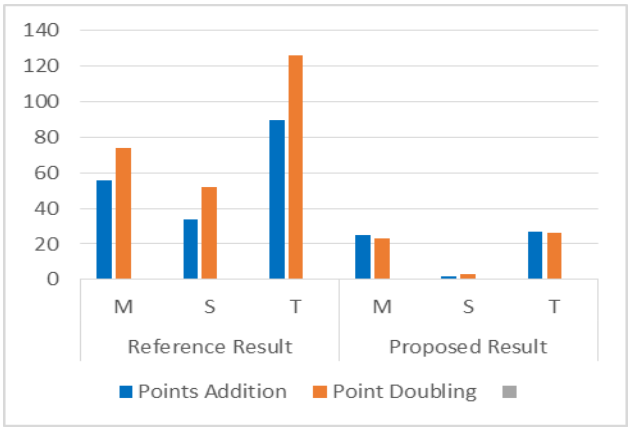

Fig. 3: Comparing the arithmetic operations in ECC over the Ring A_4 for point addition and point doubling

IX. CONCLSION

In this approach we have analysed the ECC (Elliptic Curves Cryptography) over the Ring $A_4 = F_{2^d}[\varepsilon]$ where $\varepsilon^4 = 0,$ and the coding over the EC (Elliptic Curve) $E_{a,b}(A_4)$ well established. There are mainly two useful operations elaborated in ECC over the Ring $A_4 = F_{2^d}[\varepsilon]$ first one is the addition and other is doubling. In these major operations, all multiplications and squares are calculated by AIVM Sutras to push forward the all scalar multiplications. Our approach described the squaring of an n-digit number and multiplications in point doubling and addition. The AIVM techniques based multiplications give best performance and results than the arithmetical multiplication. AIVM technique provides a minimum manipulation in the operations of ECC such as, point addition other is point doubling which are the major parts of ECC to manage the keys of encryption and decryption process. This approach is effectual in the terms of processing time, speed, time of key generation, key size, strength, and as well as area.

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