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Simplification of MIMO Dynamic Systems using Differentiation and Cauer Second Form

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Abstract— A simplification method for multi-inputs and multi-outputs (MIMO) dynamic system via reducing the order of the original large-scale system is presented in this paper. The common denominator of the original system is reduced by using differentiation method while the numerator coefficients are obtained by applying Cauer second form. The proposed method is computationally simple and capable to retain the properties of the original system. The viability of the proposed method has been checked via one numerical example.

***Keywords*—** Differentiation, Cauer Second Form, Order Reduction, Simplification, Stability

I. Introduction

The modelling of large-scale dynamic system plays an important role in the diverse field of science and engineering such as complex chemical processes, electrical engineering, pneumatic, thermal, mechanical, etc. and in specially in design and analysis, where control engineers quite often is faced with controlling of a physical system for which an analytic model is represented as a high order system. A mathematical model of a high order system may pose difficulties in its analysis, synthesis and identification. Therefore, it is desirable to approximate it by low order model which retains the main qualitative properties of the original high order system. Many simplification methods have been proposed to reduce the complexity of a high order system from last four decades.

Many research papers are available for simplification in frequency [1-5] and time domain [6-7]. Several methods, which are suggested by mixing two order reduction techniques, are available in the literature. In this paper, authors proposed a mixed simplification method in which denominator polynomial is obtained by differentiation technique [8] while the coefficients of numerator are obtained by using Cauer second form [9]. The advantages of the Cauer second form are to retain steady-state value and transient nature of the original system.

Advantages of using simplified models: The following are the advantages of using simplified model

- Easy understanding of the system's behaviour
- Simple and economic hardware implementation
- Less processing time
- Less computer memory required
- Optimization can be done easily
- Efficient controller design, etc.

In this paper, section –I contains introduction of the paper and advantages/contribution of the simplied models. Section-II describes the problem statement; Section-III clearly descries the proposed simplification method in details. Section-IV contains one numerical example and results. For error comparison between original and simplified model a formula is suggested. Section-V consists of conclusions and scope for future work.

II. Problem Statement

Let the n^{th} MIMO system with $'p'$ inputs and $'q'$ outputs be described by its transfer function matrix as follows:

$$[G(s)] = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1p}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{2p}(s) \\ \dots & \dots & \dots & \dots \\ g_{q1}(s) & g_{q2}(s) & \dots & g_{qp}(s) \end{bmatrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) & \dots & A_{1p}(s) \\ A_{21}(s) & A_{22}(s) & \dots & A_{2p}(s) \\ \dots & \dots & \dots & \dots \\ A_{q1}(s) & A_{q2}(s) & \dots & A_{qp}(s) \end{bmatrix} \quad (1)$$

Where $D(s)=a_{11}+a_{12}s+...+a_{1,n+1}s^n$ is the common denominator of the original transfer function matrix and numerator element $A_{ij}=a_{21}+a_{22}s+...+a_{2,n}s^{n-1}$.

The k^{th} order reduced simplified model in frequency domain with the same number of inputs and outputs is taken as

$$[R_k(s)]=\begin{bmatrix} r_{11}(s) & r_{12}(s) & ... & r_{1p}(s) \\ r_{21}(s) & r_{22}(s) & ... & r_{2p}(s) \\ ... & ... & ... & ... \\ r_{q1}(s) & r_{q2}(s) & ... & r_{qp}(s) \end{bmatrix}$$

$$=\frac{1}{D_k(s)}\begin{bmatrix} B_{11}(s) & B_{12}(s) & ... & B_{1p}(s) \\ B_{21}(s) & B_{22}(s) & ... & B_{2p}(s) \\ ... & ... & ... & ... \\ B_{q1}(s) & B_{q2}(s) & ... & B_{qp}(s) \end{bmatrix} \quad (2)$$

Where $D_k(s)=b_{11}+b_{12}s+b_{13}s^2+\ldots+b_{1,k+1}s^k$ is the common denominator of the reduced order model and the numerator element $B_{ij}=b_{21}+b_{22}s+b_{23}s^2+\ldots+b_{2,k}s^{k-1}$.

The objective of this work is to realize the simplified model in the form of equation (2) from the original system (1) such that it retains the important features of the original system (1).

III. Proposed Simplification Method

The proposed method consists of the following two steps:

Step-1: Differentiation method [8] for getting reduced denominator polynomial $D_k(s)$.

