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The Numerical Solution of Nonlinear Nonhomogeneous System of Differential Equations By Differential Transform Technique

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Abstract— There are several methods available; analytical (Exact), approximate and numerical; for solving differential equations. Most of these methods are computationally exhaustive because they require a lot of time and space. The Zhou's differential transform technique has an edge over the traditional methods as it uses the polynomial as the approximation to exact solution. In this paper differential transform technique is employed to solve some nonlinear nonhomogeneous, initial value problems in system of differential equations which are often encountered in applied sciences and engineering. The solutions produced by differential transform method are compared with the exact solutions achieved by Laplace transform technique. It is observed that numerical results obtained by differential transform method are in good agreements with the analytical solutions.

Keywords: Differential transform technique, System of differential equations, Laplace transforms technique, Exact solution.

I. INTRODUCTION

The differential transformation method (DTM) is the numerical technique based on Taylor's series expansion. This method assumes the polynomial as an exact solution of given differential equation. This is one of the numerical methods for solving initial value problem of ordinary differential equation, linear differential equations and nonlinear, nonhomogeneous system of differential equations. This method was first proposed by J.K. Zhou [1] in 1986 to solve electrical engineering problems. Chen and Liu [2] have used DTM technique for solving non-linear heat conduction problems and extended the study to obtain solution of two point boundary value problems [3]. Chen and Ho [4] applied DTM to eigenvalue problems. Chen and Liu [5] used DTM method for solving two point boundary value problems. Ayaz [6, 7] has applied DTM for solving differential algebraic equations and wave equations. Ravi Kanth and Aruna [8] found the solutions of linear and non-linear Schrodinger equation by DTM. Arikoglu and Ozkol [9] show that how DTM is applicable for solving fractional differential equation. Biazar and Eslami [10] solved Riccati equation by DTM. Jang et. al. [11] used this method for solving initial value problems. Chang and Chang [12] have designed a new algorithm for calculating one-dimensional differential transform of nonlinear functions.

In this paper, three systems of nonlinear and nonhomogeneous, initial value problems in system of differential equations are considered by a differential transformation technique, an approximate solution is obtained and the numerical solutions are compared with the exact solutions that are calculated from the Laplace transform method. The basic definitions and fundamental properties of DTM are adopted from [13, 14].

II. DEFINITION OF ZDTM

An arbitrary function p (t) can be expanded in Taylor series about t = 0

$$P(t) = \sum_{K=0}^{\infty} \frac{t^k}{K!}\left[\frac{d^k p}{dt^k}\right]_{k=0}$$

DTM of p (t) is

$$P(k) = \frac{1}{K!}\left[\frac{d^k p}{dt^k}\right]_{k=0}$$

III. THEOREMS ON DTM METHOD

Original Function	Transformation
x(t) + y(t)	X (k) + Y (k)
$\propto x(t)$	$\propto X(k)$
$\frac{d}{dt}x(t)$	(k + 1) X(k + 1)

$\frac{d^m}{dt^m}x(t)$	$(k+1)(k+2)\ldots(k+m)X(k+m)$
t^m	$\delta(k-m)$ =1, if $k = m$ =0, if $k \neq m$
$e^{\lambda t}$	$\frac{\lambda^k}{k!}$
$\sin(\propto t+\beta)$	$\frac{\propto^k}{k!}\sin\left(\frac{k\pi}{2}+\beta\right)$
$\cos(\propto t+\beta)$	$\frac{\propto^k}{k!}\cos\left(\frac{k\pi}{2}+\beta\right)$
$x(t)\cdot y(t)$	$\sum_{m=0}^{K}Y(m)X(k-m)$
$(1+t)^m$	$\frac{m(m-1)\ldots\ldots(m-k+1)}{k!}$

IV. EXPERIMENTATION OF THEOREMS DTM

Example 1

In a heat exchange, temperature u and v of two liquids satisfy the equations

$$4\frac{du}{dt}=v-u=2\frac{dv}{dt}$$

Subject to

u (0) = 20

v (0) = 100

Exact solution by Laplace transform technique is given by

u = -60+80 $e^{t/4}$

v= -60+160 $e^{t/4}$

By taking differential transformation of both sides of the resulting equation using above stated results, the following recurrence relations are obtained: Differential transformation method for solving differential equations:

4(K + 1) U (K + 1) =V (K) – U (K)

2 (K + 1) V (K + 1) = V (K) – U (K)

Put K = 0, 1, 2, 3, 4 ………………….

U (0) =20 and V (0)=100

U (1) =20 V (1)=40

U (2) =5/2 V (2)= 5

U (3) =5/24 V (3)=5/12

U (4) =5/384 V (4)=5/192

U (5) = -1/1536 V (5) = -1/768

U (6) = -1/12288 V (6) = -1/6144

Solution by DTM is given by

$u(t) = U(0) + U(1)t + U(2)t^2 + U(3)t^3 + \ldots$

$= 20 + 20t + 5/2\,t^2 + 5/24\,t^3 + \frac{5}{384}t^4 - \frac{1}{1536}t^5 - \frac{1}{12288}t^6 + \cdots$

$v(t)=V(0) + V(1)t + V(2)t^2 + V(3)t^3 + \ldots$

$=100 + 40t + 5t^2 + 5/12\,t^3 + \frac{5}{192}t^4 - \frac{1}{768}t^5 - \frac{1}{6144}t^6 + \cdots$

The numerical results of DTM for system of differential equations 1 is presented in Table 1.

