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An Exact Analytical Solution of Blast Wave Problem in Gas-Dynamics at Stellar Surface

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Abstract- In the present analysis, an analytical approach is used to construct an exact solution of a problem of one-dimensional unsteady adiabatic flow of a blast wave propagation with generalized geometries at stellar surface in a plasma whose density ahead of the shock front is assumed to vary as a power law of the distance from the source of the point of explosion. The plasma is assumed to be an ideal gas. An analytical solution of the problem is find out in terms of flow parameters velocity, density and the pressure, which exhibits space-time dependence. In addition, an analytical expression has been derived for the total energy of the blast wave propagation at the stellar surface.

***Keywords*:** Blast wave, Ideal gas-dynamics, Rankine-Hugoniot conditions, Stellar surface.

1. Introduction

Blast wave phenomena arise when large amount of energy is suddenly released in a small enough region over a short time interval, as in the case of spark discharges in air. Studies of blast wave propagation has been a field of great interest from both mathematical and physical points of view. The practical importance of these waves has increased in last decades due to their particular applications in astrophysics, plasma physics, nuclear science, geophysics, treatment for kidney stone disease, orthopaedics and cancer treatment. Due to various important applications of blast wave propagation in real situations, a continuous improvement in the subject is desirable.

The pioneering studies of blast wave phenomenon was made by [1], [2] and [3]. [2] obtained exact solutions to the equations for the motion in a gas generated by a point explosion. A number of analytical solutions for the blast wave propagation have been obtained by [4-9]. [5] and [7] used an approach, based on the shock propagation theory of [10], which permits a simple analytical solution to be obtain directly from the governing equations. [4] obtained the closed form solution for spherical blast wave problem, when the density of the gas ahead of the shock front varies as a power of the distance from the origin.

In recent decades, many authors e.g., [11-17] have carried out a further contribution towards the determination of exact solution of blast wave problem in different material media. [10] considered the dynamics of the stellar surface and studied emergence of a nonlinear wave at a stellar surface and obtained exact solution with allowance for gravity in the isentropic approximation. [14] obtained the exact solution for blast wave problem in ordinary gas-dynamics. [12-13] used substitution principle to obtain an exact solution for unsteady equation of perfect gas and ideal magnetogasdynamics equation. [15] used the method of Lie group transformation to obtain an approximate analytical solution to the system of first order quasi-linear partial differential equations that governs a one dimensional unsteady planer, cylindrically symmetric and spherically symmetric motion in a non-ideal gas, involving strong shock waves.

Rest of the paper is organized as follows: In section 2, an attempt has been made to obtain the closed form solution of the basic equations governing the one-dimensional unsteady flows of an ideal gas involving strong shock waves at the stellar surface. It is assumed that mass density distribution in the medium follows a power law of the radial distance from the point of explosion. In section 3, an analytical solution of the blast wave problem at the stellar surface is obtained in terms of flow variables velocity, density and the pressure. In addition, the analytical expression for the total energy is also determined in section 3. Finally conclude the paper at the end.

2. Basic Equations and boundary conditions:

The basic equations governing the one-dimensional unsteady planar and radially symmetric motion of an ideal gas describing the flow at the stellar surfaces are given by [1], [14] and [18]

$$\rho_t + u\rho_x + \rho u_x - \frac{n\rho u}{1-x} = 0, \qquad (1)$$

$$u_t + uu_x + \frac{1}{\rho}p_x = 0, \qquad (2)$$

$$p_t + up_x + \rho c^2\left(u_x - \frac{nu}{1-x}\right) = 0, \qquad (3)$$

where x is the spatial coordinate being either axial in flows with planar $(n = 0)$ geometry or radial in cylindrically symmetric $(n = 1)$ and spherically symmetric $(n = 2)$ flows, t is the time, u is the gas velocity, ρ is the density and p is the pressure. The entity $c = (\gamma p/\rho)^{1/2}$ is the equilibrium speed of sound with γ as the specific heat ratio.

The system of equation $(1) - (3)$ is supplemented with an equation of state $p = \rho\Re T$, where T is the absolute temperature and $\Re$ is the gas constant. The flow variables with a letter subscript denote partial differentiation with respect to the indicated variables unless stated otherwise.

Let the position of the shock front is given by $x = R(t)$ as a function of time t with the propagation velocity V at the shock front, defined as

$$\frac{dR(t)}{dt} = V, \qquad (4)$$

In the limit of an infinite shock Mach number, the boundary conditions at the shock front $x = R(t)$ is given by the Rankine-Hugoniot relations as follows

$$u = \frac{2}{\gamma+1}V, \qquad (5)$$

$$\rho = \frac{\gamma+1}{\gamma-1}\rho_s, \qquad (6)$$

$$p = \frac{2}{\gamma+1}\rho_s V^2, \qquad (7)$$

where ρ_s is the density ahead of the shock front.

