IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Issue-4, April 2019 E-ISSN: 2347-2693

Analytical Solution of Steady Hydromagnetic Flow Bounded by Two Concentric Circular Cylinders in a Porous Medium

Anup Kumar Karak

Department of Mathematics, Berhampore Girls' College, Berhampore, Murshidabad, Pin– 742101, India

Author's Mail Id: karakanup99@gmail.com

DOI: https://doi.org/10.26438/ijcse/v7i4.12361238 | Available online at: www.ijcseonline.org

Accepted: 22/Apr/2019, Published: 30/Apr/2019

Abstract- In this study the steady hydromagnetic flow of fluid between two porous concentric circular cylinders is consider. The fluid isconsider as viscous, incompressible, conducting. The equation of motion and the constitutive equations form a system of non-linear ODEs that is investigated analytically, and in a few cases the numerical results are compared with a known numerical solution. Numerical computations show the effect of the non-Newtonian quantities on the velocity and on the shear stress as the dimensionless parameters are varied. It is supposed that the rate of suction at the inner cylinder is equal to the rate of injection at the outer.

Keyword- hydromagnetic flow, porous medium, viscous, incompressible, conducting fluid.

1. INTRODUCTION

Shercliff [1,3] and Chang and Lundgren [4] considered the steady hydromagnetic flow through ducts of rectangular cross-section in presence of a transverse magnetic field while Edward [2, 5] considered the same problem when the duct section is an annulus and the impressed magnetic field is radial or circular. Singh and Rizvi [6] and Singh [7] investigate the problems of impulsive motion of viscous liquid contained between two concentric circular cylinders in presence of magnetic field. Mahapatra [8], Sengupta and Ghosh [9] and Roy Choudhury [1] also considered same type of problems when the cylinders rotate with various angular velocities.

Now the flow of fluid through porous media has drawn the attention to the research workers because of its wide applications in many scientific and engineering fields. Such type of flow can be found in petroleum engineering to study the movement of natural gas, oil and water through oil reservoirs, filtration and purification process in chemical engineering and so on. Ahmadi and Manvi [11] derived the general equations of motion for porous media and applied the results to some basic flow problems. Gulabram and Mishra [12] generalized the equations to consider the magnetohydrodynamic flow of conducting liquid through such media.

In the present paper, it is proposed to study the steady motion of an incompressible viscous conducting infinite concentric circular cylinders rotating with prescribed uniform angular velocities in presence of a uniform radial magnetic field. As particular cases, solutions are obtained when the medium is non-porous and the cylinders are imperevious. The distribution of the velocity has shown in graphical from.

2. FORMULATION OF THE PROBLEM

Let us consider two concentric porous non-conducting infinite circular cylinders of radii a, b (b>a) and suppose that an incompressible viscous conducting liquid flows steadily through the porous medium between them. Introducting the cylindrical co-ordinates (r,θ,z), we take the velocity components to be (u,v,w) and common axis of the cylinders as the z-axis. Also a uniform magnetic field H_0 is applied along the radial direction. It was now assume that the motion is rotationally symmetric and two dimensional, the axial component w of velocity vanishes and also the derivatives of transverse velocity with respect to θ and z vanish. The induced electric and magnetic field are also neglected. Then the modified governing equations of steady motion and the equation of continuity are

$$u\frac{du}{dr}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{dp}{dr}+\nu\left(\nabla^2 u-\frac{u}{r^2}\right)-\frac{u}{k}, \tag{2.1}$$

$$u\frac{dv}{dr}+u\frac{v}{r}=\nu\left(\nabla^2 v-\frac{v}{r^2}\right)-\frac{\sigma B_0^2}{\rho}.\frac{v}{r^2}-\frac{v}{k}, \tag{2.2}$$

$$\frac{d}{dr}(ru)=0 \tag{2.3}$$

where
$\nabla^2\equiv\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}$, $B_0=\mu_e H_0$

$\mu_e$is the magnetic permeability, the kinematic velocity, σ is the conductivity of the medium, ρ is the density, p is the pressure and k is the permeability of the porous medium. In the above equations, the rationalized M.K.S. units have been used. Now , let us assume that the cylinders rotate with uniform angular velocities Ω_1, and Ω_2 so that boundary conditions are

$v=a\Omega_1$ on r = a, $v=b\Omega_2$ on r = b (2.4)

Let us take

$$v = \frac{V}{r^{m+3}} \quad (2.5)$$

also the equation (2.3) leads to $u = -\frac{S}{r^2}$ (2.6)

where S (> 0) is the suction parameter and is taken as

$$S = 2\nu(m+3). \quad (2.7)$$

Then using (2.5) to (2.7), equation (2.1) and (2.2) are transformed to $\frac{S^2}{r^4} + \frac{V^2}{r^{2m+4}} = \frac{1}{\rho}\frac{dp}{dr} - \frac{S}{rk}$,
(2.8)
and

