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Reverse -Magic Graphoidal on Circle Related Graphs

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Abstract: Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G. Define f^* in ψ with $f^*(P) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where f^* is the induced labeling on ψ. Then, we say that G admits ψ - magic graphoidal total labeling of G. In this paper we formulated a reverse process of magic graphoidal called *reverse-magic graphoidal labeling* and proved C_n, Parachute $W_{n,2}$, Armed Crown $C_n \Theta P_n$, $K_{1,n} \times K_2$ are reverse magic graphoidal.

Keywords: Graphoidal Constant, Graphoidal Cover, Magic Graphoidal, reverse magic graphoidal.

1. INTRODUCTION

B.D. Acharya and E. Sampath Kumar[1] defined Graphoidal cover as partition of edge set of G in to internally disjoint paths (not necessarily open). The maximum cardinality of such cover is known as graphoidal covering number of G.

A graph $G = (V, E)$ is said to be magic if there exist a bijection $f: V \cup E \rightarrow \{1,2,3 \dots \dots . m + n\}$; where '$n$' is the number of vertices and 'm' is the number of edges of a graph. Such that for every path $P = \{v_1, v_2, \dots \dots, v_n\}$ in ψ . A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labelling of *G*.

From the paper B.D.Acharya, Sampathkumar [1],and S.Sarief Basha [9], combine these two definitions we lead to form the reverse process of magic graphoidal total labeling which is called reverse-magic graphoidal total labeling or reverse-magic graphoidal labeling. From here we introduced a new type of (ie. Reverse) magic graphoidal labeling (rmg) .

II. RELATED WORK

We framed many work relating reverse magic graphoidal labelling. We proved Path, Star, Comb, $[P_n: S_1]$, star related like $[P_n: S_2]$, Double Crowned star $K_{1,n} \odot 2K_1$, $< K_{1,n} : n >$, graph $K_2 + mK_1$, 0-constant reverse magic graphoidal like Binary tree and Coconut tree, and some certain graph like Bistar graph and Twig graph are reverse magic graphoidal.

III. BASIC DEFINITIONS

Definition 3.1

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges

Definition 3.2

A ***Cycle*** $\boldsymbol{C_n}$ is a closed path of length atleast 1 with n vertices

Definition 3.3

An ***Armed crown*** $C_n \theta P_n$ is a graph obtained from a cycle C_n by attaching a path P_n at each vertex of C_n.

Definition 3.4

The ***Direct product*** $K_{1,n} \times K_2$,whose vertex set is $V(K_{1,n}) \times V(K_2)$ and for which vertices (x,y) and (x',y') are adjacent precisely if $(xx') \in E(K_{1,n})$ and $(yy') \in E(K_2)$.

IV. MAIN RESULTS

Definition 4. 1

A reverse magic graphoidal labeling of a graph G is one-to-one map f from $V(G) \cup E(G) \to \{1,2,3, \ldots \ldots \ldots, m+n\}$, where *'n'* is the number of vertices of a graph and *'m'* is the number of the edges of a graph, with the property that , there is an integer constant *'μ''* such that

$f^*(P) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgC}$, is a contant

Then the reverse methodology of magic graphoidal labeling is called reverse magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse magic graphoidal graph.(rmgg).

Theorem 4.1

Every cycle C_n is a reverse magic graphoidal for $n \geq 3$

Proof:

Let $V(C_n) = \{v_i ; \quad 1 \leq i \leq n\}$

And $E(C_n) = \{v_1 v_2 v_3 v_4 \ldots \ldots . v_n v_1\}$

Let $\psi = \{P = (v_1 v_2 v_3 v_4 \ldots \ldots . v_n v_1)\}$

Define $f: V \cup E \to \{1,2, \ldots \ldots, m+n\}$ by

$$m + n = 2n$$

$$f(v_1) = 2n$$

$$f(v_i v_{i+1}) = i; \quad 1 \leq i \leq n-1$$

$$= \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2}$$

$$f(v_n\, v_1) = n$$

$$f^*(P) = f(v_i v_{i+1}) + f(v_n v_1) - \{f(v_1)\}$$

$$= \frac{n(n-1)}{2} + n - \{2n\}$$

$$= \frac{n(n-1)}{2} - n = \frac{n^2 - n + 2n}{2} = \frac{n^2 - 3}{2}$$

$$= \frac{n(n-3)}{2} = \mu_{rmgc} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

From equation (1), we conclude that G admits ψ -revere magic graphoidal labeling. The reverse magic graphoidal constant μ_{rmgc} of C_n is $\frac{n(n-3)}{2}$. Hence C_n $(n \geq 3)$ is a reverse magic graphoidal.

