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On Lukasiewicz Disjunction and Conjunction of Pythagorean Fuzzy Matrices

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Abstract**—** In this paper, the algebraic properties of two operations disjunction and conjunction from Lukasiewicz type over Pythagorean fuzzy matries are studied. Also, using the relation between disjunction and conjunction certain results are obtained using modal operators.

Keywords**—** Intuitionistic Fuzzy Matrix, Pythagorean Fuzzy Set, Pythagorean, Fuzzy Matrix, Disjunction, Conjunction.

I. INTRODUCTION

The concept of intuitionistic fuzzy matrix(IFM) was introduced by Pal[4] and simultaneously by Im[3] to generalize the concept of Thomason's[11] fuzzy matrix. Each element in an IFM is expressed by an ordered pair $\left\langle a_{ij}, a'_{ij} \right\rangle$ with $a_{ij}, a'_{ij} \in [0,1]$. The sum $a_{ij} + a'_{ij}$ of each ordered pair is less than or equal to 1. Since the appearance of IFM in 2001, several researchers [5,9] have importantly contributed for the development of IFM theory and its applications. In particular, Muthuraji et.al[6] introduced a new composition operator and studied their algebraic properties. Also they obtained a decomposition of an IFM. Emam and Fndh[2] defined some kinds of IFMs, the max-min and min-max composition of IFMs. Also they derived several important results by these compositions and construct an idempotent intuitionistic fuzzy matrix from any given one through the min-max composition.

Muthuraji and Sriram[7] introduced two operators conjunction and disjunction from Lukasiewic'z type over intuitionistic fuzzy matrix(IFM) and investigated their algebraic properties. Also in [8], they proved the set of all IFMs is a commutative monoid under these operations. Venkatesan and Sriram[12,13] defined Multiplicative operations of IFMs namely X_1, X_2, X_3 and X_4 and investigated their algebraic properties.

Yager[14] introduced Pythagorean fuzzy set(PFS) characterized by a membership degree and a non membership degree satisfying the condition that the square sum of its membership degree and non membership degree is equal to or less than 1, has much stronger ability than intuitionistic fuzzy set to model such uncertain information in multi-criteria decision making(MCDM) problems. Zhang and Xu[15] defined some novel operational laws of PFS and discuss its desirable properties.

The motivation of introducing PFSs is that in the real-life decision process, the sum of the support degree and the against degree to which an alternative satisfying a criterion provided by the decision maker may be bigger than 1 but their square sum is equal to or less than 1.

Silambarasan and Sriram[10] introduced Pythagorean fuzzy matrix(PFM) and its algebraic operations. Atanassov and Tcvetkov[1] introduced the operations disjunction and conjunction from Lukasiewic'z type over intuitionistic fuzzy sets and studied its algebraic properties. We extend these operations to PFMs and studied some of the basic properties of these operations with other predefined operators.

The remainder of this paper is organized as follows. In Section 2, the basic definitions of PFM are given. In Section 3, we define two new operations disjunction and conjunction on PFM and investigate their algebraic properties. The operator complement obeys the De Morgan's laws for the operations disjunction and conjunction. Also, we established the distributive properties of max-min and min-max compositions over disjunction and conjunction. In Section 4, using the relation between disjunction and conjunction certain results are obtained using modal operators.

II. PRELIMINARIES

In this section, we shall briefly review PFMs and their operations.

Definition 2.1[14]: Let a set X be a universe of discourse A Pythagorean fuzzy set (PFS) P is an object having the form $P = \left(\left\langle x, P(\mu_p(x), \nu_p(x)) \mid (x \in X) \right\rangle\right)$, where the function $\mu_p : X \to [0,1]$ and $\nu_p : X \to [0,1]$ defines the degree of membership and non-membership of the element $x \in X$ to P, respectively, and for every $x \in X$, it holds that $(\mu_p(x))^2 + (\nu_p(x))^2 \le 1$.

Definition 12.2 ([10]) : An PFM is a matrix of pairs $A = \left(\left\langle a_{ij}, a'_{ij} \right\rangle\right)$ of a non negative real numbers satisfying $0 \le a_{ij}^2 + {a'_{ij}}^2 \le 1$ for all i, j.

