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Analysis of Reliability of A Two-Non-Identical Units Cold Standby Repairable System Has Two Types of Failure

Praveen Gupta[1*] and Pooja Vinodiya[2]

School of Studies in Statistics, Vikram University, Ujjain (M.P.) – 456010

**Corresponding Author: poojavinodia10@gmail.com*

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Abstract- This paper shows the analysis of reliability of a system composed of the two- N.I.U., N_O and N_S in which N_O is operative and N_S is kept in standby mode upon failure of operative units N_O units N_S become operative instantaneously. Unit-1 has two types of failures. Let failure time distribution of type 1 and type 2 are assumed to be exponential with parameters λ_1 and λ_2 respectively, and the repair time is taken as general. When the second unit is failed it goes for replacement.

Keywords- Reliability, MTSF, Availability, Busy period, Mean Sojourn Time

1.1 INTRODUCTION

Concentrate the unwavering quality of machine repair issue is critical in our life since it is broadly utilized in modern framework and assembling framework. Any framework ends up questionable because of different reasons. In the conventional frameworks, the units of the framework have just two states up and down. Be that as it may, much of the time the units of the framework can have limited number of states. Most unwavering quality models expect that the up and down times of the parts are exponentially conveyed.

Wei, L.,et al (1998) presented stochastic investigation of a repairable framework with three units and two repair facilities. Agarwal, S.C., et al (2010) presented Reliability characteristic of cold-standby redundant system. In some unwavering quality parameters of a three state repairable framework with ecological disappointment were assessed. El-Damcese, M.A. (1997) studied "Human error and common-cause failure modelling of a two-unit multiple system.

1.2 DESCRIPTION AND ASSUMPTIONS OF THE SYSTEM

1. The system contains two N. I. U.. Initially unit first is operative and second is standby.
2. Both units has two modes N and F-Mode.
3. The o - unit is non-repairable, hence upon failure it goes for replacement
4. A single repairman is always available.
5. The failure time distributions of both the units, and time of replacement are taken as exponential although the repair time is taken general.

1.3 STATES AND NOTATIONS

(a) Symbols for the states

$N_0^i \; i=1,2$: Unit – i is in N – Mode and operative

$N_s^i \; i=1,2$: Unit – i is in N – Mode and standby

F_R^2 : Unit – 2 is in F – Mode and under replacement

F_{WR}^2 : Unit – 2 is in F – Mode and waiting for replacement

F_r^1 : Unit – 1 is in F – Mode and under repair

F_{2r}^1 : Unit – 1 is in F – Mode and under repair

UP STATES

$S_0 = (N_o^1, N_s^2)$; $S_1 = (F_r^1, N_o^2)$

$S_2 = (F^1_{2r}, N^2_o)$; $S_5 = (N^1_o, F^2_R)$

DOWN STATES

$S_3 = (F^1_r, F^2_{WR})$; $S_4 = (F^1_{2r}, F^2_{WR})$

NOTATIONS

λ_1 : Type1 of failure rate of first unit

λ_2 : Type 2 of failure rate of first unit.

$F_1(.)$: c.d.f. of repair time of type–1

$F_2(.)$: c.d.f. of repair time of type–2

λ_3 : Constant failure rate of second unit

γ : Replacement rate of second unit

TRANSITION DIAGRAM:

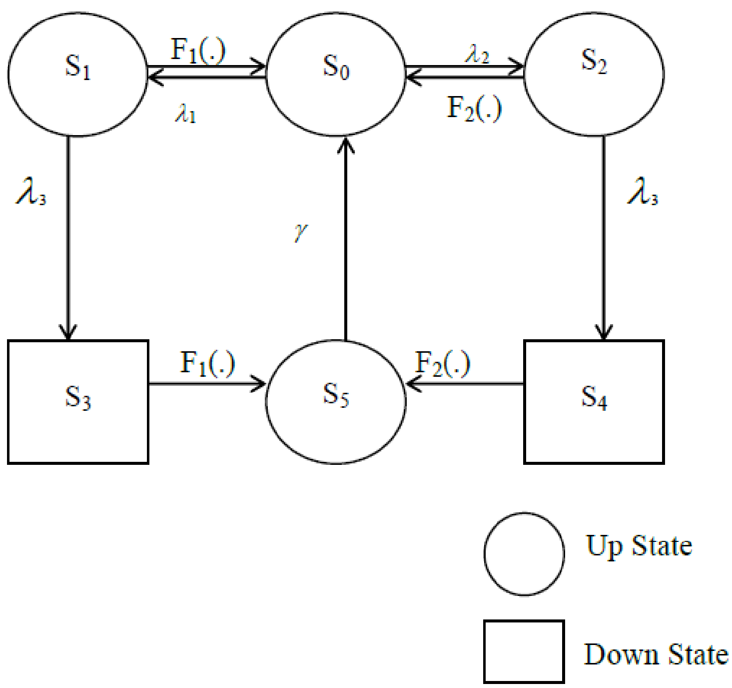

Figure- 1.1 Shown States of the system with possible transitions

1.4 TRANSITION PROBABILITIES

As defind earlier, $Q_{ij}(t)$ is the porbability that the system transits from state S_i to S_j on or before time "t " or the c.d.f. of transition time from regenrative state S_i to S_j. So by the simple probabilistic arguments the transition probabilities if the system can be obtain as follows :

$$Q_{01}(t) = \int_0^t \lambda_1 e^{-\lambda_1 u} e^{-\lambda_2 u} du$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2}\left[1 - e^{-(\lambda_1+\lambda_2)t}\right]$$

$$Q_{35}(t) = \int_0^t dF_1(u)$$

$$Q_{45}(t) = \int_0^t dF_2(u)$$

1.5 STEADY STATE TRANSITION PROBABILITIES

Let P_{ij} indicates the steady state transition probability of the system from state S_i to S_j $(i, j = 1, 2, ..., 5)$

$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2}$, $p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2}$,

$p_{10} = \tilde{F}_1(\lambda_3)$, $p_{13} = 1 - \tilde{F}_1(\lambda_3)$,

$p_{20} = \tilde{F}_2(\lambda_3)$, $p_{24} = 1 - \tilde{F}_2(\lambda_3)$,

$p_{35} = 1$, $p_{45} = 1$,

$$Q_{02}(t) = \int_0^t \lambda_2 e^{-\lambda_1 u} e^{-\lambda_2 u} du$$

$$= \frac{\lambda_2}{\lambda_1 + \lambda_2}\left[1 - e^{-(\lambda_1+\lambda_2)t}\right]$$

$$Q_{10}(t) = \int_0^t e^{-\lambda_3 u} dF_1(u)$$

$$Q_{13}(t) = \int_0^t \lambda_3 e^{-\lambda_3 u} \overline{F_1}(u) du$$

$$Q_{20}(t) = \int_0^t e^{-\lambda_3 u} dF_2(u)$$

$$Q_{24}(t) = \int_0^t \lambda_3 e^{-\lambda_3 u} \overline{F_2}(u) du$$

$$Q_{50}(t) = \int_0^t \gamma e^{-\gamma u} du$$

$$= \left[1 - e^{-\gamma t}\right].$$

$p_{50} = 1.$

The relations between probabilities

$P_{01} + P_{02} = 1$

$P_{10} + P_{13} = 1$

$P_{20} + P_{24} = 1$

$P_{35} = 1$

$P_{45} = 1$

$P_{50} = 1$

1.6 MEAN SOJOURN TIME

Mean time of stay in state S_i is called the mean sojourntime in state S_i and it is indicated by ψ_i. If T_i is the sojourn time in state S_i, then the mean sojourn time in state S_i is given by

$$\psi_i = \int P(T_i. > t)\, dt$$

$$\psi_0 = \frac{1}{\lambda_1 + \lambda_2},$$

$$\psi_1 = \int_0^t e^{-\lambda_3 u} F_1(t)dt,$$

$$\psi_2 = \int_0^t e^{-\lambda_3 u} F_2(t)dt$$

$$\psi_3 = \int_0^t \overline{F}_1(t)dt, \qquad \psi_4 = \int_0^t \overline{F}_2(t)dt,$$

$$\psi_5 = \frac{1}{\gamma}.$$

1.7 RELIABILITY AND MTSF

The probability of system that it is operable up to epoch t when it starts from state S_i is denoted by R_i (t). In order to obtain R_0 (t), we consider the following two contingencies.

