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Hybrid Quantum - Classical AI based approach to solve the Traveling Tournament Problem

Desmond Lobo[1*], Kalpita Wagaskar[2], Sarang Gawane[3], Clifford Fernandes[4]

[1,2,3,4]Department of Computer Engineering, Don Bosco Institute of Technology, Mumbai, India

[*]Corresponding Author: desmondlb@gmail.com, Tel.: +919920589379

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Abstract— Scheduling has always been deemed as a perfunctory task for most organizations. It could nevertheless become an extremely arduous task if involves the management of events and of the dynamic variables that control it. Herein, we present an idea/solution to the Travelling Tournament Problem (TTP), which is a unique combinatorial problem tackling both feasibility and optimality of the solution. As the TTP belongs to the category of NP-Hard problems, generating solutions usually tend to be extremely costly, with most of the solutions having a time complexity of as high as O(n!). However, two of the many burgeoning fields, artificial intelligence and quantum computing are making headway and we believe that quantum computers possess enough potential to be competent enough to solve such scheduling problems. The aforementioned technologies have provided us with an excellent framework, which we have consequently adopted in our implementation. In the following paper, we portray a hybrid solution that utilizes the immense computational power of a quantum computer while also tweaking the classical algorithm to dynamically reduce the number of iterations and subsequently the cost. We draw parallels between the Travelling Salesman Problem (TSP) and the TTP by utilizing the insights obtained from a detailed analysis of the existing Simulated Annealing based approach, and thus propose certain unique modifications to the best known classical only solutions.

Keywords— Quantum Computing, NP-Hard Problem, Travelling Tournament Problem, Simulated annealing.

I. INTRODUCTION

Sports League competitions are held more often than not on a global scale. Howbeit, they are subject to a rich profusion of conflicting problems primarily arising due to constraints such as venue, time, distance, cost and other unforeseeable constraints. Developing a schedule for the same becomes an increasingly daunting task even for a small number of teams courtesy of the above-mentioned constraints. An increase in the number of teams further compounds this problem due to the increased number of iterations. As stated in our previous paper [1], we have finalized on the Traveling Tournament Problem (TTP) as an abstraction of the various sports scheduling problems. The TTP, though apparently simple, is indeed deceptive as the complexity scales exponentially and for some subroutines, it does so factorially. It is therefore a problem from the NP-hard category, even amongst which it holds a rather unique place due to its immensely convoluted nature. The solution for TTP becomes particularly difficult to achieve due to the fact that it aims to achieve a feasible and an optimal solution both at the same time. Simulated annealing (SA) is primarily based on a random-seek method which exploits an analogy between the manner in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system; it forms the basis of an optimization method for combinatorial and other problems. A simulated annealing algorithm for the TTP (Traveling Tournament Problem) explores feasible and infeasible schedules, to escape local minima at very low temperatures. It can address models which are highly nonlinear, chaotic and noisy data and lots of constraints. It is a robust and general technique. Its primary benefits over different local search techniques are its flexibility and its ability to approach global optimality

In this paper, we propose an approach in which the classical algorithm is tailored to suit the Travelling tournament problem and is further enhanced with quantum circuits. Various runs of the classical algorithm suggested the presence of multiple TSP routes in the optimal schedule. It can therefore be concluded that a schedule built using the TSP routes will be fairly close in terms of the cost to the optimal solution and can be used to modify the Simulated Annealing algorithm. The Quantum Computing approach to solve the Travelling Salesman Problem is known to be quadratically faster when compared to the best-known classical implementation. We intend to use this quantum circuit to build a benchmark schedule will then be used to modify the classical TTSA. The modifications are intended to guide the search towards the optimal solution and simultaneously reduce the number of iterations it takes. This in turn will reduce the execution time.

II. RELATED WORK

This section contains the survey of different solutions made to find an approach that solves the given problem in the best way possible without irrationally inflating the complexities. We found that the Simulated Annealing approach aces as compared to others. Furthermore, we also provide a brief introduction to quantum algorithms and their relevance.