The original system consists of a denominator polynomial $D(s)$ as

$$D(s)=a_{11}+a_{12}s+a_{13}s^2+...+a_{1,n+1}s^n \quad (3)$$

The reciprocal of the $D(s)$ can be written as

$$\overset{\bullet}{D}(s)=s^nD\left(\frac{1}{s}\right)=a_{11}s^n+a_{12}s^{n-1}+...+a_{1,n+1} \quad (4)$$

The equation (4) is successively differentiated $(n-k)$ times, which yield k^{th} -order polynomial as

$$\overset{\bullet}{D}_k(s)=a_{11}s^k+a_{12}s^{k-1}+a_{13}s^{k-2}+...+a_{1,k+1} \quad (5)$$

Now, $\overset{\bullet}{D}_k(s)$ is reciprocated back to get k^{th} -order reduced denominator polynomial as

$$D_k(s)=s^k\,\overset{\bullet}{D}_k\left(\frac{1}{s}\right)=b_{11}+b_{12}s+b_{13}s^2+\ldots+b_{1,k+1}s^k \quad (6)$$

The purpose of reciprocating the polynomial is to retain the transient behaviour the system.

Step-2: Determination of the numerator $N_k(s)$ of the simplified model using Cauer second form [9].

The coefficients of Cauer second form can be evaluated by using Routh arrays as:

$$h_1=\frac{a_{11}}{a_{21}}\left\langle\begin{matrix} a_{11} & a_{12} & a_{13} & a_{1,n} & a_{1,n+1} \\ a_{21} & a_{22} & a_{23} & a_{2,n} & \end{matrix}\right.$$

$$h_2=\frac{a_{21}}{a_{31}}\left\langle\begin{matrix} a_{21} & a_{22} & a_{23} & a_{2,n} \\ a_{31} & a_{32} & a_{33} & \end{matrix}\right. \quad (7)$$

$$h_3=\frac{a_{31}}{a_{41}}\left\langle\begin{matrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{matrix}\right.$$

$\vdots$

$\vdots$

The first two rows coefficients are known and taken from the original high order system and rest of the elements are determined by the well-known Routh algorithm.

$$a_{i,j}=a_{i-2,j+1}-h_{i-2}a_{i-1,j+1} \quad (8)$$
$$i=3,4,...$$
$$j=1,2,...$$

$$h_i=\frac{a_{i,1}}{a_{i+1,1}} \qquad i=1,2,3,\ldots,k \quad (9)$$

By matching the coefficients $b_{1,j}\left(j=1,2,\ldots(k+1)\right)$ of reduced denominator and Cauer quotations $h_p\left(p=1,2,\ldots k\right)$ of equation (8), the coefficients of the reduced numerator $R_k(s)$ can be obtained. For this purpose, following inverse Routh array is used.

$$b_{i+1,1}=\frac{b_{i,1}}{h_i};i=1,2,\ldots k \quad (10)$$

$$b_{i+1,j+1}=\frac{b_{i,j+1}-b_{i+2,j}}{h_i} \quad (11)$$

for $i=1,2,\ldots(k-j)$ and $j=1,2,...(k-1)$

by applying above equations, the numerator of the reduced order model can be obtained as

$$N_k(s)=b_{21}+b_{22}s+...+b_{2,k}s^{k-1} \quad (12)$$

IV. Results and Compariosn

The proposed method for order reduction of the original system is tested on one numerical example. The comparison of the proposed method with the well- known methods is done with the help of Integral Square Error (ISE) index using SIMULINK/MATLAB.

The ISE can be defined as

$$\text{ISE} = \int_0^\infty [y(t) - y_k(t)]^2\, dt \qquad (13)$$

Where $y(t)$ and $y_k(t)$ are step response of the original and simplified model respectively.

Example: Consider a transfer function matrix of a multivariable system already taken by Shamash [10].

$$[G(s)] = \begin{bmatrix} \dfrac{s+20}{(s+1)(s+10)} \\ \dfrac{s+10}{(s+2)(s+5)} \end{bmatrix}$$

This can be rearranged as

$$[G(s)] = \frac{1}{D(s)} \begin{bmatrix} A_{11}(s) \\ A_{21}(s) \end{bmatrix}$$ where

$$D(s) = 100 + 180s + 97s^2 + 18s^3 + s^4$$

$$A_{11}(s) = 200 + 150s + 27s^2 + s^3$$

$$A_{21}(s) = 100 + 120s + 21s^2 + s^3$$

Following steps are taken to reduce the system into 2nd order system.

