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MATLAB Program to Generate Harary energy of Certain Mesh Derived Networks

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Abstract— In this paper, we compute the Harary energy of grid, cylinder, torus, extended grid networks by using MATLAB code. Also we obtained Milovanovi'c bounds for Harary energy of a graph.

Keywords—MATLAB code, Harary energy, Grid, Cylinder, Torus, Extended grid.

I. INTRODUCTION

The concept of energy of a graph was introduced by I. Gutman [1] in the year 1978. Let G be a graph with n vertices and m edges and let A = (a_{ij}) be the adjacency matrix of the graph. The characteristic equation of G is $|A - \lambda I| = 0$. The roots of this equation $\lambda_1, \lambda_2, \ldots, \lambda_n$ are called characteristic roots or eigenvalues of A (or G), which are usually taken in increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. The largest eigenvalue λ_1 is called spectral radius of G. As A is real symmetric, the eigenvalues of G are real with sum equal to zero.

Definition 1.1. The collection of eigenvalues of adjacency matrix is called the spectrum of G. If $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k$ are the distinct eigenvalues of G with multiplicities $m_1, m_2, \ldots, m_k$ respectively then spec(G) = $\begin{pmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_k \\ m_1 & m_2 & \ldots & m_k \end{pmatrix}$.

Definition 1.2. The energy E(G) of G is defined to be the sum of the absolute values of the eigenvalues of G. i.e., E(G) = $\sum_{i=1}^{n} |\lambda_i|$.

For details on the mathematical aspects of the theory of graph energy see the reviews [2], papers [3, 4, 5] and the references cited there in. The basic properties including various upper and lower bounds for energy of a graph have been established in [6, 7] and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [8, 9].

One of the important applications of graph theory is to represent practical problems by means of structural models. Graph theoretical ideas are highly utilized in computer science applications. Modeling of computer science problems leads to the development of various algorithms. The main aim of this paper is to write MATLAB program to compute Harary energy of grid, torus, cylinder and extended grid. In the last section Milovanovic bounds for Harary energy are obtained.

II. HARARY ENERGY

On addressing problem for loop switching, R. L. Graham, H. O. Pollak [10] defined distance matrix of a graph. The concept of distance energy was defined by G. Indulal et al. [11] in the year 2008. Later, A Dilek Gungor and A Sinan Cevik introduced the concept of Harary energy. Let G be a simple graph of order n with vertex set V and edge set E. Let d_{ij} be the distance between the vertices v_i and v_j then the $n \times n$ matrix $D(G) = (d_{ij})$ is called the distance matrix of G. The Harary matrix of G is the square matrix of order n whose (i, j)-entry is where d_{ij} is the distance between the vertices v_i and v_j. Let $\rho_1, \rho_2, \ldots, \rho_n$ be the eigenvalues of the Harary matrix of G. The Harary energy HE is de ned by $HE(G) = \sum_{i=1}^{n} |\rho_i|$. Detailed studies on distance energy and Harary energy can be found in [12, 13, 14, 15, 16,17].

III. HARARY ENERGY OF SOME STANDARD MESH DERIVED NETWORKS

In the year 2012, Bharati Rajan [18] computed energy of certain mesh derived networks of order $n \times n$ by using MATLAB. In this paper we compute Harary energy of grid, cylinder, torus and extended grid of order mn by using MATLAB.