The Travelling Tournament Problem (TTP) as solved in [2] is an instance of solving a scheduling problem using constraint programming and focuses on generating feasible timetables that satisfy all the specified constraints. The two subcomponents of this problem are constraint satisfaction and optimization of the travelling distance; nevertheless, it emphasizes more on the cost. [3]

In the paper published by Laxmi Thakare, Jayant Umale et.al [4] the use of Genetic Algorithm proposes to solve the TTP instances of 4, 6 and 8 teams. Genetic algorithms provide an efficient evolutionary approach to solve such a problem. However, it is based on a trial-and-error approach for encoding the problem and the selection of parameters required for the GA. [5]

In this paper [6] by Georigk et.al, a hybrid approach devised by combining integer programming and local search is used to calculate an optimal schedule. The speedup achieved is notable but not significantly higher. Also, integer programming being a rule-based approach computational complexity for a small number of teams is still pretty high enough.

The paper published by Anagnostopoulos et al. [7] suggested some radical improvements, which were instrumental in reducing the complexity of the problem. They used the Simulated Annealing (SA) algorithm to find both a feasible and optimal schedule at the same time. The unique aspect of their research was their logical bifurcation of the constraints into two separate categories, namely the ‘soft constraints’ and the ‘hard constraints’. The ‘hard constraints’ preserved the double-round robin format of the schedule and avoided any invalid schedules, whereas ‘soft-constraints’ could be relaxed for searching optimal schedules.

The field of quantum computing is on the ascendancy since the last decade as radical breakthroughs in hardware technology has enabled the creation and maintenance of stable quantum states, also known as qubits, which are fundamental unit of the quantum computer and quantum analogue of the classical bit. Quantum algorithms are believed to solve non-deterministic polynomial problems with much lesser complexity as compared to the classical computer. Quantum algorithms like Shor’s factoring algorithm and Grover’s search algorithm [8] have demonstrated ‘quantum supremacy’ or an exponential speedup against the best classical supercomputers. [9]. Furthermore, there has been a considerable amount of research happening in the field of quantum artificial intelligence, but the scale as of now, is restricted to a few qubits. [10, 11]

The IBM Quantum experience (q experience) is a cloud computing platform hosted by IBM which lets students and researchers across the world access their quantum computers, with the help of the various libraries and interfaces provided by IBM, one can design their own independent algorithm/program in the quantum framework. [12, 13]

III. PROBLEM DEFINITION

The tournament devised in the Travelling Tournament Problem as proposed by Easton, Nemhauser et al., [14] obeys the following conditions.

● Each-Venue: Every pair of teams play twice in each other's home locations.

● At-most-three: No team may have a home game or road trip for more than three consecutive games.

● No repeat: A team cannot play against the same sopponent in two consecutive games.

The tournament spans for a period of 2(n-1) days, where n is the number of teams participating in the tournament. A team plays only one game on any single day, with no days off. Therefore, the number of teams must be even. The aim is to optimize the distance traveled by the n teams.

While measuring the total distance assume that every team commences the tournament at their respective home city and in the end returns back to it. Nonetheless, a team does not return to its city after every away game, instead, if the next game is an away game as well then, the team proceed to that respective city. [14, 15]

T/R	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	6	-2	4	3	-5	-4	-3	5	2	-6
2	5	1	-3	-6	4	3	6	-4	-1	-5
3	-4	5	2	-1	6	-2	1	-6	-5	4
4	3	6	-1	-5	-2	1	5	2	-6	-3
5	-2	-3	6	4	1	-6	-4	-1	3	2
6	-1	-4	-5	2	-3	5	-2	3	4	1

Figure. 1. A typical double-round robin schedule that satisfies the above constraints. [7]

In the above figure 1, ‘T’ denotes the team number and ‘R’ denotes the round or the day/week of the scheduled game.

IV. SHORTCOMINGS OF THE SIMULATED ANNEALING APPROACH

It was observed that of all the existing solutions, simulated annealing gave the best heuristics. Nonetheless, the implementation had some serious drawbacks. The selection of operations to be performed on a given schedule to increase its optimality was completely random. Such haphazard selection often resulted in the algorithm wasted its resources in generating a greater number of infeasible schedules. Furthermore, due to the random nature of its selection, operations that resulted in the violations of the constraints were not aptly penalized, which resulted in the repeated execution of the same operations, which had previously yielded poor results. To gain a perspective of as to what extent this can aggravate the solution, we present a statistic of the number of iterations required to perform to get the solution for the least possible number of teams.