Example 2

Consider the following non-homogenous differential system:

$\frac{dx}{dt} + 2x - 3y = t$

$\frac{dy}{dt} - 3x + 2y = e^{2t}$

Subject to

X (0) = 1

Y (0) = 1

Exact solution by Laplace transform technique is given by

$x=\frac{16}{175}e^{-5t}+e^t-\frac{13}{25}-\frac{2t}{5}+\frac{3}{7}e^{2t}$

$y=-\frac{16}{175}e^{-5t}+e^t-\frac{12}{25}-\frac{3t}{5}+\frac{4}{7}e^{2t}$

By taking differential transformation of both sides of the resulting equation using above stated results, the following recurrence relations are obtained: Differential transformation method for solving differential equations:

(K + 1) X (K + 1) + 2 X (K) – 3 Y (K) = δ(K -1)

(K + 1) Y (K + 1) - 3 X (K) + 2 Y (K) = $\frac{2^K}{K!}$

Put K = 0, 1, 2, 3, 4 ………………….

X (0) =1 and Y (0)=1

X (1) =1 Y (1) =2

X (2) =5/2 Y (2) =½

X (3) =-7/6 Y (3) =17/6

X (4) =65/24 Y (4) =-47/24

Solution by DTM is given by

$X(t) = X(0) + X(1)t + X(2)t^2 + X(3)t^3 + \ldots\ldots$

$=1 + t + 5/2\,t^2 - 7/6\,t^3 + \frac{65}{24}t^4 + \ldots.$

$Y(t) = Y(0) + Y(1)t + Y(2)t^2 + Y(3)t^3 + \ldots\ldots$

$=1 + 2t + ½\,t^2 + 17/6\,t^3 - \frac{47}{24}t^4 + \ldots..$

The numerical results of DTM for system of differential equations 2 is presented in Table 2.

Example 3

Consider the following non-homogenous differential system:

$\frac{du}{dx} + v = \sin x$

$\frac{dv}{dx} + u = \cos x$

Subject to

u (0) = 1

v (0) = 0

Exact solution by Laplace transform technique is given by

u = cosh x

v = sin x – sinh x

By taking differential transformation of both sides of the resulting equations using above stated results, the following recurrence relations are obtained: Differential transformation method for solving differential equations:

(K + 1) U (K + 1) + V (K) $=\frac{1}{K!}\sin\left(\frac{K\pi}{2}\right)$

(K + 1) V (K + 1) + U (K)$=\frac{1}{K!}\cos\left(\frac{K\pi}{2}\right)$

Put K = 0, 1, 2, 3, 4 ……………

U (0) =1 and V (0) =0

U (1) =0 V (1) =0

U (2) =½ V (2) =0

U (3) =0 V (3) =-1/3

U (4) =1/24 V (4) =0
U (5) =0 V (5) =0

Solution by DTM is given by

$U(x) = U(0) + U(1)x + U(2)x^2 + U(3)x^3 + \ldots$

$= 1 + 0x + \frac{x^2}{2} + 0x^3 + \frac{1}{24}x^4 + \ldots\ldots$

$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots$

$V(x) = V(0) + V(1)x + V(2)x^2 + V(3)x^3 + \ldots\ldots\ldots\ldots$

$= 0 + 0x + 0x^2 - 1/3x^3 + 0x^4 + 0x^5 \ldots\ldots\ldots\ldots$

$= -1/3x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

The numerical results of DTM for system of differential equations 3 is presented in Table 3.

V. CONCLUSION

In this paper, we extend the application of differential transform technique to solve some nonlinear and nonhomogeneous, initial value problems in system of differential equations. This new technique avoids the difficulties and massive computational work that usually arise from the standard methods. In the present work, the calculated results are exactly the same as those obtained by Laplace transform technique, which demonstrate the reliability and efficiency of the technique. Moreover, the proposed technique offers a computationally easier approach to achieve to the exact solution by faster way.

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APPENDIX

Table 1

t	Exact Solution		DTM Solution	
	u(t)	v(t)	u(t)	v(t)
0.0	20.000	100.000	20.000	100.000
0.1	22.025	104.050	22.025	104.050
0.2	24.102	168.203	24.102	168.203
0.3	26.231	112.461	26.231	112.461
0.4	28.414	116.827	28.414	116.826
0.5	32.652	121.304	32.652	121.300

Table 2

t	Exact Solution		DTM Solution	
	x(t)	y(t)	x(t)	y(t)
0.0	1.000	1.000	1.000	1.000
0.1	1.124	1.208	1.124	1.208
0.2	1.294	1.440	1.295	1.440
0.3	1.511	1.711	1.515	1.706
0.4	1.778	2.031	1.795	2.011
0.5	2.101	2.415	2.148	2.357

Table 3

t	Exact Solution		DTM Solution	
	u(t)	v(t)	u(t)	v(t)
0.0	1.000	0.000	1.000	0.000
0.1	1.005	-0.000	1.005	-0.000
0.2	1.020	-0.003	1.020	-0.003
0.3	1.045	-0.009	1.045	-0.009
0.4	1.081	-0.021	1.081	-0.021
0.5	1.128	-0.042	1.128	-0.042