1. Grid G(m, n)

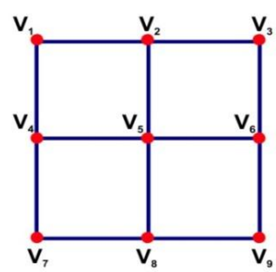

Figure : 8.1

The topological structure of a grid network, denoted by G(m; n), is de ned as the cartesian product $P_m \times P_n$ of undirected paths P_m and P_n. The spectrum of the graph does not depend on the numbering of the vertices. However here we adopt a particular numbering such that the Harary matrix has a pattern which is common for any dimension. We follow the sequential numbering from left to right as shown in the figure 8.1

MATLAB program to generate Harary energy of a grid G(m, n)

```
clc;

fprintf('HARARY ENERGY OF H GRID s \times t\n'); s=input('Enter
the value of s: ');
t=input('Enter the value of t: ');
if(s>1 && t>1)

    H=zeros(s*t);

    HM=zeros(s*t);

    c=1;

    for k=1:s

        for l=1:t
```

```
            u(k,l)=c;

            c=c+1;

        end

    end

    for k=1:s

        for l=1:t

            if(s>2 && t>2)

            if((u(k,l)+1 > 0) && (u(k,l)-1 > 0) && (u(k,l)-t > 0) && (u(k,l)+t <=
                H(u(k,l),u(k,l)-1)=1;

                    H(u(k,l)-1,u(k,l))=1;
                    H(u(k,l),u(k,l)+1)=1;

                    H(u(k,l)+1,u(k,l))=1;

                    H(u(k,l),u(k,l)-t)=1;

                    H(u(k,l)-t,u(k,l))=1;

                    H(u(k,l),u(k,l)+t)=1;

                    H(u(k,l)+t,u(k,l))=1;

                else

                    if(k>1 && k<s)

                        H(u(k,l),u(k,l)-t)=1;

                        H(u(k,l)-t,u(k,l))=1;

                        H(u(k,l),u(k,l)+t)=1;

                        H(u(k,l)+t,u(k,l))=1;

                    end

                    if(l>1 && l<t)

                        H(u(k,l),u(k,l)-1)=1;

                        H(u(k,l)-1,u(k,l))=1;

                        H(u(k,l),u(k,l)+1)=1;

                        H(u(k,l)+1,u(k,l))=1;

                    end
```



```
        end
    else
        if(s>2)
            if((k > 1) && (k < s))
                H(u(k,l),u(k,l)-t)=1;
                H(u(k,l)-t,u(k,l))=1;
                H(u(k,l),u(k,l)+t)=1;
                H(u(k,l)+t,u(k,l))=1;
                if(l= =1)
                    H(u(k,l),u(k,l)+1)=1;
                    H(u(k,l)+1,u(k,l))=1;
                end
            else
                if(k < s && l <= t)
                    H(u(k,l),u(k,l)+t)=1;
                    H(u(k,l)+t,u(k,l))=1;
                end
                if(mod(l,t) ~= 0)
                    H(u(k,l), u(k,l)+1)=1;
                    H(u(k,l)+1, u(k,l))=1;
                end
            end
        else
            if((l > 1) && (l < t))
                H(u(k,l),u(k,l)-1)=1;
                H(u(k,l)-1,u(k,l))=1;
                H(u(k,l),u(k,l)+1)=1;
                H(u(k,l)+1,u(k,l))=1;
```

```
                    if(k < s)
                        H(u(k,l),u(k,l)+t)=1;
                        H(u(k,l)+t,u(k,l))=1;
                    end
                else
                    if(k < s && l <= t)
                        H(u(k,l),u(k,l)+t)=1;
                        H(u(k,l)+t,u(k,l))=1;
                    end
                    if(mod(l,t) ~= 0)
                        H(u(k,l), u(k,l)+1)=1;
                        H(u(k,l)+1, u(k,l))=1;
                    end
                end
            end
        end
    end
end
D=graphallshortestpaths(sparse(H));
   for k=1:s*t
    for l=1:s*t
        if(D(k,l) > 1)
            HM(k,l)=1/D(k,l);
        else
            HM(k,l)=D(k,l);
        end
    end
end
```



```
    fprintf('Harary matrix: \n');

    disp(HM);

    eigenvaluesofgrid=eig(HM);

    fprintf('Co-efficients of characterstic polynomial are\n');

    fprintf('%4.4f\t',poly(HM));

    fprintf('\n');

    fprintf('Eigenvalues are\n');

    fprintf('%4.4f\t',eigenvaluesofgrid);

    fprintf('\n');

    energy=sum(abs(eigenvaluesofgrid));

    fprintf('Harary energy of a grid is %4.4f\n',energy); else
    fprintf('Not a grid. s and t values must be greater than 1\n');

end
```

