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Statistical Power Function of Average Control Charts for Non-Normal Data

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Abstract— A mathematical investigation has been made to examine in what way the power function of the usual control chart for mean based on the assumption of normality is affected when the characteristics of an item possesses a non-normal distribution specified by the first four terms of an Edgeworth series. Power functions for known standard deviation have been considered and expressions giving corrections due to parental skewness and kurtosis have been obtained in addition to the normal theory expression.

***Keywords*—**Average Control Chart, Power Function, Non-Normal distribution.

I. Introduction

Statistical methods have been widely applied in industrial process control. One of the key methods is the control chart techniques, which has been use in industry with the aim of improving processes by reducing variations. It detects assignable causes in process so that process investigation and corrective actions can be made before many defective products are manufactured. When designing control charts, it has been a usual assumption that the observations are independent, identical and normally distributed. In practice, this assumption is not always true. Many researches have been made to increase the sensitivity of the control chart and to see the effect of non-normal population on control chart. Yourstone and Zimmer (1992), Chou *et al.* (2000), Chou *et al.* (2005) utilized the Burr distribution to design the control limits of mean chart for non-normal data. Non-normality is not necessarily the result of an out of control process; indeed, many processes are non-normal. Considering this Duclose *et al.* (2005) proposed a new control chart called L-chart especially adapted for non-normal process. Haynes *et al.* (2008) measured the degree of non-normality in the control statistic while retaining the assumption of independence among the observations of control chart for mean under non-normal population. The power of a test statistic is the likelihood that the test effectively rejects the null hypothesis. It is influenced by the sample size, the significance level of the test, and the variability of the data. Singh and Mishra (2017) obtained an expression for the power of control chart with standardized normal variate for Singly-Truncated Binomial Distribution.

In this paper, an attempt has been made to examine the effect of non-normality on the power function of the control chart for mean when the characteristics of an item possesses a non-normal distribution specified by the first four terms of an Edgeworth series. Power functions for known standard deviation have been considered.

II. Power Function of Average Control Chart for Non-Normal Population

A process which is not in the statistical control suggests the presence of the assignable causes of variation which throws the process out of control. These causes may be traced and eliminated so that the process may return to the operation under stable statistical control. Thus, the data used for establishing the limits on the control charts comes from the process that is $N\left(\mu, \sigma^2/n\right)$. When the process shifts, the data is assumed to come from an $N\left(\mu', \sigma^2/n\right)$population. In the literature of statistical quality control, hypothesis testing is always used to summarize an inference on the mean of a population which is given as:

$$\left.\begin{array}{l} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 . \end{array}\right\} \tag{2.1}$$

For non-normal population, the density function is given by the first four terms of an Edgeworth series:

$$f(x) = \frac{1}{\sigma}\left\{\phi\left(\frac{x-\mu}{\sigma}\right) - \frac{\lambda_3}{6}\phi^{(3)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_4}{24}\phi^{(4)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_3^2}{72}\phi^{(6)}\left(\frac{x-\mu}{\sigma}\right)\right\} \tag{2.2}$$

The distribution of the sample mean is given by Gayen

(1949), as:

$$g(\bar{x}) = \frac{\sqrt{n}}{\sigma}\left\{\phi\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) - \frac{\lambda_3}{6\sqrt{n}}\phi^{(3)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) + \frac{\lambda_4}{24n}\phi^{(4)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) + \frac{\lambda_3^2}{72n}\phi^{(6)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right)\right\} \tag{2.3}$$

Now integrating equation (2.3) after replacing μ by μ' , we have:

$$\tau(\bar{x}) = \left\{\phi\left(\frac{\bar{x}-\mu'}{\sigma/\sqrt{n}}\right) - \frac{\lambda_3}{6\sqrt{n}}\phi^{(2)}\left(\frac{\bar{x}-\mu'}{\sigma/\sqrt{n}}\right) + \frac{\lambda_4}{24n}\phi^{(3)}\left(\frac{\bar{x}-\mu'}{\sigma/\sqrt{n}}\right) + \frac{\lambda_3^2}{72n}\phi^{(5)}\left(\frac{\bar{x}-\mu'}{\sigma/\sqrt{n}}\right)\right\}, \tag{2.4}$$

where

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}$$

and

$$\phi^{(r)}(X) = \frac{d^r}{dX}\phi(X) \quad .$$

If the samples of size n are taken from the population $N\left(\mu', \sigma^2/n\right)$ and the value of the mean is plotted with the control limits $\mu \pm 3\sigma/\sqrt{n}$, then the power of detecting the change of process is given by the following formula:

$$P_{\bar{X}} = P_r\left\{\bar{X} \geq \mu + 3\sigma/\sqrt{n}\right\} + P_r\left\{\bar{X} \geq \mu - 3\sigma/\sqrt{n}\right\} \tag{2.5}$$

