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Some Generalized Results on TOPSIS Method Involving Multi-Attributes Decision Making Problem

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***Abstract*—** In the competitive situation of an aircraft is no longer dominated by economic criteria. To an economic consideration, there are several criteria needed to be taken into account in aircraft design and evaluation of decision making processes. To solve complex real-world decision making problems, multi-attribute decision making (MADM) methods have been developed. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is among the most widely used methods at present which provides valuable outputs in different application areas. Here we improve the performance of the overall system, identify the set of important parameters of the decision making system using TOPSIS. Moreover, we calculate the distance of each alternative from the positive ideal solution (PIS) and negative ideal solution (NIS). We use a numerical experiment to demonstrate the methodology of the suggested approach.

***Keywords*—**Decosion Making, MADM, NIS. PIS, TOPSIS.

I. INTRODUCTION

MADM have been widely used to select a finite number of alternatives generally characterized by multiple conflicting criteria. Several MADM approaches have been devised to resolve a large variety of problems involving real world. In the competitive situation of an aircraft is no longer dominated by economic criteria. To an economic consideration, there are several criteria needed to be taken into account in aircraft design and evaluation of decision making processes. For environmental aspects and level of comfort we consider these multiple criteria such as aircraft design and aircraft evaluation in multi-criteria decision making problems.

The demands on air travel are increasing, not only regarding lower costs, but also better service quality, higher safety, and more environmental friendliness. The imperatives of air transport have evolved from Higher, Further, Faster to More Affordable, Safer, Cleaner and Quieter [1]. In order to sustain the growth of air transport in a long term, the aerospace industry is faced with the challenge of designing more competitive aircraft satisfying these multiple criteria simultaneously.

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was utilized to the selection of technology alternatives in conceptual and preliminary aircraft design [2]. However, TOPSIS has the limitations that it assumes that each criterion's utility is monotonic and is rather sensitive to the weighting factors. A multi-criteria interactive decision-making advisor for the selection of the most appropriate decision making method was developed [3]. TOPSIS is used to assess the performance of alternatives through the similarity with the ideal solution given by Hwang and Yoon [4]. According to the technique of Hwang and Yoon [4], the most suitable alternative is one which is nearest to the PIS and at maximum apart from the NIS [5,6,7] . The PIS makes the benefit criteria maximum while minimizing the cost criteria. The NIS on the other hand enhances the cost criteria to a maximum level while minimizing the benefit criteria.

For examining qualitative attributes, we first arrange the words such as few, average, many, so many and then assign the number to each one of them. We compare attributes to identify the significance of each of them in the selection of options. Finally, after identifying the weights of attributes in decision making, the selection is made by considering how much benefit an option offers over another option.

Decision criteria and weighting factors are main input data in the decision making process. It is observed that there are always uncertainties existing in the decision criteria due to incomplete information or limited knowledge, while the weighting factors are often highly subjective, considering the fact that they are elicited based on the decision making experience or intuition [8,9]. Therefore, uncertainty assessment for the decision criteria and the weighting factors should be prudently performed.

We identify the different criteria corresponding to the alternatives such as return on operation profit to capital, comfort and cleanness of airport terminals, trolleys approach travellers, aerodrome control, security measures and airport scale, aircraft take-off and loading time, traffic connecting city, courtesy of crew and parking lots, noise pollution control, flight safety control etc.

In the following section II, we discuss briefly on fundamental terminologies involving TOPSIS methodology, and an algorithm associated with it. Section III contain the some parameter of the evaluation of Euclidean and weighted distance values of ten alternatives, the average linguistic performance of the ten airports, the linguistic weights for ten criteria, Section IV contains the application and result discussion and Section V contains conclusion of research work with future scope.

Also in this paper, we find the distances of PIS and NIS from the respective alternatives by applying the TOPSIS methodology [10]. Moreover, here we define a closeness coefficient to determine the ranking order of the alternative.

II. TOPSIS METHODOLOGY

The TOPSIS method was first developed by Hwang and Yoon in 1981 [4]. This is a simple ranking method from the point of view of concept and application. Further, in this method the ranking of all alternatives considered in the study are identified. In the standard TOPSIS method attempts are made to select alternatives both nearest to the PIS as well as farthest from the NIS. The role of PIS is to maximize the benefit criteria and minimize the cost criteria whereas, the role of NIS is just the opposite. With the above hypotheses, calculations involving eigenvector, square rooting and summations are used for obtaining a relative closeness value of the criteria tested. TOPSIS ranks these values of relative closeness of the whole system by assigning the highest value of the relative closeness to the best attributes in the system. By various linguistic rating applied to represent the performances under certain alternative criteria, are medium low (ML), medium (M), medium good (MG), good (G), and very good (VG). The linguistic weights [11] for performing the importance of weight criteria are very low (VL), low (L), medium (M), high (H) and very high (VH) [12,13,14,15,16, 17]. For calculation of TOPSIS values, we have to go through the following Algorithm [4].