Example: *Harary matrix of grid G(3,3) is*

$$\begin{pmatrix} 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & 1 & 0 & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{3} & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & \frac{1}{3} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{3} & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 \end{pmatrix}$$

Characteristic polynomial is $\rho^9 - 16.5139\rho^7 - 37.6389\rho^6 - 20.8973\rho^5 + 16.5982\rho^4 + 19.246\rho^3 + 1.375\rho^2 - 3.4887\rho - 0.9118$.

Harary eigenvalues are -1.7459, -1.3261, -1.3261, -0.75, -0.5, -0.4796, 0.5761, 0.5761, 4.9756.

Harary energy of a grid G(3; 3) is 12.2554.

2. Cylinder C(m, n)

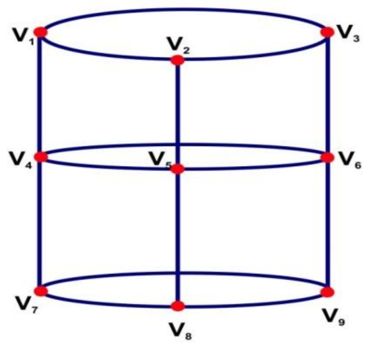

Figure : 8.2

The topological structure of a cylinder network is denoted by C(m, n) and is defined as the cartesian product $P_m \times C_n$ of undirected path P_m and an undirected cycle C_n. The numbering of vertices adopted for cylinder is same as that of a grid.

