

Apex Magic Square Labeling for Complete Bipartite Graph and their application using Enigma

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Available online at: www.ijcseonline.org

Accepted: 24/May/2018, Published: 31/May/2018

Abstract: Achieving better phases of security in transferring an image over LAN or internet and performance of enigma is depends on its efficiency based on time taken and the way it generates different graphs from a pixel. The most widely used algorithms are RSA, public key algorithm and other such algorithms does not provide full security to the cipher text, RSA is not going to assure the enigma while transforming the images. A better approach is to use Apex magic total Labeling (AMTL) of complete graph. It increases the efficiency by adding more extra security to the enigma systems. This method provides more security due to its difficulty in encryption in AMTL formation that cannot be easily found. This proposed research work provides an additional level of security to public key algorithms such as RSA, DH algorithm, etc. this approach is experimented in simulated systems like MATLAB.

Keywords–RSA, bipartitegraph, Apexmagic square, Enigma, SAMTL

I. INTRODUCTION

A graph whose vertices can divided into two disjoint sets such that every edge connects a vertex. Let $G(A,E)$ is graph where A is called the apex set or node set, while set E is the edge set of graph G . A complete bipartite graph is a graph whose vertices is partitioned into two upset sets such that more than one graph vertices should not adjacent to same set. A graph labeling is the naming of labels, traditionally represented by integers, to the edges, or both, of a graph. Apex labeling is a function of E to a set of labels. A graph with such a function defined is called an Apex-labeled graph. Likewise, an edge labeling is a function mapping E to a set of labels. In this case, G is called an edge-labeled graph.

The labeling of a graph is any adjusting the maps to some set of graph elements to a set of numbers (usually positive or non negative integers). If the area is the edge set then it is called edge labeling. If the area is the vertex set then it is called apex labeling. If the field consists of both edge set and apex set then it is called as total labeling.

An edge magic labeling is one in which the sum of all labels with respect to each vertex is a constant with respect to other edge. Similarly apex magic labeling is defined. AMTL is a one-to-many mapping $f: A \cup E \rightarrow \{1, 2, \dots, m+n\}$. There

will be a constant k such that at any apex $m, f(m) + \sum f(m, n) = k$, where the sum is over all apex n which is adjacent to m . For any labeling the weighted graph is associates a value with every edge in the graph, sum of all the labels at a apex is called as the weight of the apex, denoted by $wf(m)$, hence for VMTL we require that all vertices weight must be the same, namely k is called the magic number for the labeling.

Edge magic total labeling on a graph with apex 'a' and edges 'e' will defined as one-to-one mapping taking the apex and edges onto the integers $1, 2, \dots, a+e$, with the property that the sum of all the label on an edges is constant.

Graph coloring is nothing but naming of labels called colors to the apex of graph such that no two adjacent vertices share the same color this is called a vertex coloring. Similarly, an edge coloring is naming of colors to each edge so that no two adjacent edges share the same color, and the main idea of face coloring is to assign each face of a graph a color such that two faces must get different color but it share same edge.

Cryptography is the study of techniques for protecting communication in the presence of third parties. It is also responsible for converting ordinary message to secure message and vice versa. More generally, cryptography is process of forming and analyzing protocols that prevent unauthorized parties from accessing private messages.

Various aspects of an information security such as data confidentiality, data integrity, authentication.

Here data is cipher by using the cryptography and appears as irregular stream of the data to the third party by using a technique called substitution and transposition but the reality of the message is known to the hacker, where as in technique of Enigma the existence of the message is not evident. As defined by Cachin, Enigma is the art and science of communicating in such a way that the presence of an images cannot be detected.

Here the shared image must only be visible to a subset of users known as the authorized users. We shall attempt to make the secret shares verifiable using the concept of colored graphs (Apex magic labeled graphs). The main idea is that each colored graph can be inserted into selected pixel of images, not only the colored graph other graphs also inserted into pixels of images, then that images can be send to across the network.

The public key cryptography algorithms are not going to assure the enigma while transforming the images. But the images can secure with the help of one-to-one maps onto the appropriate set.