Table 1. Minimum iterations for optimal cost using SA

Number of Teams	Number of Iterations	Optimal Cost
4	36,000	7876
6	89,000	27,567
8	Approx. 140,000 – 150,000	49,423

As we can see that the complexity of the algorithm is appreciably high and hence has a scope for improvement, we intend to do certain modifications to enhance the performance of the algorithm and reach the optimal cost faster. For achieving the same, we suggest the following approach developed with the aid of quantum computing, a promising new-age technology.

V. CLASSICAL-QUANTUM HYBRID APPROACH

In the light of the above analysis and after studying about various classical as well as quantum models, we realized that it would be best to use a hybrid approach where the unique aspects of both complement each other. In the following sub-section, we have explained in detail the different aspects of our solution and how the serve the ultimate purpose of achieving both optimization and feasibility.

A. TSP using Variational Quantum Eigensolver

The aim of the Variational Eigensolver is to minimize the value of certain matrix which might represent a physical or quantifiable value. The solution for the Travelling Salesman Problem (TSP) using the Variational Quantum Eigensolver (VQE) is premised on the supposition that the Hamiltonian intrinsic to quantum computing is logically equivalent to the cost matrix of travelling between the cities in question.[16, 17]. The basis states of the quantum systems are used to represent a possible route that can be traversed among the cities whereas the respective diagonal displays the cost.

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{matrix} |000> \\ |001> \\ |010> \\ |011> \\ |100> \\ |101> \\ |110> \\ |111> \end{matrix}$$

Matrix.1. Depicting the basis states

The vectors representing the individual routes are later multiplied with the Hermitian matrix that gives us the cost or energy value of that route. Of the various energy values, the route corresponding to the least value is selected.

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrix.2. Calculating the eigenvector

This methodology uses the routes as eigenvectors to the Hamiltonian matrix and tries to find the lowest eigenvector, thereby finding the lowest or least costly route.

By iteratively changing the number of cities various sets of optimal routes are generated and fed to the classical algorithm. [18]

B. TSP Benchmark Schedule

The solution we suggest herein focuses on overcoming the drawbacks mentioned above, of the simulated annealing approach. We intend to modify the acceptance probability of the schedule generated in a given iteration. This is done using a separate schedule, TSP_BM, which is generated using the solution generated for a travelling salesman problem with the distance matrix for the contending teams as the input. This TSP route is generated using the quantum eigen-solver. The TSP route generated is the least cost route connecting all the cities, given by the following equation.

$$T_R = \min\left(cost(R_i)\right) \qquad (1)$$

T_R = TSP route generated for the given distance matrix
$R_i = i^{th}$ route from a list of all possible routes between N cities.

Using T_R, the TSP_BM schedule is generated using the following method.

If Feasible:

$T_{ij} = T_R$ (j) for 0 < j < len(T_R)

Else:

Try

$$T_{ij} = T_{i(j+1)} \qquad (2)$$

Where T_{ij} represents the j^{th} column of the i^{th} team in the schedule.

The reason behind integrating this TSP based approach is that analysis conducted during the multiple runs of the classical algorithm showed the presence of TSP routes present in the optimal solution as shown in the figure below.

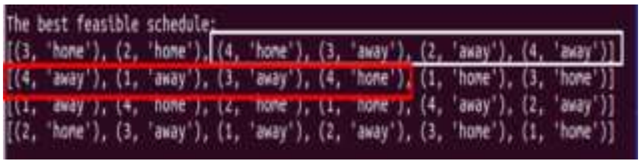

Figure. 2. Optimal TSP routes hidden in the best schedule

Therefore, a schedule built using TSP routes will definitely be fairly close in terms of cost to the optimal schedule. This TSP benchmark schedule generated with the help of quantum eigensolver is then used to further modify the classical simulated annealing algorithm as follows:

We initialize the algorithm by setting the best-found feasible schedule as the TSP_BM schedule

$$\text{Best Feasible Schedule=TSP\_BM} \tag{3}$$

Then we modify the acceptance probability of the current schedule to ensure that our algorithm spends relatively more time in the feasible region as compared to the infeasible reason. This is done with the help of the TSP_BM schedule in the following way.

If the number of violations = 0 and the cost of the current schedule is better than the previous schedule then accept the current schedule.

