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Standard Representation of Set Partitions of Γ_1 non-deranged permutations

M. Ibrahim[1*], M. Muhammad[2]

[1]Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria
[2]Department of Statistics Federal Polytechnic Kaura Namoda, Zamfara, Nigeria

**Corresponding Author: muhammad.ibrahim@udusok.edu.ng, Tel.: +234-806-2381-016*

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Abstract— Some further theoretic properties of the scheme called Γ_1 non-deranged permutation Group, especially in relation to ascent block were identified and studied in this paper. This was done first through some computations on this scheme using prime numbers $p \geq 5$. A recursion formula for generating maximum number of block and minimum number of block were developed and it's also observed that $arc(\omega_i)$ is equidistributed with $asc(\omega_i)$ for any arbitrary permutation group and it in decreasing order for Γ_1 non-deranged permutations it also established that the number of ascent block in ω_i is $_i$.

Keywords— Ascent Number, Ascent set ,Ascent block and $\Gamma_1 -$ non deranged permutations.

I. INTRODUCTION

Permutation statistics were first introduced by [4] and then extensively studied by[13].in the last decades much progress has made, both in the discovery and the study of new statistics, and in extending these to other type of permutations such as words and restricted permutation.The concept of derangements in permutation groups (that is permutations without a fix element) has proportion in the underlying symmetric group S_n. [5] used the concept to develop a scheme for prime numbers $P \leq 5$ and $\Omega \subseteq N$ which generate the cycles of permutations (derangements) using $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp}...(1+(p-1)i)_{mp})$ to determine the arrangements. It is difficult for a set of derangements to be a permutation group because of the absence of the natural identity element (a non derangement),The construction of the generated set of permutations from the work of [5] as a permutation group was done by [17].They achieved this by embedding an identity element into the generated set of permutation(strictly derangements) with the natural permutation composition as the binary operation (the group was denoted as G_p)

With no doubt, patterns in permutations have been well studied for over a century. As seem to be the case, these patterns were studied on permutations arbitrary. The symmetric group S_n is the set of all permutations of a set Γ of cardinality n. There are several types of other smaller permutation groups (subgroup of S_n) of set Γ,a notable one among them is the alternating group A_n. On the other hand, [9] studied the representation of Γ_1-non deranged permutation group $G_P{}^{\Gamma_1}$ via group character, hence established that the character of every $\omega_i \in G_p{}^{\Gamma_1}$ is never zero. Also the non standard Young tableaux of Γ_1-non deranged permutation group $G_P{}^{\Gamma_1}$ has been studied by Garba *et al*.(2017), they established that the Young tableaux of this permutation group is non standard. [1] studied pattern popularity in Γ_1-non deranged permutations they establish algebraically that pattern τ_1 is the most popular and pattern τ_3, τ_4 *and* τ_5 are equipopular in $G_P{}^{\Gamma_1}$ they further provided efficient algorithms and some results on popularity of patterns of length-3 in $G_P{}^{\Gamma_1}$.[2] studied Fuzzy on Γ_1-non deranged permutation group $G_P{}^{\Gamma_1}$ and discover that it is a one sided fuzzy ideal (only right fuzzy but not left) also the

$\alpha-$level cut of f coincides with $G_P^{\Gamma_1}$ if $\alpha=\frac{1}{p}$ [10] studied ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and discover that the union of ascent of all Γ_1-non derangement is equal to identity also observed that the difference between $Asc(\omega_i)$ and $Asc(\omega_{p-1})$ is one. More recently [11] provide very useful theoretical properties of Γ_1-non deranged permutation s in relation to excedance and shown that the excedance set of all ω_i in $G_P^{\Gamma_1}$ such that $\omega_i \neq e$ is $\frac{1}{2}(p-1)$. Hence we will in this paper we study standard representation of Γ_1 non-deranged permutations by using partitioning the permutation set, using ascent block. A recursion formular for generating maximun block and minimum block were generated by using ascent block we also established that the number of ascent block in ω_i is i.