1. The probability that System stays up in state S_0 and not make any transformation to other state up to epoch t is given by

$$e^{-(\lambda_1+\lambda_2)t} = Z_0(t)$$

2. The probability that the System enters to the state S_1 from S_o amid $(u, u+du),\ u \le t$ and after that beginning from S_1, it stays up consistently amid residual time (t-u), is given by

$$\int_0^t q_{01}(u)du R_1(t-u) = q_{01} © R_1(t)$$

Then

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t)$$
$$R_1(t) = Z_1(t) + q_{10}(t) © R_0(t)$$
$$R_2(t) = Z_2(t) + q_{20}(t) © R_0(t) \qquad (1\text{-}3)$$

where

$$Z_0 = e^{-(\lambda_1+\lambda_2)t}, \qquad Z_1 = e^{-\lambda_3 t}\overline{F}_1(t)$$

$$Z_2 = e^{-\lambda_3 t}\overline{F}_2(t)$$

Taking Laplace transforms of the above equations

$$R_0^*(s) = Z_0^*(s) + q_{01}^*(s) © R_1^*(s) + q_{02}^*(s) © R_2^*(s)$$
$$R_1^*(s) = Z_1^*(s) + q_{10}^*(s) © R_0^*(s)$$
$$R_2^*(s) = Z_2^*(s) + q_{20}^*(s) © R_0^*(s) \qquad (4\text{-}6)$$

For briefness, We require mislaid argument 's' from $q_{ij}^*(s), Z_i^*(s)$ and $R_i^*(s)$. Solving the above matrix equation for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (7)$$

where

$$N_2(s) = Z_0^* + q_{01}^*\, Z_1^* + q_{20}^*\, Z_2^*$$

$$D_2(s) = 1 - q_{01}^*\, q_{10}^* + q_{02}^*\, q_{20}^*.$$

Reliability of the system can be obtained by taking the inverse Laplace transform of equation (7).

Formula for MTSF

$$E(T_0) = \int R_0\ (t)\ dt$$
$$= \lim_{s \to 0} R_0^*\ (s)$$
$$= \frac{N_1(0)}{D_1(0)} \qquad (1\text{-}3)(8)$$

To determine the $N_1(0)$ and $D_1(0)$, we first obtain $Z_i^*(0)$ by using the following result

$$\lim_{s \to 0} Z_i^*(s) = \int Z_i\ (t)\ dt$$

Therefore,

$Z_0^*(0) = \psi_0$ *and* $Z_i^*(0) = \psi_i \;\; (i = 0, 1, 2, 3, 4, 5)$

Thus using $q_{ij}^*(0) = p_{ij}$ we get $N_1(0)$ and $D_1(0)$.

We can get the MTSF of the system by putting these values of $N_1(0)$ and $D_1(0)$ in (8).

1.8 AVAILABILITY ANALYSIS

1. When unit is in normal (N) mode and operative

Let $A_i^N(t)$ be the probability that starting from state S_i the system is up, due to a unit in N-mode at epoch t. By using similar probabilistic arguments as in earlier situation, one can easily obtain the following recurrence relations:

$$A_0^N(t) = Z_0(t) + q_{01}(t) © A_1^N(t) + q_{02}(t) © A_2^N(t)$$

$$A_1^N(t) = Z_1(t) + q_{10}(t) © A_0^N(t) + q_{15}^{(2)}(t) © A_5^N(t)$$

$$A_2^N(t) = Z_2(t) + q_{20}(t) © A_0^N(t) + q_{25}^{(4)}(t) © A_5^N(t)$$

$$A_3^N(t) = q_{35}(t) © A_5^N(t)$$

$$A_4^N(t) = q_{45}(t) © A_5^N(t)$$

$$A_5^N(t) = Z_5 + q_{50}(t) © A_0^N(t)$$

After taking Laplace Transformation and Solving these equation for $A_0^{N*}(s)$

We get

$$A_0^{N*}(s) = \frac{N_2(s)}{D_2(s)}$$

where

$$N_2(s) = Z_0^* + q_{01}^* Z_1^* + q_{01}^* q_{15}^* Z_5^* + q_{02}^* Z_2^* + q_{02}^* q_{25}^* Z_5^*$$

$$D_2(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - (q_{01}^* q_{15}^* + q_{02}^* q_{25}^*) q_{50}^*.$$

Therefore, the steady state availability is given by

$$A_0^N = \lim_{t\to\infty} A_0^N(t)$$

$$\lim_{s\to 0} S A_0^{*N}(s)$$

$$\lim_{s\to 0} S \frac{N_2(s)}{D_2(s)}$$

So by the following results

$$Z_i^*(0) = \int Z_i(t) dt = \psi_i$$

and $q_{ij}^*(0) = p_{ij}$, we have

$$D_2(0) = 0.$$

1.9 BUSY PERIOD ANALYSIS

1. When repairman is occupied in the repair of failed unit

Let $B_i^F(t)$ be the likelihood that the repairman occupied in fixing the failed unit. $B_0^F(t) = q_{01}(t) © B_1^F(t) + q_{02}(t) © B_2^F(t)$

$$B_1^F(t) = Z_1(t) + q_{10}(t) © B_0^F(t) + q_{15}^{(2)}(t) © B_5^F(t) \quad (9\text{-}14)$$

$$B_2^F(t) = Z_2(t) + q_{20}(t) © B_0^F(t) + q_{25}^{(4)}(t) © B_5^F(t)$$

$$B_3^F(t) = Z_3(t) + q_{35}(t) © B_5^F(t)$$

$$B_4^F(t) = Z_4(t) + q_{45}(t) © B_5^F(t)$$

$$B_5^F(t) = q_{50}(t) © B_0^F(t)$$

After taking Laplace Transformation and solving for $B_0^{F*}(s)$

$$B_0^{F*}(s) = \frac{N_3(s)}{D_2(s)}$$

$$N_3(s) = Z_2^* \ q_{01}^* (q_{35}^* + q_{50}^*) + Z_1^* \ q_{25}^* \ q_{45}^* (1 - q_{50}^* \ q_{01}^*) + Z_3^* \ q_{01}^* (q_{10}^* + q_{15}^* \ q_{35}^*)$$

$$D_2(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - (q_{01}^* q_{15}^* + q_{02}^* q_{25}^*) q_{50}^*.$$

2. When repairman is occupied in replacement of a failed unit

Let $B_i^R(t)$ be the probability that starting from state S_i, the repairman is occupied in the replacement of a failed unit.

$$B_0^R(t) = q_{01}(t) © B_1^R(t) + q_{02}(t) © B_2^R(t)$$

$$B_1^R(t) = q_{10}(t) © B_0^R(t) + q_{15}^{(2)}(t) © B_5^R(t)$$

$$B_2^R(t) = q_{20}(t) © B_0^R(t) + q_{25}^{(4)}(t) © B_5^R(t)$$

$$B_3^R(t) = q_{35}(t) © B_5^R(t)$$

$$B_4^R(t) = q_{45}(t) © B_5^R(t)$$

$$B_5^R(t) = Z_5(t) + q_{50}(t) © B_0^R(t)$$

After taking Laplace Transformation and solving for $B_0^{F*}(s)$ we get

$$B_0^{F*}(s) = \frac{N_4(s)}{D_2(s)}$$

$$N_4(s) = q_{25}^* \ q_{45}^* (1 - q_{50}^* \ q_{01}^*) + q_{01}^* (q_{10}^* + q_{15}^* \ q_{35}^*) Z_5^*$$

$$D_2(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - (q_{01}^* q_{15}^* + q_{02}^* q_{25}^*) q_{50}^*$$

1.10 CONCLUSION

This model was constructed for a repairable system with two N. I. U.. Availability, reliability, MTSF and also find difference between type-1 and type-2 failure the result were shown graphically. Result indicates that the MTSF and the system reliability depend on the failure rate.