Else if the number of violations > 0 but the cost is lesser than the best infeasible schedule then accept the schedule with a probability given by

$$e^{-\frac{abs(tspbm-bi)}{t_0}} \tag{4}$$

Where tspbm = cost of the TSP_BM schedule

bi = cost of the best infeasible schedule

t_0 = initial temperature

Thus because of the negative exponent, greater the difference between tspbm and bi, lesser is the probability of the schedule getting accepted, this modification does not let the algorithm stray away from the feasible region more than necessary.

$$e^{-\frac{abs(previous-current)*nbv}{t_0}} \tag{5}$$

$previous$ = cost of the schedule in the previous iteration

$current$ = cost of the schedule in the current iteration

nbv = number of violations in the current schedule

The number of violations present in the schedule decide its acceptance probability by varying the exponent value.

A dynamic counter is implemented to speed up the algorithm and reduce the number of iterations. It is implemented as given below

Only if the best feasible schedule found is better than the TSP_BM schedule then for every iteration increase the counter value by 2 with the following probability

$$e^{-\frac{abs(tspbm-bf)}{t_0}} \tag{6}$$

C. *Quantum Adder*

The need for a faster summation methodology is felt when we observe the sheer number of calculations that goes in carrying out the subroutines. Hence, we realized that a fundamentally faster technique is required to speed up the calculations intrinsic to the iterations.

After exploring the bitwise implementation of the quantum adder, we inferred that no meaningful speedup was achieved and that for some instances both the space and time complexity in fact increases. We subsequently realized that we need to premise our calculations on insights that are far more deeply seated within the realm of quantum computation, which also naturally meant building a radically different intuition of the logical operations.

Our efforts culminate in utilizing the Quantum Fourier Transform technique for carrying out the operations. The underlying principles behind this application are simple and fundamental to ones understanding of quantum computation.

Assume that a wave signal is to be classically analysed to understand its constituent elements, or conversely a new signal is constructed by adding together multiple sinusoidal functions with varying frequencies. It is achieved by using the Fourier series transformation as given below.

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi ikj/N} \tag{7}$$

Where y_k is the function in a particular domain which can be construed of comprising of elements summed over a specific range.

Mathematically it can be interpreted in many different ways, however the essentiality in our peculiar case is to view the working principle of the Fourier transform; [19, 20]

In our model instead of taking the inputs as numbers or functions we take them as vectors. Wherein each element in the vector represents the probability of the state being measured as a combination of 0 and 1, often represented by a Ψ also known as the wavefunction. Therefore, if say we assume that we have two quantum bits at our disposal then the quantum states formed by the linear combination of the two is known as the quantum superposition state which is expressed as follows:

$$|\Psi> = \alpha|0> + \beta|1> \quad (8)$$

Where α and β are complex numbers whose sum of products with their complex conjugates yields us the number 1. They are referred to as the probability amplitude of the basis states of a quantum ensemble and are defined as follows:

$$\alpha^*\alpha + \beta^*\beta = 1 \quad (9)$$

The basis states are the states that define the number of dimensions that are available for us to use and the number of basis states is directly proportional to number of physical quantum states available for computation. Any quantum state that is a superposition of two or more basis states collapses to one of the basis states on measurement depending upon the values of their respective probability amplitudes. [19]

The quantum version of the classical discrete Fourier Transform or more commonly known as the Quantum Fourier Transform (QFT), will use these basic quantum principles in its implementation. Mathematically, the QFT is identical to the classical Fourier transform.

$$QFT|x> = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{i\frac{2\pi xk}{N}}|k> \quad (10)$$

Where the state |x> like our previous state |Ψ> is a Superposition or linear combination of multiple basis states {|0>, |1>, …., |N-a>}.

Now in order to work with real numbers or natural numbers as we do in our day-to-day life, we will have to encode the input in the quantum format where they appear as vectors. The simplest way to do it is to perform a mapping of say all numbers x into a $|x>$ basis states. The fundamental difference here being that for every 'n' qubit represent 2^n classical values thereby expressing exponential greater numbers.

When QFT is applied what it essentially does that it performs a change in the basis state of the quantum wave function, say from $|x>$ to $|k>$ where every input is taken with the phases of $e^{i\frac{2\pi xk}{N}}$.