II. PRELIMINARIES

Definition 2.1 [2]
Let Γ be a non empty set of prime cardinality greater or equal to 5 such that $\Gamma \subset$ A bijection ω on Γ of the form

$$\omega_i=\begin{pmatrix}1 & 2 & 3 & . & . & . & p\\ 1 & (1+i)_{mp} & (1+2i)_{mp} & & & & (1+(p-1)i)_{mp}\end{pmatrix}$$

is called a Γ_1-non deranged permutation. We denoted G_p to be the set of all Γ_1-non deranged permutations.

Definition 2.2 [2]
The pair G_p and the natural permutation composition forms a group which is denoted as $G_P^{\Gamma_1}$.This is a special permutation group which fixes the first element of Γ.

Definition 2.3 [10]
An descent of permutation

$$f=\begin{pmatrix}1 & 2 & 3 & . & . & . & n\\ f(1) & f(2) & f(3) & . & . & . & f(n)\end{pmatrix}$$

is any positive $i<n$ (where i and n are positive integers) where the current value is less than the next, that is i is an ascent of a permutation $f(i)<f(i+1)$.

The ascent set of f, denoted as $Asc(f)$, is given by $Asc(f)=\{i: f(i)<f(i+1)\}$ the ascent number of f, denoted as $asc(f)$, is defined as the number of ascent and is given by $asc(f)=|Asc(f)|$.

Definition 2.4
$Run(\omega_i)$ is the number of ascent block in ω_i

Definition 2.5
$min(\omega_i)$ is the minimum number in each of the block of the $Run(\omega_i)$ while $|\min(\omega_i)|$ is the cardinality of minimum number of a block(s)

Definition 2.6
$\max(\omega_i)$ is the minimum number in each of the block of the $Run(\omega_i)$ while $|\max(\omega_i)|$ is the cardinality of minimum number of a block(s)

Definition 2.7
$Isv(\omega_i)$ is the number of isolated vertex in ω_i

III. RELATED WORK

There are many research articles devoted to Mahonian statistics and their generalizations, for example see [3,7] for Mahonian statistics for words, [15,16] for Mahonian statistics and Laquerre polynomial,[14] for a major index statistic for set partitions, [8] for inversion and major index for standard young tableaux. [12] established that the intersection of descent set of all Γ_1-non derangement is empty, also observed that the descent number is strictly less than ascent number by $p-1$. [18] show that inversion number and major index are not equidistributed also the difference between sum of the major index and sum of the Inversion number is equal to the sum of descent number in Γ_1 non-deranged permutations.

IV. MAIN RESULTS

In this section, we present some results on partitioning the permutation sets using ascent block of subgroup $G_P^{\Gamma_1}$ of S_p (Symmetry group of prime order with $p \geq 5$).

Lemma 4.1
Suppose that $G_P^{\Gamma_1}$ is Γ_1-non deranged permutations. Then the $Run(\omega_i)=i$.

Proof.

Since the ascent is separating the block orderly, then for ω_i ,the ascent number is $p-1$,therefore the block will be $p-(p-1)=1$. For ω_i ,the ascent number is $p-2$,and the Run will be $p-(p-2)=2$. Then the $Run(\omega_i)=p-(p-i)$.Therefore $Run(\omega_i)=i$

□

Theorem 4.2

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, then the

$$|\min(\omega_i)|=|\max(\omega_i)|=i.$$

Proof.

From lemma 3.1 the

$$Run(\omega_i)=i.$$

And

$$|\min(\omega_i)|=Run(\omega_i)=i$$

Since the

$$|\min(\omega_i)|=|\max(\omega_i)|$$

then

$$|\max(\omega_i)|=i$$

Therefore

$$|\min(\omega_i)|=|\max(\omega_i)|=i.$$

□

Remark 4.3

Note that theorem 3.2 holds for any arbitrary permutation in symmetric group S_n.