MATLAB program to generate Harary energy of cylinder C(m, n)

```
clc;

fprintf('HARARY ENERGY OF A CYLINDER s \times t\n');
s=input('Enter the value of s: ');
t=input('Enter the value of t: ');
if(s>1 && t>1)

    H=zeros(s*t);

    HM=zeros(s*t);

    c=1;

    for k=1:s

        for l=1:t

            u(k,l)=c;

            c=c+1;

        end

    end

    for k=1:s

        for l=1:t

            if(l == 1)

                H(u(k,l),u(k,l)-1+t)=1;

                H(u(k,l)-1+t,u(k,l))=1;
```

```
end

if(s>2 && t>2)

    if((u(k,l)+1 > 0) && (u(k,l)-1 > 0) && (u(k,l)-t > 0) && (u(k,l)+t <=
      if(l>1 && l<t)
        H(u(k,l),u(k,l)-1)=1;

            H(u(k,l)-1,u(k,l))=1;

            H(u(k,l),u(k,l)+1)=1;

            H(u(k,l)+1,u(k,l))=1;

        end

        H(u(k,l),u(k,l)-t)=1;

        H(u(k,l)-t,u(k,l))=1;

        H(u(k,l),u(k,l)+t)=1;

        H(u(k,l)+t,u(k,l))=1;

        if(l == 1)

            H(u(k,l),u(k,l)-1+t)=1;

            H(u(k,l)-1+t,u(k,l))=1;

        end

    else

        if(k>1 && k<s)

            H(u(k,l),u(k,l)-t)=1;

            H(u(k,l)-t,u(k,l))=1;

            H(u(k,l),u(k,l)+t)=1;

            H(u(k,l)+t,u(k,l))=1;

        end

        if(l>1 && l<t)

            H(u(k,l),u(k,l)-1)=1;

            H(u(k,l)-1,u(k,l))=1;

            H(u(k,l),u(k,l)+1)=1;

            H(u(k,l)+1,u(k,l))=1;
```

```
            end
        end
    else
        if(s>2)
            if((k > 1) && (k < s))
                H(u(k,l),u(k,l)-t)=1;
                H(u(k,l)-t,u(k,l))=1;
                H(u(k,l),u(k,l)+t)=1;
                H(u(k,l)+t,u(k,l))=1;
                if(l==1)
                    H(u(k,l),u(k,l)+1)=1;
                    H(u(k,l)+1,u(k,l))=1;
                end
            else
                if(k < s && l <= t)
                    H(u(k,l),u(k,l)+t)=1;
                    H(u(k,l)+t,u(k,l))=1;
                end
                if(mod(l,t) ~= 0)
                    H(u(k,l), u(k,l)+1)=1;
                    H(u(k,l)+1, u(k,l))=1;
                end
            end
        else
            if((l > 1) && (l < t))
                H(u(k,l),u(k,l)-1)=1;
                H(u(k,l)-1,u(k,l))=1;
                H(u(k,l),u(k,l)+1)=1;
```



```
                    H(u(k,l)+1,u(k,l))=1;
                    if(k < s)
                        H(u(k,l),u(k,l)+t)=1;
                        H(u(k,l)+t,u(k,l))=1;
                    end
                else
                    if(k < s && l <= t)
                        H(u(k,l),u(k,l)+t)=1;
                        H(u(k,l)+t,u(k,l))=1;
                    end
                    if(mod(l,t) ~= 0)
                        H(u(k,l), u(k,l)+1)=1;
                        H(u(k,l)+1, u(k,l))=1;
                    end
                end
            end
        end
    end
end
D=graphallshortestpaths(sparse(H));
%fprintf('Harary matrix: \n');
%disp(D);
for k=1:s*t
    for l=1:s*t
        if(D(k,l) > 1)
            HM(k,l)=1/D(k,l);
        else
            HM(k,l)=D(k,l);
```

```
            end
        end
    end
    fprintf('HARARY matrix: \n');
    disp(HM);
    eigenvaluesofgrid=eig(HM);
    fprintf('Co-efficients of characterstic polynomial are\n');
    fprintf('%4.4f\t',poly(HM));
    fprintf('\n');
    fprintf('Eigenvalues are\n');
    fprintf('%4.4f\t',eigenvaluesofgrid);
    fprintf('\n');
    energy=sum(abs(eigenvaluesofgrid));
    fprintf('harary energy of a cylinder is %4.4f\n',energy);
    else
fprintf('not a cylinder. s and t values must be greater than 2\n'); end
```

Example 3.2. *Harary matrix of cylinder C(3, 3) is*

$$\begin{pmatrix} 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial is $\rho^9 - 19.4167\rho^7 - 50.8333\rho^6 - 40.0903\rho^5 + 11.8148\rho^4 + 32.5823\rho^3 + 15.8962\rho^2 + 3.0697 + 0.21$.

Harary eigenvalues are -1.6287, -1.6287, -1.1667, -1.1667, -0.3046, -0.2047, -0.2047, 0.8333, 5.4713.

Harary energy of a cylinder C(3, 3) is 12.6092.

3. Torus T(m, n)

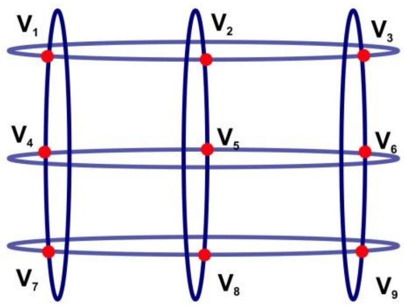

Figure : 8.3

The topological structure of a torus network is denoted by T(m, n) and is defined as the Cartesian product $C_m \times C_n$ where C_m and C_n are undirected cycles. The numbering adopted for torus is same as that of a grid.