The RSA, the widely used public key algorithm does not guarantee full security. RSA mainly deals with encrypting individual images. The AMTL (Apex Magic Total Labeling) table approach acts as an add-on to the existing RSA algorithm. It acts like a wrapper. The objective of using AMTL is to increase the complexity.

The existing work done by using LSB techniques such as substitution method for Enigma. There are various techniques of Enigma but the images are mostly used to hide or embed the secret information. These types of techniques are known as image Enigma techniques. Image Enigma method can be categorized in to two groups i.e. the transform domain technique group and the spatial domain technique group. In the Spatial domain method the intensity values of the pixels of the image are used to hide the information directly, while in the transform domain methods frequency domain of images which are previously transformed, used to embed the information.

In the rest of the paper, organized as follows: In section II Literature survey, in section III methodology, in section IV Algorithm, in section V Implementation and in section VI conclusion.

II. LITERATURE SURVEY

In a labeling of a graph $G(V, E)$ is a mapping from the set of edges, vertices or both edges and Vertices to the set of labels. The different types of labeling are Apex labeling, edge

labeling and total labeling. The values considered for the labeling are only positive values.

When you consider $n \times n$ matrix, Magic square is the arrangement of 1 to n numbers such that no numbers are repeated in the $n \times n$ matrix. Different methods for the construction of Magic Square were proposed and found in [1].

As time passed M.Miller [2] described magic labeling as Super Magic when consecutive integers are used to denote edge labels and shown below. Later Super Apex Magic Labeling.

Magic labeling was introduced by Slamir in 1963[3]. In magic labeling the sum of labels related to an Apex (an Apex magic labeling) or to an edge (an edge magic labeling) are to be constant all over the graph. In an anti-magic labeling all the sums (weights) are different according to the conjecture by K.A.Sugeng and W.Xie [4].

In [6] Gopinath Ganapathy and K. Mani showed how to achieve high level of security by using magic squares of order 16. Krishnappa H.K., N.K.Srinath and Ramakanth Kumar P. [5] showed that how to use magic squares to realize VMTL of complete graph K_n . After proving that labeling is available for different graphs, we shall go on to display a practical application of graph labeling in the field of cryptography.

Algorithms for Apex Magic Total Labeling on complete graphs for K_n when n is odd and K_n when n is even considering 3 variations of N i.e. $N = 2 \pmod{4}$, $N = 4 \pmod{8}$, $N = 0 \pmod{8}$ in [2]. In Apex labeling matrix, the columns, according to the existing algorithm [7] add up to single magic constant.

Super edge magic total labeling is discussed in [4]. In this three issues are addressed that when super edge magic total labeling can be done for complete graph K_n . For complete graph K_n , the Apex magic labeling can be done when n is odd, and $n = 0 \pmod{4}$ and $n > 4$. SAMTL cannot be done for the condition $n = 2 \pmod{4}$.

Possible magic constants for a given complete graph will range from $(n(n^2+3))/4$ to $(n(n+1)^2)/4$. Hence the maximum magic constant is $(n(n+1)^2)/4$. A theorem is proposed to prove that, when n is odd SAMTL can be done. In this algorithm, in a matrix of $n \times n$, the elements are filled in a fashion that filling process proceeds in down-right manner. If the present position is already filled, then we have to move back and then again start filling with same procedure. When all elements of S , where $S = n + m$, are filled $n(n-1)/2$ boxes will be empty. These empty spaces are filled with the symmetry of other side elements $M[i,j]=M[j,i]$. The

examples like K3, K5 will show that SAMTL can be made for complete graphs with n is odd.

Similarly, for $n \equiv 0 \pmod{4}$, $n > 4$, the maximum magic constant $(n(n+1)^2)/4$ will be evaluated to integer hence for this condition of n , SAMTL can be done whereas for $n \equiv 2 \pmod{4}$, SAMTL cannot be done, because the maximum magic constant $(n(n+1)^2)/4$ will evaluate to non-negative integer.