Definition 2 2.3 ([10]): For any two PFMs $A, B \in \mathrm{F}_{mn}$, we have

(i) $A \le B$ iff $a_{ij} \le b_{ij}$ and $a'_{ij} \ge b'_{ij}$,

(ii) $A = B$ iff $a_{ij} = b_{ij}$ and $a'_{ij} = b'_{ij}$,

(iii) $A^C = \left(\left\langle a'_{ij}, a_{ij} \right\rangle\right)$,

(iv) $A \wedge B = \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \right\rangle\right)$,

(v) $A \vee B = \left(\left\langle \max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij}) \right\rangle\right)$,

(vi) $A \oplus_P B = \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, a'_{ij} b'_{ij} \right\rangle\right)$,

(vii) $A \,\mathrm{e}_{P}\, B = \left(\left\langle a_{ij} b_{ij}, \sqrt{{a'_{ij}}^2 + {b'_{ij}}^2 - {a'_{ij}}^2 {b'_{ij}}^2} \right\rangle\right)$,

(viii) $A \otimes_{1P} B = \left(\left\langle \sqrt{\max(a_{ij}^2, b_{ij}^2)}, a'_{ij} b'_{ij} \right\rangle\right)$,

(ix) $A \otimes_{2P} B = \left(\left\langle a_{ij} b_{ij}, \sqrt{\max({a'_{ij}}^2, {b'_{ij}}^2)} \right\rangle\right)$,

(x) The $m \times n$ zero PFM O is an PFM all of whose entries are $\langle 0,1 \rangle$, The $m \times n$ universal PFM J is an PFM all of whose entries are $\langle 1,0 \rangle$.

III. ALGEBRAIC PROPERTIES OF LUKASIEWICZ DISJUNCTION AND CONJUNCTION OF PFMS

Atanassov and Tcvetkov[1] introduced the operations disjunction and conjunction from Lukasiewicz type over intuitionistic fuzzy sets and studied its algebraic properties. We extend these operations to PFMs and studied some of the basic properties of these operations with other predefined operators.

Definition 3.1: 3 Using intuitionistic fuzzy form of Lukasiewicz implication, we will introduce a disjunction

$$A \vee_L B = \left(\left\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)} \right\rangle\right).$$

We will call the new disjunction Lukasiewicz Pythagorean fuzzy disjunction.
Also, we can construct,

$$A \wedge_L B = \left(\left\langle \sqrt{\max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{\min(1, {a'_{ij}}^2 + {b'_{ij}}^2)} \right\rangle\right).$$

We will call the new conjunction Lukasiewicz Pythagorean fuzzy conjunction.

Remark 3.2: 4 For both new operations, having in mind that disjunction is obtain from conjunction

$$\begin{aligned}
&(A^C \wedge_L B^C)^C \\
&= \left(\left\langle \sqrt{\max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{\min(1, {a'_{ij}}^2 + {b'_{ij}}^2)} \right\rangle\right) \\
&= (A \vee_L B).
\end{aligned}$$

Property 3.3:5 For any two PFMs $A, B \in \mathrm{F}_{mn}$, $A \vee_L B$ and $A \wedge_L B$ are PFMs.

Proof. Let A and B be any two PFMs.
If ${a'_{ij}}^2 + {b'_{ij}}^2 \le 1$, then

$$\begin{aligned}
0 &\le \left(\sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}\right)^2 + \left(\sqrt{\max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right)^2 \\
&\le \min(1, a_{ij}^2 + b_{ij}^2) \\
&\le 1.
\end{aligned}$$

If ${a'_{ij}}^2 + {b'_{ij}}^2 > 1$, then

$$\begin{aligned}
0 &\le \left(\sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}\right)^2 + \left(\sqrt{\max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right)^2 \\
&\le a_{ij}^2 + b_{ij}^2 + {a'_{ij}}^2 + {b'_{ij}}^2 - 1 \\
&\le 2 - 1 \\
&\le 1.
\end{aligned}$$

Thus $A \vee_L B$ is a PFM.

Similarly we can prove $A \wedge_L B$ also PFM.

The following properties are obvious.

Property 3.4: 6 For any PFM $A \in \mathrm{F}_{mn}$, we have

$(i) A \vee_L O = A,$

$(ii) A \vee_L J = J,$

$(iii) A \wedge_L O = O,$

$(iv) A \wedge_L J = A.$

The operations disjunction andconjunction are commutative as well as associative

.