MATLAB program to generate Harary energy of torus T(m, n)

```
clc;

fprintf('HARARY ENERGY OF H TORUS m×n\t'); s=input('Enter the value
of s: ');
t=input('Enter the value of t: ');
 if(s>1 && t> 1)
        H=zeros (s*t);

        HM=zeros(s*t);

        c=1;

        for k=1:s

                for l=1:t

                        u(k,l)=c;

                        c=c+1;

                end

        end
```



```
for i=1:s
    for j=1:t
        if(j == 1)
            H(u(k,l),u(k,l)-1+t)=1;
            H(u(k,l)-1+t,u(k,l))=1;
        end
        if(i == 1)
            H(u(k,l),u(k,l)+((s-1)*t))=1;
            H(u(k,l)+((s-1)*t),u(k,l))=1;
        end
        if(j < t)
          H(u(k,l), u(k,l)+1)=1;
            H(u(k,l)+1, u(k,l))=1;
        end
        if(i < s)
            H(u(k,l),u(k,l)+t)=1;
            H(u(k,l)+t,u(k,l))=1;
        end
    end
end
D=graphallshortestpaths(sparse(H));
%fprintf('Harary matrix: \n');
%disp(D);
for k=1:s*t
    for l=1:s*t
        if(D(k,l) > 1)
            HM(k,l)=1/D(k,l);
        else
```



```
                    HM(k,l)=D(k,l);
                end

            end

        end

        fprintf('Harary matrix: \n');

        disp(HM);

        eigenvaluesofgrid=eig(HM);

        fprintf('Co-efficients of characterstic polynomial are\n');

        fprintf('%4.4f\t',poly(HM));

        fprintf('\n');

        fprintf('Eigenvalues are\n');

        fprintf('%4.4f\t',eigenvaluesofgrid);

        fprintf('\n');

        energy=sum(abs(eigenvaluesofgrid));

        fprintf('harary energy of a torus is %4.4f\n',energy); else
        fprintf('not a torus. s and t values must be greater than 1\n');
end
```

Example 3.3. Harary matrix of torus T (3, 3) is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 \end{pmatrix}$$

Characteristic polynomial is $\rho^9 - 22.5\,\rho^7 - 67.5\,\rho^6 - 75.9375\,\rho^5 - 30.375\,\rho^4$.
Harary eigenvalues are -1.5, -1.5, -1.5, -1.5, 0, 0, 0, 0, 6.

Harary energy of a torus T (3, 3) is 12.

4. Extended grid EX(m, n)

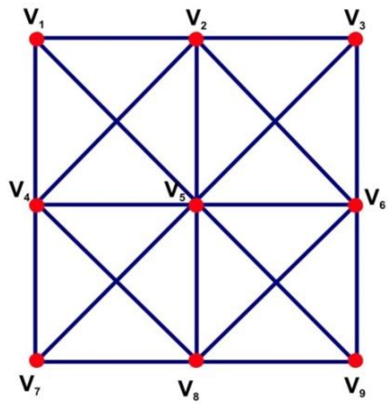

Figure : 8.4

By making each 4 cycle in a m n mesh into a complete graph we obtain an architecture called an extended mesh denoted by EX(m, n). The number of vertices in EX(m,n) is mn and the number of edges is 4mn− 3m− 3n + 2. We follow the sequential numbering from left to right.