A process of finding and relating particular groups or communities is done in [6]. Here the communities which are not much detected by Yahoo or other search engines are first detected. These groups or communities though not familiar to all, but it may contain any valuable or reliable information in it. Hence detecting such communities will serve a purpose of finding information. Also these groups may become well-known in future. For this type of extraction, graph theory is used. Dense Bipartite Graphs (DBG) are used in this work.

Edge Magic Total Labeling [EMTL] have been studied by Wallis, Baskoro, Miller and Slamin [11]. Krishnappa.H.K. N.K.Srinath, Ramakanth Kumar.P and Manjunath.S proposed an algorithm to construct EMTL for $K_{m,n}$.

In [11] R. Rajeswari, R. Parameshwari, proposed Total Magic Cordial Labeling of Star related graphs, method is used to achieve the labeling if a graph. They also derived and proved the Star, bistar, split of star and bistar are total magic cordial graph.

In [12] Hao Lu; Mahantesh Halappanavar, Daniel Chavarria-Miranda, proposed an algorithm for balanced coloring models in the context of parallel computing. The goal is to achieve a balanced coloring of an input graph without increasing the number of colors that an algorithm oblivious to balance would have used.

In [13] A Gupta, E Sharma, N Sachan, N Tiwari, proposed Door Lock System through Face recognition using MATLAB. The goal is to every image in the training set can be represented as a linear combination of weighted eigenvectors, This paper addresses the building of face recognition system by using Principal Component Analysis (PCA) method. PCA is a statistical approach used for reducing the number of variables in face recognition.

In [14] M Bandyopadhyaya, S Chattopadhyaya, A Das, Emphasis on Genetic Algorithm over Different PLD Tuning Methods using MATAB, This paper presents details on the algorithm and implementation, including the major components in our design: recombination, mutation, fitness function. The algorithm was implemented with Genetic

Algorithm tool in MATLAB R2010 for performance evaluation.

III. METHODOLOGY

The primary data for this study is mainly focused on finding classes of graphs which admits a particular type of labelling. Here we consider a particular class of graph which demonstrates Apex magic square labelling. The classes we considered here is a complete graph. Various graph labeling that generalise the idea of a magic square have been discussed.

In particular Apex magic square labelling on a graph with v vertices and e edges will be defined as a one to many map taking the vertices and edges onto the integers. The magic square in normal is represented using 4×4 matrixes. The required total across horizontal, vertical and diagonal squares at corner and centre should produce same required total.

The required total should deduct with constant key value, the result divide by 4 and the result called as quotient (n). the quotient n should fill up the bottom right square fill up diagonal square from right bottom incremented each square by 5, fill up left bottom square by adding 3 to quotient N i.e. $N+3$ fill up diagonal square left bottom incrementing each square by 3, The remaining blank square to be filled starting from first row with quotient+1 i.e. $N+1$.(we have to move from left to right for each row) skip the square if already filled, after filling all the square the resultant matrix is magic square .the summation of each row, column and diagonal is equal to required total.

IV. ALGORITHM

Apex_Magic_Square_Labeling $K_n(\text{int } n)$

This algorithm takes integer parameter n , here n indicates the number of apex of complete graph K_n . This fills the $n \times n$ matrix by the number 1 to M , Sum of Label of each apex is going to be same for all the vertices.

Step 1: compute the number of edges

$$m \leftarrow n(n-1)/2$$

Where m is the number of vertices, n is the number of edges.

Step 2: consider a key value

$$\text{Find finalvalue}(s) = (\text{Targetvalue}-\text{keyvalue}) / (\text{number of rows}(\text{or number of columns}))$$

Step 3: Initialize diagonal 1 value as

$$D_1_val=30;$$

And diagonal 2 value as

$$D_2_val=33$$

And non_diagonal_val=30;

//for diagonal 1 and put the value

Step 4: Repeat $i \leftarrow \text{row}-1$ to $i \geq 0$
 Decrement $i --$ and $j --$
 $j \leftarrow \text{col}-1$ to $j \geq 0$
 Then if $(i == j)$ then
 $\text{Matrix}[i][j] = D_1_val$;
 $D_1_val = D_1_val + 5$;