Theorem 4.2

Parachute $W_{n,2} = P_{2,n-2}$ is reverse magic graphoidal for$n > 2$.

proof :

Let G be the Parachute $W_{n,2}$.

Let $V(G) = u_i$; $1 \leq i \leq n+1$

And $E(G) = \{(u_i u_{i+1} ;\ 1 \leq i \leq n) \cup (u_{n+1}u_1) \cup (u_1u_3)\}$

Define $f: V \cup E \rightarrow \{1,2, \ldots \ldots, m+n\}$ by

$f(u_1) = 1$

$f(u_3) = 2n + 3$

$f(u_iu_{i+1}) = i + 1;$ $1 \leq i \leq n$

$f(u_{n+1}u_1) = n + 2$

$f(u_1u_3) = n + 3$

Let $\psi = \{P = (u_iu_{i+1} \ldots\ldots u_{n-1}u_nu_{n+1}u_1u_3)\}$

So,

$$f^*(P) = f(u_iu_{i+1}) + + f(u_{i+1}u_{i+2}) + \cdots \ldots \ldots \ldots + f(u_{n+1}u_1) + f(u_1u_3) - \{f(u_1) + f(u_3)\}$$

$$1 \leq i \leq n$$

$$= i + 1 + i + 2 + \cdots \ldots \ldots \ldots . + n + (n+1) + (n+2) + (n+3) - \{1 + 2n + 3\}$$

$$= \sum_{i=2}^{n+3} i - \{2n + 4\}$$

$$= \frac{(n+3)(n+4)}{2} - 1 - (2n+4)$$

$$= \frac{n^2+7n+12-2-4n-8}{2} = \frac{n^2+3n+2}{2} = \mu_{rmgc} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

From (1), we conclude that G admits ψ - reverse magic graphoidal labeling. The reverse magic graphoidal constant μ_{rmgc} of parachute $W_{n,2}$ is $\frac{n^2+3n+2}{2}$. Hence parachute $W_{n,2}$ is reverse magic graphoidal.

Theorem 4.3

An Armed Crown is a reverse magic graphoidal

Proof :

Let G be an armed crown

Let $V(G) = u_i$; $0 \leq i \leq n^2$

$$E(G) = \begin{cases} (u_i\, u_{i+1}), (u_{i+1}\, u_{n+i}), (u_{n+i}\, u_{2n+i}), \ldots, u_{n(n-2)+i} u_{n(n-1)+i}); 1 \leq i \leq n-1 \\ (u_n\, u_1), (u_1\, u_{2n}), (u_{2n}\, u_{3n}), \ldots\ldots (u_{n(n-1)}\, u_{n^2}); \end{cases}$$

Here, $m + n = 2n^2$

Define $f: V \cup E \rightarrow \{1,2, \ldots \ldots, 2n^2\}$ by

$f(u_i) = i$; $1 \leq i \leq n-1$

$f(u_{n(n-1)+i}) = 2n^2 + 1 - i$; $1 \leq i \leq n-1$

$f(u_{n^2}) = 2n^2 - n + 1$; $1 \leq i \leq n-1$

$f(u_i\, u_{i+1}) = n + i$; $1 \leq i \leq n-1$

$f(u_{i+1}\, u_{n+i}) = 2n + i$; $1 \leq i \leq n-1$

$f(u_{n+i} u_{2n+i}) = 3n + i$; $1 \leq i \leq n-1$

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$f(u_{n(n-2)+i}\, u_{n(n-1)+i}) = 2n^2 - 1 - (n-1)\, i$; $1 \leq i \leq n-1$

$f(u_n\, u_1) = 2n$

$f(u_1\, u_{2n}) = 3n$

$f(u_{2n}\, u_{3n}) = 4n$

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$f(u_{n(n-1)} u_{n^2}) = 2n^2 - 1 - n(n-1)$

Let $\psi = \{ P_1 = (u_i\, u_{i+1}) \cup (u_{i+1} u_{n+i}) \cup (u_{n+i}\ u_{2n+i}), \ldots . \cup (u_{n(n-2)+i}\, u_{n(n-1)+i})$;

$1 \leq i \leq n-1$

$$P_2 = (u_n u_1) \cup (u_1 u_{2n} \cup (u_{2n} u_{3n}) \cup \ldots \ldots \cup \left(u_{(n-1)i}\, u_{ni}\right); \quad 1 \leq i \leq n-1\}$$