In the present problem, the density ρ_s is assumed to vary according to the power law of the radius of the shock front $R(t)$ and is given by

$$\rho_s = \rho_0 R^{\alpha}, \qquad (8)$$

where ρ_0 and α are constants. The constant α is to be determined later.

The total energy E carried by the blast wave is equal to the energy supplied by the explosive and this is constant. Therefore, we have

$$E = 4\pi\int_1^R \left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma-1}p\right)(1-x)^n \, dx, \qquad (9)$$

which represents the sum of the kinetic and internal energy of the gas.

3. New exact solution of the shock wave problem

With the help of Rankine-Hugoniot relations (5), (6) and (7), we construct a relation among the flow variables as given by

$$p = \frac{\gamma-1}{2}\rho u^2. \qquad (10)$$

After using equation (10), the governing equations (2) and (3) can be transformed to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \frac{(\gamma-1)u}{2\rho}\left(\frac{\partial \rho}{\partial t} - \rho\frac{\partial u}{\partial x} - \frac{n\rho u}{1-x}\right) = 0, \qquad (11)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{(\gamma-1)u}{2}\left(\frac{\partial u}{\partial x} - \frac{nu}{1-x}\right) = 0. \qquad (12)$$

Combining Eqs.(11) and(12), and integrating the resulting equation we obtain

$$\rho u(1-x)^{-n} = \Gamma(t), \qquad (13)$$

where $\Gamma(t)$ is an arbitrary function of integration.

Using the equation(13), equation (1) reduces of the form

$$\frac{1}{u}\frac{\partial u}{\partial t} + \frac{2n}{1-x}u - \frac{1}{\Gamma(t)}\frac{d\Gamma(t)}{dt} = 0, \qquad (14)$$

On solving equation (12) and(14), we have

$$u = \xi\frac{(1-x)}{\Gamma(t)}\frac{d\Gamma(t)}{dt}, \qquad (15)$$

where $\xi = 2[(n+1)\gamma + 3n + 1]^{-1}$.

Inserting equation (15) in equation (14) and then integrating, we obtained

$$\Gamma(t) = \Gamma_0 t^{-\lambda}, \qquad (16)$$

where $\lambda = 1 + 4n[(n+1)\gamma - n + 1]^{-1}$ and Γ_0 is an arbitrary constant.

With the help of Rankine–Hugoniot relation (5), we can obtain the analytical expression for the radius of the shock front as

$$R = 1 - t^{\frac{\gamma+1}{2}\xi\lambda}. \qquad (17)$$

Rankine–Hugoniot condition (6) yields the following value of the constant α as

$$\alpha = \frac{n(\gamma-3)}{\gamma+1} - 1. \qquad (18)$$

Consequently, the analytical solution of the blast wave problem at stellar surface is given by

$$\rho = -\frac{\Gamma_0}{\xi\lambda}(1-x)^{n-1}t^{1-\lambda}, \quad u = -\xi\lambda\frac{(1-x)}{t},$$

$$p = -\left(\frac{\gamma-1}{2}\right)\Gamma_0\xi\lambda t^{-1-\lambda}(1-x)^{n+1}, \quad \rho_s(R) = -\left(\frac{\gamma-1}{\gamma+1}\right)\frac{\Gamma_0}{\xi\lambda}(1-R)^{\alpha}. \qquad (19)$$

After determining the flow variables density, velocity and pressure, we can also calculate the total energy carried by the blast wave at stellar surface given by

$$E = \frac{2\pi}{(n+1)}\Gamma_0\xi\lambda. \qquad (20)$$

It may be observed here that in the absence of the stellar surface, the analytical solutions (19) and (20) obtained is a well-known solution to the blast wave problem carried out by various approaches.

4. Conclusion

In the present article, the exact analytical solution for the problem of blast wave propagation in a one dimensional ideal adiabatic gas flow with generalized geometries at stellar surface has been derived. Here the density ahead of the shock front is assumed to vary according to a power law of the distance. An analytical expression for the density, velocity and pressure are obtained in terms of position and time. An analytical expression for the total energy carried by blast wave in an ideal gas-dynamics at stellar surface is also derived. It is noted that in the absence of the stellar surface, such an analytical solutions are coincides with earlier result obtained by [14].

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