MATLAB program to generate Harary energy of extended grid EX(m, n)

```
clc;

fprintf('HARARY ENERGY OF AN EXTENDED GRID s \times t\n'); s=input('Enter the value of s: ');
t=input('Enter the value of t: ');
if(s>1 && t>1)

     H=zeros(s*t);

     HM=zeros(s*t);

     c=1;

     for k=1:s
          for l=1:t

               u(k,l)=c;

               c=c+1;

          end

     end

     for k=1:s

          for l=1:t
```

```
if(s>2 && t>2)
    if((u(k,l)+1 > 0) && (u(k,l)-1 > 0) && (u(k,l)-t > 0) && (u(k,l)+t <= H(u(k,l),u(k,l)-1)=1;
        H(u(k,l)-1,u(k,l))=1;
        H(u(k,l),u(k,l)+1)=1;
        H(u(k,l)+1,u(k,l))=1;
        H(u(k,l),u(k,l)-t)=1;
        H(u(k,l)-t,u(k,l))=1;
        H(u(k,l),u(k,l)+t)=1;
        H(u(k,l)+t,u(k,l))=1;
        H(u(k,l), u(k,l)+t+1)=1;
        H(u(k,l)+t+1, u(k,l))=1;
        H(u(k,l), u(k,l)+t - 1)=1;
        H(u(k,l)+t - 1, u(k,l))=1;
        H(u(k,l), u(k,l)-t+1)=1;
        H(u(k,l)-t+1, u(k,l))=1;
        H(u(k,l), u(k,l)-t - 1)=1;
        H(u(k,l)-t - 1, u(k,l))=1;
    else
        if(k>1 && k<s)
            H(u(k,l),u(k,l)-t)=1;
            H(u(k,l)-t,u(k,l))=1;
            H(u(k,l),u(k,l)+t)=1;
            H(u(k,l)+t,u(k,l))=1;
            if(l<t)
                H(u(k,l),u(k,l)+t+1)=1;
                H(u(k,l)+t+1,u(k,l))=1;
            end
            if(l>1)
```



```
                    H(u(k,l),u(k,l)+t-1)=1;

                    H(u(k,l)+t-1,u(k,l))=1;

                end

            end

            if(l>1 && l<t)

                H(u(k,l),u(k,l)-1)=1;

                H(u(k,l)-1,u(k,l))=1;

                H(u(k,l),u(k,l)+1)=1;

                H(u(k,l)+1,u(k,l))=1;
                if(k<s)

                    H(u(k,l), u(k,l)+t-1)=1;

                    H(u(k,l)+t-1, u(k,l))=1;

                    H(u(k,l), u(k,l)+t+1)=1;

                    H(u(k,l)+t+1, u(k,l))=1;

                end

            end

        end

    else

        if(s>2)

            if(k>1 && k<s)

                H(u(k,l),u(k,l)+1)=1;

                H(u(k,l)+1,u(k,l))=1;

                H(u(k,l),u(k,l)-1)=1;

                H(u(k,l)-1,u(k,l))=1;

                H(u(k,l),u(k,l)+t)=1;

                H(u(k,l)+t,u(k,l))=1;

                H(u(k,l),u(k,l)-t)=1;

                H(u(k,l)-t,u(k,l))=1;

                if(l > 1)
```

```
                H(u(k,l),u(k,l)- t - 1)=1;
                H(u(k,l)- t - 1,u(k,l))=1;
                H(u(k,l),u(k,l)- t + 1)=1;
                H(u(k,l)- t + 1,u(k,l))=1;
            end
            if(l<t)
                H(u(k,l),u(k,l)+ t - 1)=1;
                H(u(k,l)+ t - 1,u(k,l))=1;
                H(u(k,l),u(k,l)+ t + 1)=1;
                H(u(k,l)+ t + 1,u(k,l))=1;
            end
        else
            if(l<t)
                H(u(k,l),u(k,l)+1)=1;
                H(u(k,l)+1,u(k,l))=1;
            end
        end
    else
        if(t>2)
            if(l > 1 && l < t)
                H(u(k,l),u(k,l)+1)=1;
                H(u(k,l)+1,u(k,l))=1;
                H(u(k,l),u(k,l)-1)=1;
                H(u(k,l)-1,u(k,l))=1;
                if(k > 1)
                    H(u(k,l),u(k,l)- t - 1)=1;
                    H(u(k,l)- t - 1,u(k,l))=1;
                    H(u(k,l),u(k,l)- t + 1)=1;
```

```
                H(u(k,l)- t + 1,u(k,l))=1;
            end
            if(k<s)
                H(u(k,l),u(k,l)+ t - 1)=1;
                H(u(k,l)+ t - 1,u(k,l))=1;
                H(u(k,l),u(k,l)+ t + 1)=1;
                H(u(k,l)+ t + 1,u(k,l))=1;
            end
        end
        if(k<s)
            if(l>1 && l<t)
                H(u(k,l),u(k,l)+t)=1;
                H(u(k,l)+t,u(k,l))=1;
            else
                H(u(k,l),u(k,l)+t)=1;
                H(u(k,l)+t,u(k,l))=1;
            end
        end
    else
        if(l<t)
            H(u(k,l),u(k,l)+1)=1;
            H(u(k,l)+1,u(k,l))=1;
            if(l>1)
                H(u(k,l),u(k,l)-1)=1;
                H(u(k,l)-1,u(k,l))=1;
            end
            if(k>1)
                H(u(k,l),u(k,l)-t)=1;
```



```
                        H(u(k,l)-t,u(k,l))=1;
                    end
                end
                if(k< s)
                    H(u(k,l),u(k,l)+t)=1;
                    H(u(k,l)+t,u(k,l))=1;
                    if(l == 1)
                        H(u(k,l),u(k,l)+t + 1)=1;
                        H(u(k,l)+t + 1,u(k,l))=1;
                    else
                        H(u(k,l),u(k,l)+t - 1)=1;
                        H(u(k,l)+t - 1,u(k,l))=1;
                    end
                end
            end
        end
    end
end
D=graphallshortestpaths(sparse(H));
%fprintf('Harary matrix: \n');
%disp(D);
for k=1:s*t
    for l=1:s*t
        if(D(k,l) > 1)
            HM(k,l)=1/D(k,l);
        else
            HM(k,l)=D(k,l);
        end
```

```
        end
    end

    fprintf('Harary matrix: \n');

    disp(HM);

    eigenvaluesofgrid=eig(HM);

    fprintf('Co-efficients of characterstic polynomial are\n');

    fprintf('%4.4f\t',poly(HM));

    fprintf('\n');

    fprintf('Eigenvalues are\n');

    fprintf('%4.4f\t',eigenvaluesofgrid);

    fprintf('\n');

    energy=sum(abs(eigenvaluesofgrid));

fprintf('harary energy of a extended grid is %4.4f\n',energy); else

fprintf('not an extended grid. s and t values must be greater than 2\t'); end
```