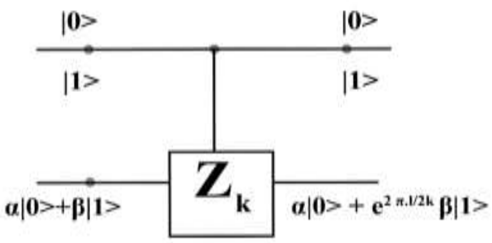

Figure. 3. Schematic of applying transformation to a superposition state

Let QFT $|x>$ be denoted as $|\varphi(x)>$, hence we have it as the tensor product of the basis states.

$$\sum_{j=0}^{N-1} e^{i\frac{2\pi xk}{N}} = |\varphi_n(x)> \otimes \ldots \otimes |\varphi_2(x)> \otimes |\varphi_1(x)> \quad (11)$$

To put it analogously, as in the classical Fourier transform, we change the do-main of the wave functions from frequency to time and vice versa, in the very same fashion by applying phase shifts on the quantum states we change the nature of the superposition of quantum states and map it differently that the original basis states.

The addition between two numbers or states using QFT happens similarly by applying phase rotation between the two-qubits by using conditional gates. The quantum addition is hence performed by using a sequence of conditional rotations. The controlled R gates are usually applied for performing the same. Unfortunately, as the controlled R gates do not exist in the repository of gates available on Qiskit, the platform used for implementation, we had to employ the controlled - U1 gate, which identical in nature to the Controlled R gates. [21]

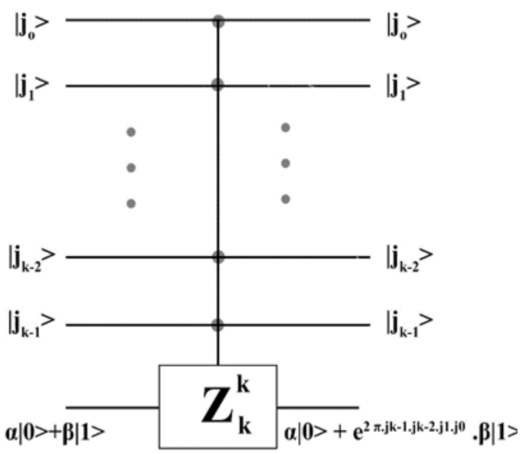

Figure. 4. Circuit depicting controlled manipulation of the phases using bits of the second number as reference

The major advantage of using these controlled gates that saves us the task of converting the second number to be added into its Fourier transform. This is because the bits of the second number are used as control bits on the applied controlled - U1 gates.
This allows us to compute addition in novel fashion where, for two number a and b, we have

After applying the IQFT or the Inverse Quantum Fourier Transform one can retrieve the binary or decimal value of our classical inputs

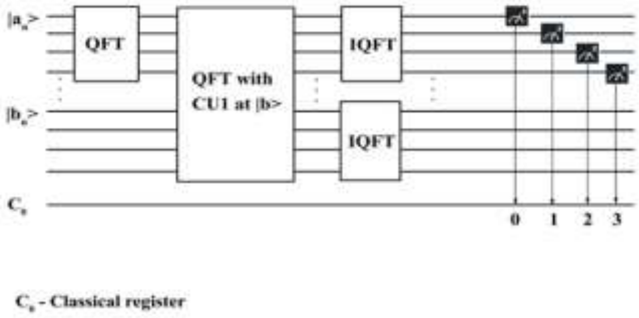

Figure. 5. A representation of the QFT circuit implemented on the Quantum Computer. [22]

The result obtained from the above circuit is then sent to the modified TTSA algorithm. One can also view this as a subroutine that is called whenever logical operations are needed to be performed within the algorithm. This most significant aspect of this approach being that as the digits are transformed and manipulated in a different domain in conjunction with each other, they act like binary sinusoidal waves who perform a destructive interference at certain inputs and constructive interference at others, this eliminates the need of the carry operation thereby saving bits as well as time of operation.