Step-1: Determination of 2nd –order denominator of the simplified 2nd –order model:

Applying Step-1 of the proposed simplification method 2nd –order denominator is obtained as

$$D_2(s) = 194s^2 + 1080s + 1200$$

Step-2: Determination of numerator coefficients of the simplified model using Cauer second form:

Routh array for getting h coefficients as:

$$h_1 = 0.5 \left\langle \begin{array}{lllll} 100 & 180 & 97 & 18 & 1 \\ 200 & 150 & 27 & 1 & \end{array} \right.$$

$$h_2 = 1.9048 \left\langle \begin{array}{llll} 200 & 150 & 27 & 1 \\ 105 & 83.5 & 17.5 & \end{array} \right.$$

$\vdots$

Inverse Routh array can be constructed through h_1 and h_2 as

$$h_1 = 0.5 \left\langle \begin{array}{lll} 1200 & 1080 & 194 \\ 2400 & -359.94 & \end{array} \right.$$

$$h_2 = 1.9048 \left\langle \begin{array}{ll} 2400 & -359.94 \\ 1259.97 & \end{array} \right.$$

$\vdots$

Hence, $B_{11}(s) = 2400 - 359.94s$

Similarly, Routh array for getting h coefficients to realize $B_{21}(s)$

$$h_1 = 1 \left\langle \begin{array}{lllll} 100 & 180 & 97 & 18 & 1 \\ 100 & 120 & 21 & 1 & \end{array} \right.$$

$$h_2 = 1.667 \left\langle \begin{array}{llll} 100 & 120 & 21 & 1 \\ 60 & 76 & & \end{array} \right.$$

$\vdots$

Inverse Routh array can be constructed through h_1 and h_2 as

$$h_1 = 1 \left\langle \begin{array}{lll} 1200 & 1080 & 194 \\ 1200 & 360.14 & \end{array} \right.$$

$$h_2 = 1.667 \left\langle \begin{array}{ll} 1200 & 360.14 \\ 719.86 & \end{array} \right.$$

$\vdots$

Hence, $B_{21}(s) = 1200 + 360.4s$

Finally, 2nd –order reduced simplified model is obtained as

$$[R_2(s)] = \frac{1}{194s^2 + 1080s + 1200} \begin{bmatrix} 2400 - 359.94s \\ 1200 + 360.4s \end{bmatrix}$$

The step response and frequency response characteristics are compared graphically and shown in figure 1 and 2 respectively.

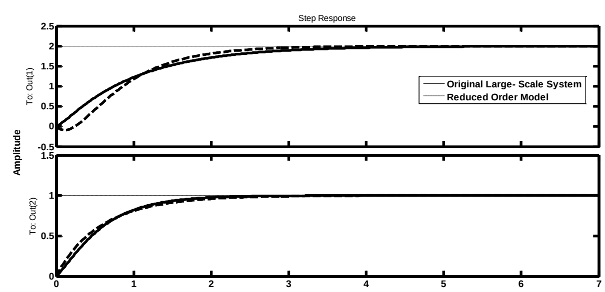

Fig.1 Step response comparison between original and reduced model

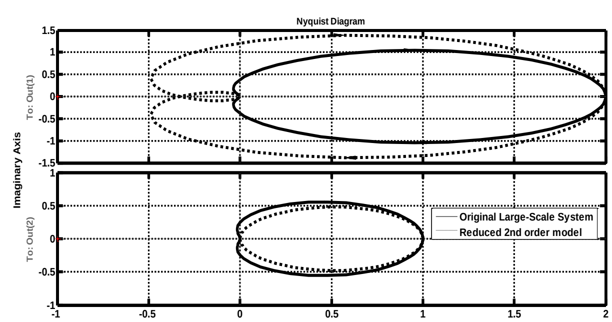

Fig.2 Frequency response comparison between original and reduced model

Table-1 Comparison of Time Response Specifications

Model	**Output (1)**			**Output (2)**		
	Rise Time (Second)	**Settling Time (Sec.)**	**Steady State Value**	**Rise Time (Second)**	**Settling Time (Sec.)**	**Steady State Value**
Original System	2.21	3.97	2	1.2	2.1	1
Reduced Model	1.57	2.97	2	1.36	2.46	1

Table-2 Error Comparison between Original and Reduced model

Reduction Method	**Output (1)**	**Output (2)**
	ISE	
Proposed Method	0.078	0.00242
C.B. Vishwakarma [11]	0.054	0.00738

From the above comparison, it may be concluded that time response specifications of simplified model is near to the original model's specification and hence, the simplified model may be taken in place of the original system for analysis and controller design. The ISE index is also comparable with other method shown in Table-2. The low value of ISE indicates that simplified model response is near to the response of the original system.

V. Conclusions and Future Scope

The author proposed a mixed method for reducing the order of MIMO linear dynamic systems. The numerator of the reduced MIMO system is computed by Cauer second form while the common reduced denominator is obtained by differentiation method. The proposed has been tested on the MIMO system of the 4th order. The second order simplified model is obtained by the proposed method as shown in the example. The time and frequency response comparison between the original and simplified model is graphically shown in figure 1 and 2. The time response specifications and ISE index comparison between the original and simplified model are tabulated in Table 1 and 2 respectively. Finally, It may be concluded that the proposed method is capable to retain transient and steady-state behaviour of the original system in the simplified model.

The proposed work may be extended by using evolutionary optimization techniques such as Genetic Algorithm (GA), Ant Colony Optimization (ACO), etc.

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