Example 3.4. Harary matrix of Extended grid EX(3, 3) is :

$$\begin{pmatrix} 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 & 0 \end{pmatrix}$$

Characteristics polynomial is $\rho^9 - 24\rho^7 - 80\rho^6 - 110.1250\rho^5 - 66.5\rho^4 - 7.5\rho^3 + 7.625\rho^2 + 1.3789\rho - 0.375$.

Harary eigenvalues are −1.5, −1.3909, −1.2071, −1.2071, −0.9124, −0.5, 0.2071, 0.2071, 6.3034.

Harary energy of a extended grid EX(3, 3) is 13.4352.

IV. PROPERTIES OF HARARY EIGENVALUES

Lemma 4.1: Let G be a simple graph with vertex set $V=\{v_1,v_2,...,v_n\}$, edge set E. If $\rho_1,\rho_2,...,\rho_n$ are the eigenvalues of Harary matrix H(G) then

$$(i)\ \sum_{i=1}^{n} p_i = 0 \text{ and } (ii)\ \sum_{i=1}^{n} p_i^2 = 2\sum_{i<j}\frac{1}{d(v_i,v_j)^2}$$

□

Lemma 4.2: Let G be a simple (n, m) graph then $E_H(G) \le \sqrt{n\left(2\sum_{i<j}\frac{1}{d(v_i,v_j)^2}\right)}$

V. MILOVANOVIC BOUNDS FOR HARARY ENERGY

In this section we establish bounds for Harary energy which is in sequal to the work of Milovanovic et al. [19].