VI. Results & Discussion

The results obtained from our hybrid model showed coherent distinction from the results of the former classical model. As displayed in Table 1, the classical implementation of TTSA for four teams took a staggering 36,000 iterations and around 90,000 iterations for six teams whereas our model arrived at the very same schedule within 18,000 iterations and approximately 38,000 iterations for four and six teams respectively which is approximately 55% faster than the previous models. The approach that we have endeavoured to delineate here is unique of its kind and probably one of the first quantum assisted speed-up of classical algorithms for scheduling of sports or any public event for that matter. The TSP-BM or TSP benchmark estimation deviates very slightly from the final optimal result and hence can be deemed as an immensely reliable estimate. It also insinuates the fact the pre-processing done to the algorithm does indeed yield favourable results. The Quantum Fourier Transform or the QFT is a yet another novel way of expediating logical operations in a classical algorithm. As the quantum systems that we use are still considered as cutting-edge technology and presently under the purview of research domain, very limited resources are available for experimentation. Nonetheless, even with rudimentary setups the model produces significantly better results. As quantum technologies continue to improve, a more seamless integration shall become conceivable. The speedup achieved by QFT is notable, an estimate of all the bit-wise operation carried out to perform all the iterations sums up to tentatively 15 million operations which the QFT approach brings to mere 2 million a staggering 7 times faster.

The variational quantum eigensolver approach used in the pre-processing to procure the optimal routes is also quadratically faster than all its classical alternatives. Thus, for the implementation 4 and 6 teams the following difference in the time complexity can be seen.

We generated a dataset, which tells us the number of iterations it takes to reach the final solution, from various runs required for a better understanding of how the selection of moves affect the other parameters and cost of the solutions. Also, the generation of the dataset helps us with the selection of parameters required for optimising the choice of the moves in our proposed approach. The following graph shows the difference between the number of iterations it takes for the Old Simulated annealing algorithm and for our approach.

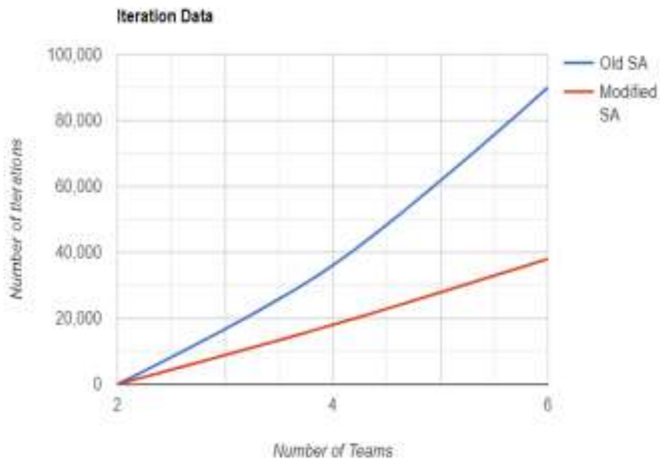

Figure. 6. Results for 4 and 6 teams.

A. *Novel features*

The novelties of this paper include the use of quantum computing technique. It is one of the few instances where an attempt has been made to blend classical computations with quantum architectures. The approach undertaken for doing so is also unique, due to various logistical and technological constraints. Apart from dividing the problem through different computing architectures, the problem was also dissected logically and solved in an incremental manner. To elaborate this further, on analysing the problem we realized that the TTP itself is composed of various sub-problems which cumulatively gives it its convoluted form. Each of these subproblems gives a leeway to solve different folds of the problems, the relevance has been discussed in section 5 of this paper. This solution also acts as a use-case demonstration of how the integration of quantum computing techniques can drastically transform our perspective towards logical problems. Though in its infancy, quantum computing shows promising signs of facilitating the expansion of our computational capacities. Our insistence on exploring quantum solutions was also guided by the notion to test whether the quantum devices indeed are helpful in expediting the computations.

B. A Note on Complexity

The TTP is arguably one of the most computationally intractable problem known to us. The brute force method of inefficiently solving this problem gives almost an O (n^2!) complexity. It is very difficult to find an optimal or even a suboptimal solution for a problem whose complexity scales upward so rapidly. Hence, every effort put into finding a better solution is like an infinitesimal improvement to the solution. Our work builds itself on the suboptimal solution provided by the simulated annealing algorithm. The algorithm in its original form navigates the sample space of all possible schedules with a certain degree of randomness and then depending on the recurrence of certain solutions settles a global (or at times even local) minimum. While investigating this solution we realized that a lot of time and effort was spent by the algorithm in exploring the infeasible regions of the sample space and its exploration of the solution space was being carried out in a very unintelligent manner. In this project it has been our effort to minimize the effort in this particular section of the solution.