So,

$$f^*(P_1) = \{f(u_i\, u_{i+1}) + f(u_{i+1} u_{n+i}) + f(u_{n+i} u_{2n+i}) + \cdots \ldots.. + f(u_{n(n-2)+i} u_{n(n-1)+i})\} - \{f(u_i) + f(u_{n(n-1)+i})\}$$

$$= n + i + 2n + i + 3\, n + i + .. + (n-1)n + i + 2n^2 - 1 - (n-1)i - \{\, i + 2n^2 + 1 - i)$$

$$= n + 2n + 3n + \ldots \ldots n(n-1) + i + i + \cdots \ldots .. + i + 2n^2 - 1 - (n-1)i - \{2n^2 + 1\}$$

$$= n\big(1 + 2 + \ldots + (n-1)\big) + (n-1)i + 2n^2 - 1 - (n+1)i - \{2_n{}^2 + 1\}\,;\ 1 \leq i \leq n-1$$

$$= \frac{n[n(n-1)]}{2} + 2n^2 - 1 - \{2n^2 + 1\}$$

$$= \frac{n^2(n-1)}{2} + 2n^2 - 1 - \{2n^2 + 1\}$$

$$= \frac{n^3 - n^2 + 4n^2 - 2 - 4n^2 - 2}{2}$$

$$= \frac{n3 - n2 - 4}{2} = \mu_{\,rmgc} \underline{\qquad\qquad\qquad\qquad\qquad\qquad} (1)$$

$$f^*(P_2) = f(u_n\, u_1) + f(u_1 u_{2n}) - f(u_{2n}\, u_{3n}) +) \ldots \ldots \ldots + f(u_{(n-1)n}\, u_{n^2})$$

$$-\{f(u_n) + f(u_{n2})\}$$

$$= \{2n + 3n + 4n + \cdots \ldots .. + n^2 + 2n^2 - 1 - n\,(n-1)\} - \{\, i + 2n^2 + 1 - i\}$$

$$= n + (2 + 3 + 4 \ldots \ldots + n) + 2n^2 - 1 - n(n-1) - (2n^2 + 1)$$

$$= n\ \Big[\frac{n(n-1)}{2} - 1\Big] + n^2 + n - 1 - (2n^2 + 1)$$

$$= \frac{n^3 + n^2 - 2n + 2n - 2}{2} - 2n^2 + 1$$

$$= \frac{n^3 + 3n^2 - 2}{2} - 2n^2 + 1$$

$$= \frac{n^3 - 3n^2 - 2 - 4n^2 - 2}{2}$$

$$= \frac{n^3 - n^2 - 4}{2} = \mu_{rmgc} \underline{\qquad\qquad\qquad\qquad\qquad\qquad} (2)$$

From the above equation (1) & (2)we conclude that G admits ψ - revere magic graphoidal total labeling. The reverse magic graphoidal constant μ_{rmgc} of armed crown is $\frac{n^3 - n^2 - 4}{2}$. Hence binary tree is reverse magic graphoidal.

Theorem 4.4

The graph $K_{1,n} \times K_2$ is reverse magic graphoidal.

Proof

Let G be the graph $K_{1,n} \times K_2$

Let $V(G) = \{a, b, a_i, b_i\}$; $1 \le i \le n$

And $E(G) = \begin{cases} (ab) \\ [(aa_i) \cup (bb_i) \cup (a_i b_i); \quad 1 \le i \le n] \end{cases}$

Define $f: V \cup E \rightarrow \{1,2,\dots\dots, m+n\}$ by

$f(a) = 5n + 2$

$f(b) = 5n + 3$

$f(ab) = 5n + 1$

$f(aa_i) = i$; $1 \le i \le n$

$f(a_i b_i) = n + i$; $1 \le i \le n$

$f(b_i b) = 4n + 1 - 2i$; $1 \le i \le n$

Let $\psi = \{ P_1 = (aa_i b_i b; \quad 1 \le i \le n), \quad P_2 = (ab) \}$

So,

$f^*(P_1) = f(aa_i) + + f(a_i b_i) + f(b_i b) - \{f(a) + f(b)\}$

$= i + n + i + 4n + 1 - 2i - \{5n + 2 + 5n + 3\}$

$= 5n + 1 - \{10n + 5\}$

$= -5n - 4$

$= -(5n + 4) = \mu_{rmgc}$ ____________________(1)

$f^*(P_2) = f(ab) - \{f(a) + f(b)\}$

$= 5n + 1 - \{5n + 2 + 5n + 3\}$

$= -5n - 4$

$= -(5n + 4) = \mu_{rmgc}$ ____________________(2)

From (1) & (2) we conclude that G admits ψ - reverse magic graphoidal labeling. The reverse magic graphoidal constant μ_{rmgc} of $K_{1,n} \times K_2$ is $-(5n + 4)$. Hence $K_{1,n} \times K_2$ is reverse magic graphoidal.

V. CONCLUSION

The graphoidal labeling is one the most important techniques in graph theory. As all the graphs graphoidal techniques is very interesting to investigate graphs or graph families which admit reverse graphoidal labeling. We have reported reverse graphoidal labeling of various graphs.

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