Standardizing the above equation (2.5),

$$Z = \frac{\bar{x}-\mu'}{\sigma/\sqrt{n}} \tag{2.6}$$

The power function for normal distribution is obtained by converting equation (2.5) into the standardized form, we have:

$$P_{\bar{X}} = \left\{P_r\left(Z \geq (\mu-\mu')\sqrt{\frac{n}{\sigma^2}} + 3\right) + P_r\left(Z \leq (\mu-\mu')\sqrt{\frac{n}{\sigma^2}} - 3\right)\right\}, \tag{2.7}$$

$$P_{\bar{X}} = \left\{P_r\left(Z \geq -d\sqrt{n} + 3\right) + P_r\left(Z \leq -d\sqrt{n} - 3\right)\right\}, \tag{2.8}$$

$$P_{\bar{X}} = \left\{P_r\left(Z \leq d\sqrt{n} - 3\right) + P_r\left(Z \leq -d\sqrt{n} - 3\right)\right\}, \tag{2.9}$$

$$P_{\bar{X}} = \left\{\varphi\left(-d\sqrt{n} + 3\right) + \varphi\left(-d\sqrt{n} - 3\right)\right\}, \tag{2.10}$$

where $d = \dfrac{\mu-\mu'}{\sigma}.$

The Power Function of the control chart when the underlying population is non-normal is obtained by putting above value of equation (2.10) in equation (2.4):

$$\tau(\bar{x}) = \left\{\begin{array}{l} \left\{\varphi\left(-d\sqrt{n}+3\right) + \varphi\left(-d\sqrt{n}-3\right)\right\} - \\ \dfrac{\lambda_3}{6\sqrt{n}}\phi^{(2)}\left\{\varphi\left(-d\sqrt{n}+3\right) + \varphi\left(-d\sqrt{n}-3\right)\right\} + \\ \dfrac{\lambda_4}{24n}\phi^{(3)}\left\{\varphi\left(-d\sqrt{n}+3\right) + \varphi\left(-d\sqrt{n}-3\right)\right\} + \\ \dfrac{\lambda_3^2}{72n}\phi^{(5)}\left\{\varphi\left(-d\sqrt{n}+3\right) + \varphi\left(-d\sqrt{n}-3\right)\right\} \end{array}\right\} \tag{2.11}$$

III. Numerical Illustration and Results

In order to see how the normal theory power Function is distorted in a situation of non-normality, we consider few specific values of standardized cumulants, λ_3 and λ_4 shown in Table-1. For the negative value of λ_4 the power Function becomes steeper while the reverse is the case when λ_4 is positive. Now, we consider the case when λ_3 is positive *i.e.* $\lambda_3 = 0.5$ and $\lambda_4 = 0$, it is clear from the table values that power Functions becomes steeper and reverse case happens for negative value of λ_3. It may be noted from Table-1 that the contribution of λ_4 to power function is small as compared to that of λ_3 . To a lepto-kurtic population, when the normal theory control chart is applied an overall improvement is likely to result and for the values of λ_4 it will be really a marked improvement. On the other hand, in case of platy-kurtic population, the power function deteriorates. Positive skewness tends to improve the power Function. Hence, the presence of skewness and kurtosis would effect a plan depends on the magnitudes of λ_3 and λ_4 in a particular case and their effect being additive, the combined result is likely to be such as not to permit a conclusion of general nature to be derived.

Table- 1: Values of Power Function for Average Control Chart under non-normal population when .

$(\lambda_3, \lambda_4) \rightarrow$ / $d \downarrow$	$(0, 0)$	$(0, 0.5)$	$(0, 1.0)$	$(0, 2.0)$	$(0.5, 0)$	$(-0.5, 0.5)$	$(0.5, 0.5)$	$(0, -0.5)$	$(0.5, 2.0)$
0.5	0.02994	0.03132	0.03269	0.03544	0.02365	0.03799	0.02503	0.0286	0.02915
0.8	0.11292	0.11337	0.11383	0.11473	0.10925	0.11638	0.10970	0.1125	0.11106
1.0	0.22245	0.22206	0.22166	0.22087	0.22647	0.21683	0.22608	0.2228	0.22489
1.3	0.46291	0.46275	0.46260	0.46229	0.47746	0.44795	0.47731	0.4631	0.47685
1.5	0.63837	0.63885	0.63934	0.64030	0.65102	0.62707	0.65151	0.6379	0.65296
1.8	0.84730	0.84725	0.84720	0.84709	0.84734	0.84818	0.84729	0.8474	0.84714
2.0	0.92951	0.92854	0.92757	0.92564	0.92373	0.93451	0.92276	0.9305	0.91986
2.3	0.98394	0.98265	0.98137	0.97878	0.97831	0.98778	0.97703	0.9852	0.97316
2.5	0.99520	0.99434	0.99349	0.99176	0.99203	0.99710	0.99117	0.9961	0.98859
2.8	0.99944	0.99919	0.99893	0.99842	0.99867	0.99982	0.99841	0.9997	0.99764
3.0	0.99990	0.99981	0.99973	0.99957	0.99968	0.99999	0.99959	1.0000	0.99935
3.3	0.99999	0.99998	0.99998	0.99995	0.99997	1.00000	0.99996	1.0000	0.99994
3.5	1.00000	0.99998	0.99998	0.99999	1.00000	1.00000	0.99999	1.0000	0.99999

Furthermore, It is seen that, for negative and positive values of *d*, the values of the power function results to be exactly same. In general, it can be said that the power function of the average control chart is slightly affected by the non-normality of the population. The value of the power function increases or decreases as the departure from normality depends on the signs of the skewness and kurtosis.

Overall, this paper advances the research of non-normality to the determination of the power function for the average control chart.

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