Though the inherent randomness in the Simulated Annealing algorithm causes it to explore infeasible regions of the solution space, a certain degree of randomness is still imperative as it plays a crucial role in escaping the local minima and avoiding suboptimal schedules.

The logical complexity of this problem is two-fold. One, is to find a schedule that appropriately fits the requirement of the ‘Home’ and ‘Away’ game requirements to maintain an element of fairness in the tournament. Two, is to have matches scheduled in a manner that the cumulative travelling costs of all teams is minimum. The aspect that makes this problem devilishly intractable is that both these two parts are in a way interwoven into each other and have direct bearing on one another.

Incidentally, this also where the key to minimizing its complexity lies, while analysing all the feasible as well as infeasible schedules generated by the algorithm, the most optimal ones had more less similar combinations or cities travelled in more or less the same order. This sequence of cities travelled was nothing more than a miniature version of the canonical np-hard traveling salesman problem. We realized that tackling the optimality first or least partially would lead us to quicker to a feasible solution than the other way around.

We used this newfound insight to construct a new metric called the TSP benchmarking schedule, whose exact mathematical formulation and details have been discussed in section 5 of this paper. This benchmark guided the exploration of the solution space in the SA algorithm. With the help of this benchmark, the solution spaces containing traversal sequences whose cost was considerably disparate from the TSP solutions were discarded. This reduced the size of the solution space that algorithm needed to explore and expedited the process of arriving on the best feasible schedule.

These processes were supplemented with the Quantum Architectures that were employed. For instance, the TSP solutions provided by the quantum eigensolver we arrived at quadratically faster against a classical approach. A similar effect is achieved by the quantum adder.

The overall run-time of the algorithm is reduced by half and the optimal results are obtained much quicker.

VII. Future Scope

The techniques and modifications presented here are only the tip of the iceberg when it comes to modifications that can be done either in the classical or quantum framework. For instance, there are multiple other parameters that can be extemporized with to analyse its impact on the solution. The quantum encoding presented here is only one such methodology from an assortment of techniques and approaches that can be experimented with. Due to the hardware limitation of only 36 qubits our development was quite contrived, and as the capabilities of quantum systems improve, a greater part of the solution can be integrated into the quantum part.

VIII. Conclusion

In the work presented, we explored the field of quantum computing as well as classical metaheuristic algorithms and engendered a novel approach which combines the quantum and the classical domains. Albeit, still in nascency, quantum algorithms have an immense potential to solve various problems that the purely classical approaches have difficulty solving, namely, the NP hard and the NP complete category of problems like the one we explored in this paper. We believe that our attempt at developing the above model is only the beginning of a great series of development for which there exists ample scope in the world of computation. The field of Quantum Computing has opened a new vista of research in this field and we believe that all existing models can be vastly im-proved and far better designs can be constructed. Nonetheless, as we learn more about this intriguing universe, we need to furnish ourselves to adapt by beginning to experiment with these new models.

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AUTHORS PROFILE

Mr. Desmond Lobo completed his Bachelor’s in Computer Engineering from Don Bosco Institute of Technology (DBIT) affiliated to the University of Mumbai. He is currently working at Cere Labs Pvt. Ltd. as an AI Engineer. His is interested in Quantum Computing, Meta-heuristic algorithms, Machine Learning, Artificial Intelligence and Natural Language Processing.

Ms. Kalpita Ajinkya Wagaskar is working as Assistant Professor in the Department of Computer Engineering at Don Bosco Institute of Technology, Mumbai. She has over 12 years of experience in Industry and academics. Her projects have received 2 minor grants from Mumbai University. Her areas of interest include Artificial Intelligence, Machine learning and Quantum Computing.

Mr. Sarang Gawane completed his Bachelor’s in Computer Engineering from Don Bosco Institute of Technology (DBIT) affiliated to the University of Mumbai. He is currently pursuing his MS in Computer Science degree from the University of Illinois at Chicago. His interests lie in Machine Learning, Artificial Intelligence and Data Driven Engineering.

Mr. Clifford Fernandes is currently working as an IT Project Manager, Department of Computer Science Engineering (CSE), Indian Institute of Technology, Bombay (IIT-Bombay), Mumbai, India. He is further looking to pursue his Post Graduate Diploma from Seneca College, (Ontario), Canada. His interests lie in Data Analytics and Technology Management.