Algorithm [4]:

Step-1 Choose decision matrix D which is consists of alternative and criteria is described by

$$D = A_\alpha \overset{C_\beta}{\left(x_{\alpha\beta}\right)}_{m\times n}$$

where A_α , $\alpha = 1,\cdots,m$ are alternatives and C_β , $\beta = 1,\cdots,n$ are criteria, $x_{\alpha\beta}$ are original scores indicates the rating of the alternative A_α with respect to criteria C_β. The weight vector $w = (w_1, w_2, \cdots, w_n)$ is composed of the individual weights w_β $(\beta = 1,2,\cdots,n)$ for each criteria C_β . Generally, the criteria are classified into two types: benefit and cost. The benefit criterion is higher value while a cost criterion is valid for opposite value.

Step-2 Construct normalized decision matrix $N_{\alpha\beta}$, where $\mathrm{N}_{\alpha\beta} = x_{\alpha\beta} \Big/ \sqrt{\sum x_{\alpha\beta}^2}$ for $\alpha = 1,\cdots,m;\ \beta = 1,\cdots,n$, where $x_{\alpha\beta}$ and $\mathrm{N}_{\alpha\beta}$ are original and normalized score of decision matrix, respectively.

Step-3 Construct the weighted normalized decision matrix: $V_{\alpha\beta} = w_\beta \mathrm{N}_{\alpha\beta}$, where w_β is the weight for β^{th} criteria and $\sum w_\beta = 1$.

Step-4 Determine the positive ideal solution and negative ideal solution.

$$\mathrm{A}^+ = \left(v_1^+, v_2^+, \cdots, v_n^+\right) \text{ and } \mathrm{A}^- = \left(v_1^-, v_2^-, \cdots, v_n^-\right),$$

$$\text{where } v_\beta^+ = \{\max_\alpha V_{\alpha\beta} \big| \beta \in J_1; \min_\alpha V_{\alpha\beta} \big| \beta \in J_2\}$$

$$\text{and } v_\beta^- = \{\min_\alpha V_{\alpha\beta} \big| \beta \in J_1; \max_\alpha V_{\alpha\beta} \big| \beta \in J_2\}$$

where J_1 and J_2 represents the benefit criteria and cost criteria respectively.

Step-5 Compute the Euclidean distances from the positive ideal A^+ and negative ideal A^- solutions for each alternative A_α respectively:

$$d_\alpha^+ = \sqrt{\sum\nolimits_\beta \left(\Delta_{\alpha\beta}^+\right)^2} \text{ and } d_\alpha^- = \sqrt{\sum\nolimits_\beta \left(\Delta_{\alpha\beta}^-\right)^2}$$

where $\Delta_{\alpha\beta}^+ = \left(v_\beta^+ - V_{\alpha\beta}\right)$ and $\Delta_{\alpha\beta}^- = \left(v_\beta^- - V_{\alpha\beta}\right)$ with $\alpha = 1,\cdots,m$

Step-6 Compute the relative closeness Ω_α for each alternative A_α with respect to positive ideal solution A^+ as given by

$$\Omega_\alpha = d_\alpha^- \big/ \left(d_\alpha^- + d_\alpha^+\right), \text{ where } \alpha = 1,\cdots,m.$$

Palpably, $0 \le \Omega_\alpha \le 1$, where $\alpha = 1,\cdots,m$. If $\Omega_\alpha = 0$, alternative A_α would be negative ideal solution. In contrast,

$\Omega_\alpha = 1$ denotes A_α to be positive ideal solution. An alternative A_α gets closer to the negative ideal solution as Ω_α approaches 0, whereas alternative A_α gets closer to the ideal solution and farther from the negative ideal solution as Ω_α approaches 1.

III. EVALUATION FRAMWORK

In the multi-criteria decision making (MCDM) problem, a number of alternatives can determine and compared to using the different criteria. The aim of MCDM problem is to provide support to the decision-maker in the process of making the choice between alternatives. The ranking order of a set of alternatives according to their closeness coefficients and best alternative is found from the set of alternatives.

In Table-1 we define Negative Weighted distance (NWD) for each alternative, Positive Weighted distance (PWD) for each alternative, Negative Euclidean distance (NED) for each alternative and Positive Euclidean distance (PED) for each alternative. In Table-2 we define the average linguistic performance of the ten airports (i.e. alternatives) corresponding to the criteria for all experts. In Table-3 we define the linguistic weights for ten criteria for each and individual experts.