Step 5: // For diagonal 2 and put the value
 If $(j == (\text{col} - i) - 1)$
 Then $\text{matrix}[i][j] = D - 2_val$;
 $D_2_val = d_2_val + 3$;

Step 6: //For non diagonal value
 for $i \leftarrow 0$ to $i < \text{row}$
 for $j \leftarrow 0$ to $j < \text{col}$
 Then if $(i == 0 \ \&\& \ j < 0)$
 Then continue
 Then increment non-diagonal value
 if $(\text{matrix}[i][j] == 0)$
 Then $\text{matrix}[i][j] = \text{non_diagonal_val}$

Step 7: for printing the matrix
 for $i \leftarrow 0$ to $i < \text{row}$
 for $j \leftarrow 0$ to $j < \text{col}$
 Then print $\text{matrix}[i][j]$

Step 8: stop

V. IMPLIMENTAION

To design an algorithm that will optimally find the sets of rows and columns and diagonals that will result in various magic constants of complete graph and complete bi- partite graph. The different ways in which magic constant is achieved in Apex magic square by using 4×4 matrix but here care should be taken it doesn't affect magic constant obtained for apex square of complete graph.

45	31	32	42
34	40	39	37
38	36	35	41
33	43	44	30

Table 1: The matrix of labels for K_n . The row has the vertex labels and the column has the edge i to $n - 1$.

The magic square in normal is represented using 4×4 matrixes. 16 consecutive numbers have to be used to fill up 16 squares so as to produce same required total across horizontal, vertical and diagonal squares at corner and centre should produce same required total.

A more suitable algorithm using techniques like Dynamic Programming or Brute force algorithm to obtain the above results by considering the efficiency.

Provide high level of security by encrypting /decrypting and hiding the secret information in images. This is achieved by the following steps:

- A. At the sender side :**
- To encrypt the given image which has to be sent in a secured way we first obtain the pixel position of a given image.
 - This pixel is considered as a position to insert the graph.
 - The required graph is encrypted and send across the network.

B. At the receiver end:

The decryption takes place in the reverse order i.e.:

- The receiver will receive the image containing the graphs.
- In the image, the encrypted pixels can be choosed.
- The selected pixels can be taken and it is decrypted.

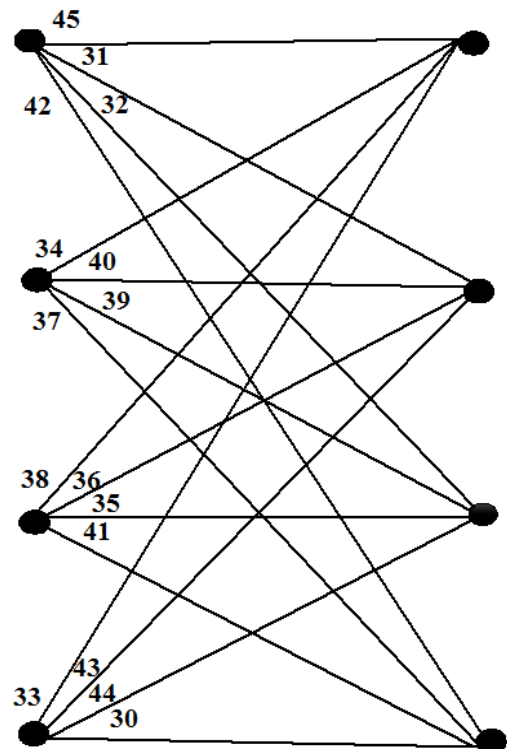


Figure 1. The construction of the labeling for K_n

VI. CONCLUSION

In this paper we proposed an approach for finding classes of graphs which admits a particular type of labeling and optimally find the sets of rows and columns and diagonals that will result in various magic constants of complete graph and complete bi-partite graph. This will add on one more layer security, the magic square in normal is represented using $n \times n$ matrix. n consecutive number have to be used to fill up n squares so as to produce same required total across horizontal, vertical and diagonal squares at corner and centre should produce same required total.

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