$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - \left(\frac{n^2}{r^3} + q^2\right)V = 0 \quad (2.9)$$

where

$$n^2 = m^3 + \omega_0^2,\ \omega^2 = \frac{\sigma B_0^2}{\rho},\ q^2 = \frac{1}{k}. \quad (2.10)$$

The boundary conditions (2.4) reduce to
$V(r) = \Omega_1 a^{m+4}$ on r = a,
$V(r) = \Omega_2 b^{m+4}$ on r = b . (2.11)

3. SOLUTION OF THE PROBLEM

The solution of the equation (2.9) subject to the boundary condition (2.11) is given by

$$V(r) = \frac{1}{\Lambda}\begin{bmatrix}\Omega_1 a^{m+4}\{I_n(qr)K_n(qb) - I_n(qb)K_n(qr)\} - \\ \Omega_2 b^{m+4}\{I_n(qr)K_n(qa) - I_n(qa)K_n(qr)\}\end{bmatrix} \quad (3.1)$$

where I_n and K_n are modified Bessel functions of first and second kinds of order n and
$\Lambda = I_n(qa)K_n(qb) - I_n(qb)K_n(qa)$ (3.2)

Putting S = 0 i.e. m = −1, the corresponding solution for the velocity of the fluid through impervious concentric circular cylinders is obtained as

$$V(r) = \frac{1}{\Lambda}\begin{bmatrix}\Omega_1 a\{I_n(qr)K_n(qb) - I_n(qb)K_n(qr)\} - \\ \Omega_2 b\{I_n(qr)K_n(qa) - I_n(qa)K_n(qr)\}\end{bmatrix} \quad (3.3)$$

On the other hand, if we suppose that the medium through which the conducting fluid flow is non-prous i.e. impermeable, then K→ ∞ and the solutions of the equation (2.9) subject to the boundary conditions (2.11) are given by

$$V(r) = \frac{1}{a^{2n+1} - b^{2n+1}}\begin{bmatrix}(\Omega_1 a^{m+n+4} - \\ \Omega_2 b^{m+n+4})r^n - \\ \begin{pmatrix}\Omega_1 b^{2n} a^{m+n+5} - \\ \Omega_2 a^{2n} b^{m+n+4}\end{pmatrix} r^{-n}\end{bmatrix},$$

for $n \neq 0$ (3.4)

and

$$V(r) = \frac{1}{2(\log a - \log b)}[(\Omega_1 a^{m+4} - \Omega_2 b^{m+4})\log r + \Omega_2 b^{m+4}\log a - \Omega_1 a^{m+4}\log b],$$

for $n = 0$ (3.5)

For impervious concentric circular cylinders, the solutions are obtained from the above equations by putting S = 0 i.e. m = −1 in the forms

$$V(r) = \frac{1}{a^{2n+1} - b^{2n+1}}[(\Omega_1 a^{n+3} - \Omega_2 b^{n+3})r^n - (\Omega_1 b^{3n} a^{n+3} - \Omega_2 a^{2n} b^{2n+3})r^{-n}],$$

for $n \neq 0$ (3.6)

$$V(r) = \frac{1}{3(\log a - \log b)}[(\Omega_1 a - \Omega_2 b)\log r + b\Omega_2 \log a - a\Omega_1 \log b],$$

for $n = 0$ (3.7)

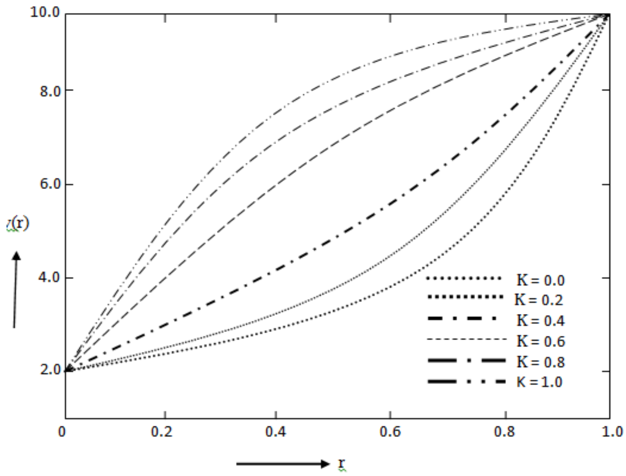

Fig. 1 (n ≠ 0)

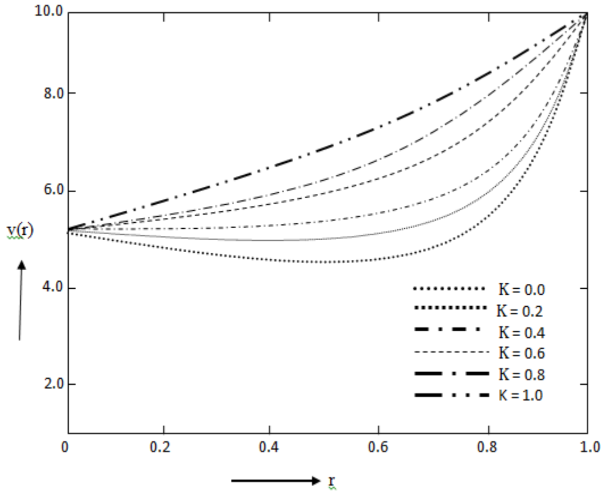

Fig. 2 (n = 0)