Table 1. The Euclidean and Weighted Distance Values of Ten Alternatives

Alternative	NWD	PWD	NED	PED
A_1	v_1^-	v_1^+	d_1^-	d_1^+
A_2	v_2^-	v_2^+	d_2^-	d_2^+
⋮	⋮	⋮	⋮	⋮
A_{10}	v_{10}^-	v_{10}^+	d_{10}^-	d_{10}^+

Table 2. The Average Linguistic Performance of the Ten Airports

AlT.\ Cri	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	Experts
A_1	VG	G	MG	M	ML	G	ML	VG	VG	MG	(E_1,E_2,E_3,E_4,E_5)
A_2	G	ML	MG	G	ML	VG	MG	ML	VG	MG	
A_3	M	G	VG	MG	G	ML	VG	VG	M	G	
A_4	VG	M	G	VG	VG	ML	MG	G	ML	VG	
A_5	VG	M	ML	G	VG	VG	G	VG	VG	MG	
A_6	VG	G	VG	M	MG	G	ML	G	VG	G	
A_7	VG	VG	M	VG	G	VG	VG	MG	ML	VG	
A_8	VG	VG	VG	VG	MG	VG	M	G	VG	G	
A_9	M	VG	VG	G	MG	MG	MG	G	G	VG	
A_{10}	VG	MG	G	M	M	G	MG	VG	G	VG	

Table 3. The Linguistic Weights for Ten Criteria

Cri.\Exp.	E_1	E_2	E_3	E_4	E_5
C_1	VH	H	M	L	VL
C_2	VH	VL	M	L	H
C_3	M	H	ML	VH	L
C_4	H	MH	L	H	VH
C_5	M	ML	VH	H	H
C_6	L	H	MH	L	ML
C_7	L	VH	H	MH	MH
C_8	VH	VL	H	VL	VH
C_9	M	VH	ML	H	L
C_{10}	H	ML	VH	MH	ML

IV. APPLICATION

In this section, we work out a numerical example to illustrate the TOPSIS method for decision making problem with crisp data. Assume that ten airports $A_1,\cdots,A_{10}$ are evaluated by five experts E_1, E_2, E_3, E_4 and E_5 under crisp environment for operation performance against ten criteria. Suppose that we have ten criteria $C_1,\cdots,C_{10}$ are identified and ten alternatives $A_1,\cdots,A_{10}$ are identified as the evaluation criteria for these alternatives. Ten criteria are considered: return on operation profit to capital (C_1), comfort and cleanness of airport terminals (C_2), trolleys approach travellers (C_3), aerodrome control (C_4), security measures and airport scale (C_5), aircraft take-off and loading time (C_6), traffic connecting city (C_7), courtesy of crew and parking lots (C_8), noise pollution control (C_9), and flight safety control (C_{10}). TOPSIS method is proposed for evaluating the performance of the airports, considering the different criteria and weights of the criteria. The proposed method is applied to solve this problem.

Table 4. The Decision Matrix and Weights of Ten Alternatives

Al\ Cri	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	9	8	7	6	5	8	5	9	10	7
A_2	8	5	7	8	6	9	7	5	10	7
A_3	6	8	10	7	8	5	10	9	6	8
A_4	10	6	8	10	9	6	7	8	5	9
A_5	9	6	5	8	10	9	8	9	9	7
A_6	10	8	9	6	7	8	5	8	9	8
A_7	9	10	6	9	8	10	9	7	5	9
A_8	9	9	9	9	7	9	6	8	9	8
A_9	6	9	9	8	7	7	7	8	8	9
A_{10}	9	7	8	6	6	8	7	9	8	10
Weight	0.1	0.08	0.12	0.07	0.15	0.08	0.2	0.06	0.09	0.05

Table 5. The Normalized Decision Matrix

Al\ Cri	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0.3306	0.3266	0.2789	0.2427	0.2126	0.315	0.2178	0.3519	0.3901	0.268
A_2	0.2939	0.2041	0.2789	0.3236	0.2551	0.3544	0.3049	0.1955	0.3909	0.268
A_3	0.2204	0.3266	0.3984	0.2832	0.3402	0.1969	0.4356	0.3519	0.2341	0.3063
A_4	0.3674	0.2449	0.3187	0.4046	0.3827	0.2362	0.3049	0.3128	0.1951	0.3446
A_5	0.3306	0.2449	0.1992	0.3236	0.4252	0.3544	0.3485	0.3519	0.3511	0.268
A_6	0.3674	0.3266	0.3586	0.2427	0.2977	0.315	0.2187	0.3128	0.3511	0.3063
A_7	0.3306	0.4082	0.239	0.3641	0.3402	0.3937	0.392	0.2737	0.1951	0.3446
A_8	0.3306	0.3674	0.3586	0.3641	0.2977	0.3544	0.2614	0.3128	0.3511	0.3063
A_9	0.2204	0.3674	0.3586	0.3236	0.2977	0.2756	0.3049	0.3128	0.3121	0.3446
A_{10}	0.3306	0.2858	0.3187	0.2427	0.2551	0.315	0.3049	0.3519	0.3121	0.3829