Property 3.5: 7 For any three PFMs $A, B, C \in F_{mn}$, we have

$(i) A \vee_L B = B \vee_L A,$

$(ii) (A \vee_L B) \vee_L C = A \vee_L (B \vee_L C),$

$(iii) A \wedge_L B = B \wedge_L A,$

$(iv) (A \wedge_L B) \wedge_L C = A \wedge_L (B \wedge_L C).$

The operator complement obey the De Morgan's laws for the operations disjunction and conjunction.

Property 3.6: 8 For any two PFMs $A, B \in F_{mn}$, we have

$(i) (A \vee_L B)^C = A^C \wedge_L B^C,$

$(ii) (A \wedge_L B)^C = A^C \vee_L B^C.$

Proof.

$(i) (A \vee_L B)^C$

$$= \left(\left\langle \sqrt{\max(0, a'^2_{ij} + b'^2_{ij} - 1)}, \sqrt{\min(1, a^2_{ij} + b^2_{ij})} \right\rangle \right)$$

$= A^C \wedge_L B^C.$

Hence, $(A \vee_L B)^C = A^C \wedge_L B^C.$

The proof (ii) is similar to that of (i).

Property 3.7: 9 For any PFM $A \in F_{mn}$, we have

$(i) (A \vee_L A^C)^C = A \wedge_L A^C,$

$(ii) (A \wedge_L A^C)^C = A \vee_L A^C.$

Proof.

$(i) A \vee_L A^C$

$$= \left(\left\langle \sqrt{\min(1, a^2_{ij} + a'^2_{ij})}, \sqrt{\max(0, a'^2_{ij} + a^2_{ij} - 1)} \right\rangle \right).$$

$(A \vee_L A^C)^C$

$$= \left(\left\langle \sqrt{\max(0, a'^2_{ij} + a^2_{ij} - 1)}, \sqrt{\min(1, a^2_{ij} + a'^2_{ij})} \right\rangle \right)$$

$= A \wedge_L A^C.$

Hence, $(A \vee_L A^C)^C = A \wedge_L A^C.$

The proof (ii) is similar to that of (i).

The distributive properties of max-min and min-max compositions over disjunction and conjunction.

Property 3.8: 10 For any three PFMs $A, B, C \in F_{mn}$, we have

$(i) (A \wedge B) \vee_L C = (A \vee_L C) \wedge (B \vee_L C),$

$(ii) (A \vee B) \wedge_L C = (A \wedge_L C) \vee (B \wedge_L C).$

Proof. $(i) (A \wedge B) \vee_L C$

$$= \left(\left\langle \min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij}) \right\rangle \right) \vee_L \left(\left\langle c_{ij}, c'_{ij} \right\rangle \right)$$

$$= \left(\left\langle \sqrt{\min(1, \min(a^2_{ij}, b^2_{ij}) + c^2_{ij})}, \sqrt{\max(0, \max(a'^2_{ij}, b'^2_{ij}) + c'^2_{ij} - 1)} \right\rangle \right)$$

$$= \left(\left\langle \sqrt{\min(1, a^2_{ij} + c^2_{ij}, b^2_{ij} + c^2_{ij})}, \sqrt{\max(0, a'^2_{ij} + c'^2_{ij} - 1, b'^2_{ij} + c'^2_{ij} - 1)} \right\rangle \right)$$

$$= \left(\left\langle \sqrt{\min(1, a^2_{ij} + c^2_{ij})}, \sqrt{\max(0, a'^2_{ij} + c'^2_{ij} - 1)} \right\rangle \right) \wedge \left(\left\langle \sqrt{\min(1, b^2_{ij} + c^2_{ij})}, \sqrt{\max(0, b'^2_{ij} + c'^2_{ij} - 1)} \right\rangle \right)$$

$= (A \vee_L C) \wedge (B \vee_L C)$

Hence, $(A \wedge B) \vee_L C = (A \vee_L C) \wedge (B \vee_L C).$

The proof (ii) is similar to that of (i).

Similarly, we can prove the following property.