4. NUMERICAL RESULT

Taking $\frac{\Omega_1}{\Omega_2} = 2$, $\omega = 2.5$, m = 2, a = 1, b = 2, the distribution of the velocity for porous concentric circular cylinders has been shown in figure -1 for various values of k within the region $a \leq r \leq b$. It is seen that the permeability of the medium increases the velocity. Figure -2 gives the nature of the velocity when the medium is non-porous. Here also the permeability increases the velocity gradually.

ACKNOWLEDGEMENT

The author remain thankful to Prof. D.C. Sanyal, retired Professor of Mathematics, University of Kalyani for his kind help, guidance and encouragement during the preparation of the paper.

REFERENCES

[1] Z. Abbas, J. Hasnaina, and M. Sajid, “Hydromagnetic mixed convective two-phase flow of couple stress and viscous fluids in an inclined channel,” *Zeitschriftfür Naturforschung A*, **vol. 69**, no. 10-11**, pp. 553–561**, **2014.**

[2] J. C. Umavathi, A. J. Chamkha, A. Mateen, and A. Al-Mudhaf, “Unsteady two-fluid flow and heat transfer in a horizontal channel,” *Heat and Mass Transfer*, **vol.42**, no.2, **pp. 81–90**, **2005.**

[3] ShercliffJ.A.Proc. Comb. Phil. Soci, 49 (1953), **136**.

[4] Chang C.C. and Lundgren, T.S., ZAMP, 12 (1961), **100.**

[5] Edward, R.D’SA ZAMS 17,**140, 1966.**

[6] Singh D. and Rizvi, Phys. Fluids, **7**, **285, 1964.**

[7] Singh, G.S.Jour. Sci. Engg. Res., 9, **284, 1965.**

[8] MahapatraJ.R.Appl. Sci. Res, 27, **274, 1973.**

[9] Sengupta P.R. and Ghosh, S.K., Gzech, Jour. Phys. 25, **514, 1975.**

[10] Roy Chowdhury, S.K. Ind, Jour. Theo. Phys, 31, **137, 1983.**

[11] Ahmadi O. and Manvi, R. Ind. Jour. Tech, 9, **441, 1971.**

[12] Gulabram and Mishra, R.S.Ind. Jour. Pure. Appl: Math., **8, 637, 1977.**

[13] Biswas, D., Blood Flow Models: A Comparative Study, Mittal Publications, New Delhi, India, **2000.**

[14] Pedley, T.J., The Fluid Mechanics of Large Blood Vessels, Cambridge University Press, London, **1980.**

[15] Haynes, R.H., Physical Basis of The Dependence of Blood Viscosity on Tube Radius, Am. J.Physiol, 198(6), **1193-1200**, **1960.**

[16] Cowin, S.C., The Theory of Polar Fluids, Adv. Appl. Mech, 14, **179-347, 1974.**

[17] Skalak, R., Theoretical Models on Capillary Flow, Blood Cells, **8, 147-152, 1982.**

[18] Mann, D.E. and Trarbell, J.M., Flow of Non-Newtonian Blood Analog Fluids in Rigid Curved and Straight Artery Models, Biorheology, **27**, **711-733**, **1990.**

[19] Biswas, D. and Nath, A., Poiseuille Flow of Power-Law Fluid Model for Blood Flow with Velocity Slip, Journal of Assam University, **6(1), 1-13**, **2001.**

[20]Bugliarello, G. and Hayden, J.W., High speed Microcinematographic Studies of Blood Flow in Vitro, Science, **138, 981-983, 1982.**

[21]Bugliarello, G and Sevilla, J., Velocity Distribution and Other Characteristics of Steady and Pulsatile Blood in Fine Glass Tubes, Biorheology, **7, 85-107, 1970.**

[22]Young, D.F. and Tsai, F.Y., Flow Characteristics in Models of Arterial stenosis-I: Steady Flow, J.Biomech. **6, 395-411, 1973.**

Authors Profile

Dr. Anup Kumar Karak pursed Bachelor of Science from Burdwan University, India in 1994 and Master of Science from University of Kalyani, India in 1997. He received his P.h.D. award in 2007 from University of Kalyani. He is currently working as Assistant Professor in Department of Mathematics, Berhampore Girls’ College, Berhampore, West Bengal, India since 2010. He is a life member of Calcutta Mathematical Society. His main research work focuses on Bio-Mathematics. He has 16 years of teaching experience and 16 years of research experience.