Table 6. The Weighted Normalized Decision Matrix

Al\ Cri	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0.0331	0.0261	0.0335	0.017	0.0319	0.0252	0.0436	0.0211	0.0351	0.0134
A_2	0.0294	0.0163	0.0335	0.0227	0.0383	0.0283	0.061	0.0117	0.0351	0.0134
A_3	0.022	0.0261	0.0478	0.0198	0.051	0.0157	0.0871	0.0211	0.0211	0.0153
A_4	0.0367	0.0196	0.0382	0.0283	0.0574	0.0189	0.061	0.0188	0.0176	0.0172
A_5	0.0331	0.0196	0.0239	0.0227	0.0638	0.0283	0.0697	0.0211	0.0316	0.0134
A_6	0.0367	0.0261	0.043	0.017	0.0447	0.0252	0.0436	0.0188	0.0316	0.0153
A_7	0.0331	0.0327	0.0287	0.0255	0.051	0.0315	0.0784	0.0164	0.0176	0.0172
A_8	0.0331	0.0294	0.043	0.0255	0.0447	0.0283	0.0523	0.0188	0.0316	0.0153
A_9	0.022	0.0294	0.043	0.0227	0.0447	0.022	0.061	0.0188	0.0281	0.0172
A_{10}	0.0331	0.0229	0.0382	0.017	0.0383	0.0252	0.061	0.0211	0.0281	0.0191

Table 7. Closeness Coefficients

I.S.\Alt.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
d_α^+	0.0581	0.045	0.0309	0.0382	0.0339	0.0503	0.031	0.041	0.0385	0.0419
d_α^-	0.0282	0.0314	0.0552	0.0399	0.048	0.0344	0.0485	0.0371	0.0354	0.0322

Table 8. Ranking Order

Alternative	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Ω_α	0.3266	0.4114	0.6411	0.5104	0.5866	0.4058	0.61	0.4754	0.479	0.4346
Rank	10	8	1	4	3	9	2	6	5	7

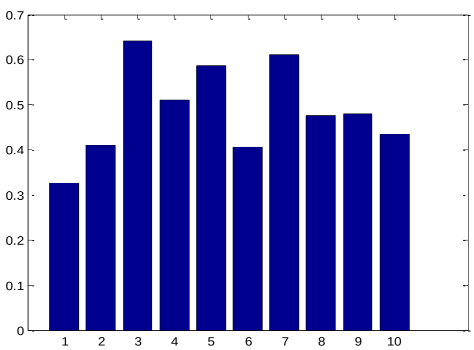

Figure 1. Ranking the Alternative with respect to Relative Closeness by applying the TOPSIS Decision Making

These data and also the vector of corresponding weight, of each criteria, the normalized decision matrix and weighted normalized decision matrix are given in Table 4, Table 5, and Table 6, respectively. The closeness coefficients, which are defined to determine the ranking order of all alternatives by calculating the distance to both the PIS and NIS, are given in Table 7. According to the closeness coefficient, ranking the order preference, order of these alternatives is also given in Table 8.

Table 8 shows the results obtained for the above example by using the proposed approach and Fig.1 shows the best airport represented by using different criteria, and finite number of alternatives. So the ranking order of 10 airports is selected as follows:

$$A_3 > A_7 > A_5 > A_4 > A_9 > A_8 > A_{10} > A_2 > A_6 > A_1$$

The best selection in the given alternatives, the selected airport is A_3.

V. CONCLUSION AND FUTURE SCOPE

MADM finds wide applications in the solution of real world decision making problem. Most MADM problems include both quantitative and qualitative criteria which are often assessed using imprecise data and human judgments. Here we provide a thorough and systematic review of the existing MADM methods. Theoretical background as well as the algorithm is presented for this method. Here, we consider the distance of PIS and NIS. i.e. the less distance from the PIS and the more distance from the NIS. In this paper, we propose a new methodology to provide a simple approach to find best alternative airport and help decision makers to select the best one.

There is enormous scope of research on TOPSIS in various directions. Several opportunities can be created involving the distance from the positive and negative solutions and the relative closeness to the ideal solution. Although several techniques have been earlier integrated with the classical TOPSIS, many other new techniques involving TOPSIS have not yet been explored. These techniques enhance the significance of classical TOPSIS in handling various new problems.

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