Property 11 3.9: For any three PFMs $A, B, C \in F_{mn}$, we have

$(i) (A \vee B) \vee_L C = (A \vee_L C) \vee (B \vee_L C),$

$(ii) (A \wedge B) \wedge_L C = (A \wedge_L C) \wedge (B \wedge_L C).$

While, the following equalities are not valid.

$(i) (A \vee_L B) \vee C = (A \vee C) \vee_L (B \vee C),$

$(ii) (A \wedge_L B) \vee C = (A \vee C) \wedge_L (B \vee C),$

$(iii) (A \vee_L B) \wedge C = (A \wedge C) \vee_L (B \wedge C),$

$(iv) (A \wedge_L B) \wedge C = (A \wedge C) \wedge_L (B \wedge C),$

$(v) (A \vee_L B) \wedge_L C = (A \wedge_L C) \vee_L (B \wedge_L C),$,

$(vi) (A \wedge_L B) \vee_L C = (A \vee_L C) \wedge_L (B \vee_L C).$

IV. MODAL OPERATORS ON PFM

Pal[9] defined the necessity and possibility operators(modal operators) for IFMs. Murugadas et al.[5] studied the relations between $\Box$ and $\Diamond$ operators for IFMs. Analogous to these definitions Silambarasan and Sriram[10] defined the necessity and possibility operators for PFMs. In this section, using the relation between disjunction and conjunction certain results are obtained using modal operators.

Definition 4.112 ([10]): For any PFM $A \in F_{mn}$, we have

$$(i)\Box A = \left(\left\langle a_{ij}, \sqrt{1-a_{ij}^2}\right\rangle\right),$$

$$(ii)\Diamond A = \left(\left\langle \sqrt{1-a_{ij}'^2}, a_{ij}'\right\rangle\right).$$

.

Property 4.2: 13 For any two PFMs $A, B \in F_{mn}$, we have

$$(i)\Box(A \vee_L B) = \Box A \vee_L \Box B,$$

$$(ii)\Box(A \wedge_L B) = \Box A \wedge_L \Box B,$$

$$(iii)\Diamond(A \vee_L B) = \Diamond A \vee_L \Diamond B,$$

$$(iv)\Diamond(A \wedge_L B) = \Diamond A \wedge_L \Diamond B.$$

Proof. (i) $\Box(A \vee_L B)$

$$= \Box\left(\left\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\max(0, a_{ij}'^2 + b_{ij}'^2 - 1)}\right\rangle\right)$$

$$= \left(\left\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{1-\min(1, a_{ij}^2 + b_{ij}^2)}\right\rangle\right)$$

$$= \left(\left\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\max(1-1, 1-a_{ij}^2 - b_{ij}^2)}\right\rangle\right)$$

$$= \left(\left\langle \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\max(0, (1-a_{ij}^2) + (1-b_{ij}^2) - 1)}\right\rangle\right)$$

$$= \Box A \vee_L \Box B.$$

Hence, $\Box(A \vee_L B) = \Box A \vee_L \Box B.$

The proof (ii), (iii) and (iv) are similar to that of (i).

Property 4.3:14 For any two PFMs $A, B \in F_{mn}$, we have

$$(i)\Box(A \vee_L B)^C = \Box A^C \wedge_L \Box B^C,$$

$$(ii)\Box(A \wedge_L B)^C = \Box A^C \vee_L \Box B^C,$$

$$(iii)\Diamond(A \vee_L B)^C = \Diamond A^C \wedge_L \Diamond B^C,$$

$$(iv)\Diamond(A \wedge_L B)^C = \Diamond A^C \vee_L \Diamond B^C.$$

Proof. $(i)\Box(A \vee_L B)^C$

$$= \left(\left\langle \sqrt{1-\min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{\min(1, a_{ij}^2 + b_{ij}^2)}\right\rangle\right)$$

$$= \Box A^C \wedge_L \Box B^C.$$

Hence, $\Box(A \vee_L B)^C = \Box A^C \wedge_L \Box B^C.$

The proof (ii), (iii) and (iv) are similar to that of (i).

V. CONCLUSION

In this paper, we define two new operations disjunction and conjunction on PFM and investigate their algebraic properties. The operator complement obey the De Morgan's laws for the operations disjunction and conjunction. Also, we established the distributive properties of max-min and min-max compositions over disjunction and conjunction. And using the relation between disjunction and conjunction certain results are obtained using modal operators.

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