1.11 PARTICULAR CASE

We consider the condition when all the repair times are follows exponential distribution i.e.

$$F_1(t) = 1 - e^{-\theta_1} \qquad F_2(t) = 1 - e^{-\theta_2}$$

Then

$$p_{10} = \frac{\theta_1}{\theta_1 + \lambda_3} \qquad p_{13} = \frac{\lambda_3}{\theta_1 + \lambda_3}$$

$$p_{20} = \frac{\theta_2}{\theta_2 + \lambda_3} \qquad p_{24} = \frac{\lambda_3}{\theta_2 + \lambda_3}$$

$$p_{57} = \frac{\beta}{\theta + \beta} \qquad p_{50} = \frac{\theta}{\theta + \beta} \quad \psi_1 = \frac{1}{\lambda_3 + \theta_1}$$

$$\psi_2 = \frac{1}{\lambda_3 + \theta_2} \qquad (34\text{-}39)$$

$$\psi_3 = \frac{1}{\theta_1} \qquad \psi_4 = \frac{1}{\theta_2}.$$

1.12 GRAPHICAL STUDY

In graphical study we can see the behaviour of the system, for this curves are plotted for MTSF at the different values of λ_1 and λ_2 and kept other parameter at fixed value.

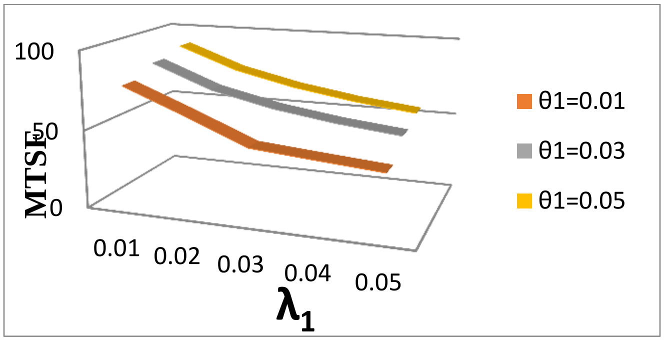

Figure 4.2 shown MTSF w.r.t. type 1 failure.

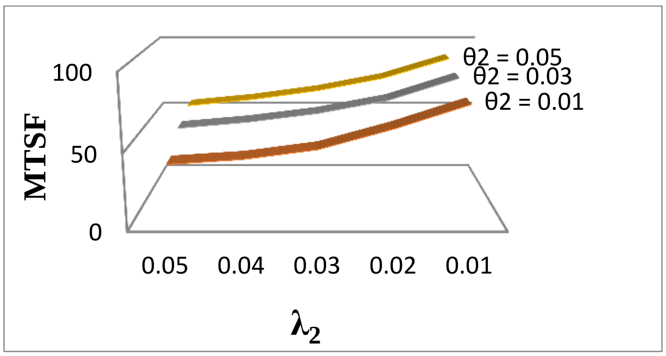

Figure 4.3 shown MTSF w.r.t. type 2 failure.

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Author Profile

Ms.Pooja Vinodiya pursed Bachelor of Science from G.D.C. College, Ujjain in 2010 and Master of Science from SOS in Statistics, Vikram University (Gold Medalist) in year 2012. She is currently pursuing Ph.D. and awarded JRF with Rajeev Gandhi National Fellowship, UGC (New Delhi) in year 2014. She is published two research papers in reputed international journals and conference including ISSAC-2016 at Aligarh Muslim University and National Conference at Savitribai Phule Pune University. Her main research work focus on Reliabiliy Analysis, Availability Analysis, Prevantive Maintenance of system. She has five years reasearch expierence.