Theorem 5.1. Let G be a graph with n vertices and m edges. Let $|\rho_1|\ge|\rho_2|\ge...\ge|\rho_n|$ be a non-increasing order of eigenvalues of H(G) then

$$E_H(G) \ge \sqrt{n\left(2\sum_{i<j}\frac{1}{d(v_i,v_j)^2}\right) - \alpha(n)(|\rho_1|-|\rho_n|)^2}$$

Where $\alpha(n) = n\left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)$ and [x] denotes the integral part of a real number.

Proof. Let $a, a_1, a_2, ..., a_n, A$ and $b, b_1, b_2, ..., b_n, B$ be real numbers such that $a \le a_i \le A$ and $b \le b_i \le B$ $\forall\ i = 1, 2, ..., n$ then the following inequality is valid.

$$\left|n\sum_{i=1}^{n} a_i b_i - \sum_{i=1}^{n} a_i \sum_{i=1}^{n} b_i\right| \le \alpha(n)(A-a)(B-b).$$

Where $\alpha(n) = n\left[\frac{n}{2}\right](1-\frac{1}{n}\left[\frac{n}{2}\right])$ and equality holds if and only if $a_1 = a_2 = \cdots = a_n$ and $b_1 = b_2 = \cdots = b_n$. If $a_i = |\rho_i|$, $b = |\rho_i|$, $a = b = |\rho_n|$ and $A = B = |\rho_1|$ then,

$$\left|n\sum_{i=1}^{n}|\rho_i|^2 - \left(\sum_{i=1}^{n}|\rho_i|^2\right)\right| \le \alpha(n)(|\rho_1|-|\rho_n|)^2.$$

But $\sum_{i=1}^{n}\rho_i^2 = 2\sum_{i<j}\frac{1}{d(v_{i,}v_j)^2}$ and $E_H(G) \le \sqrt{n\left(2\sum_{i<j}\frac{1}{d(v_{i,}v_j)^2}\right)}$ then the above inequality becomes $n\left(2\sum_{i<j}\frac{1}{d(v_{i,}v_j)^2}\right) - (E_H(G))^2 \le \alpha(n)\,(|\rho_1|-|\rho_n|)^2$

i.e., $E_H(G) \ge \sqrt{n\left(2\sum_{i<j}\frac{1}{d(v_{i,}v_j)^2}\right) - \alpha(n)\,(|\rho_1|-|\rho_n|)^2}$.

Theorem 5.2. Let m and n be the number of edges and vertices of G and $|\rho_1| \ge |\rho_2| \ge \cdots \ge |\rho_n| > 0$ be a non-increasing order of eigenvalues of $H(G)$ then

$$E_H(G) \geq \frac{2\sum_{i<j}\frac{1}{d(v_i,v_j)^2} + n|\rho_1||\rho_n|}{(|\rho_1|+|\rho_n|)}$$

Proof. Let $a_i \neq 0, b_i$, r and R be real numbers satisfying $ra_i \leq b_i \leq Ra_i$, then the following inequality holds . (Theorem 2, [20])

$$\sum_{i=1}^{n} b_i^2 + rR\sum_{i=1}^{n} a_i \leq (r+R)\sum_{i=1}^{n} a_i\, b_i.$$

Put $b_i = |\rho_1|,\ a_i = 1, r = |\rho_n|$ and $R = |\rho_1|$ then

$$\sum_{i=1}^{n} |\rho_i|^2 + |\rho_1||\rho_n|\sum_{i=1}^{n} 1 \leq (|\rho_1| + |\rho_n|)\sum_{i=1}^{n} |\rho_i|.$$

i.e., $$2\sum_{i<j}\frac{1}{d(v_i,v_j)^2}\ |\rho_1||\rho_n|n \leq (|\rho_1| + |\rho_n|)E_H(G)$$

$$\therefore\ E_G(G) \geq \frac{2\sum_{i<j}\frac{1}{d(v_i,v_j)^2} + n|\rho_1||\rho_n|}{(|\rho_1| + |\rho_n|)}$$

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