Proposition 4.4

Let $\omega_i \in G_P^{\Gamma_1}$. where $i<\dfrac{p+1}{2}$ Then the

$$\min(\omega_i)\cap max(\omega_i)\neq\phi.$$

Proof.

Given that $i<\dfrac{p+1}{2}$, then there is no isolated vertex in (ω_i). Therefore

$$\min(\omega_i)\cap\max(\omega_i)=\phi$$

Theorem 4.5

Suppose that $G_P^{\Gamma_1}$ is Γ_1-non deranged permutations. Then the

$$Isv(\omega_i)=\min(\omega_i)\cap\max(\omega_i).$$

Proof.

Since the only vertex $v\in\min(\omega_i)$ and $v\in\max(\omega_i)$ is isolated and

$$Isv(\omega_i)=\min(\omega_i)\cap\max(\omega_i)$$

, then $v\in Isv(\omega_i)$. This implies that

$$Isv(\omega_i)=\min(\omega_i)\cap\max(\omega_i)$$

Proposition 4.6

Let $\omega_i \in G_P^{\Gamma_1}$. where $i=\dfrac{p+1}{2}$,then the $Isv(\omega_i)=\{i\}$.

Proof.

For any $\omega_i \in G_P^{\Gamma_1}$, $\omega_{\frac{p+1}{2}}=a_1,a_2,\ldots,a_p$, where $a_p=\dfrac{p+1}{2}$ and it is not less than or greater than any value, since in the block we know that any number has to be less than or greater than, and any number that is not less than or greater than is isolated.

Hence $Isv(\omega_i)=\{i\}$.

Remark 4.7

For any $G_P^{\Gamma_1}$, $\omega_{\frac{p+1}{2}}$ has only one isolated vertex, and also the isolated vertex is in increasing order.

Proposition 3.8

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, then the

$$\min(\omega_i)=\bigcup_{k=1}^{i}\{k\}$$

Proof.

From theorem 4.2 we see that $|\min(\omega_i)|=i$. For $i=1,2,\ldots,p-1$ we have

$$\min(\omega_i)=\{i\}$$
$$\min(\omega_{i+1})=\{i\}\cup\{i+1\}$$

$$\min(\omega_{i+2}) = \{1\} \cup \{i+1\} \cup \{i+2\}.$$

For any integer $k \geq 0$,

$\min(\omega_{i+k}) = \{i\} \cup \{i+1\} \cup \ldots \cup \{i+k\}$. This show that

$$\min(\omega_i) = \bigcup_{k=1}^{i} \{k\}$$

□

Proposition 4.9

Suppose that $G_P^{\Gamma_1}$ is Γ_1-non deranged permutations. Then the $\max(\omega_i) = \bigcup_{k \square 0}^{i-1} \{p-k\}$.

Proof.

It is clear from proposition 3.2 that $|\max(\omega_i)| = i$,for $i = 1$, $\max(\omega_i) = p$ therefore

$$\max(\omega_{i+1}) = \{p\} \cup \{p-1\}$$

$$\max(\omega_{i+2}) = \{p\} \cup \{p-1\} \cup \{p-2\}$$

and for an integer $k \geq 0$,

$$\max(\omega_i + k) = \{p\} \cup \{p-1\} \cup \ldots \cup \{p-k\}.$$

This implies that the

$$\max(\omega_i) = \bigcup_{k \square 0}^{i} \{p-k\}.$$

□

Corollary 4.10

For every $\omega_i = e$,where e is the identity of any of the permutations, the

$$\min(\omega_i) = \{1\}$$

and

$$\max(\omega_i) = \{p\}$$

Lemma4.11

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, then the $arc(\omega_i) = p - i$.

Proof.

It is clear the $asc(\omega_i)$ is the $|\ \{i : a_i < a_{i+1}\}\ |$ and it can be denoted by $asc(\omega_i) = p - i$ .and the $arc(\omega_i)$ can be defined as $|\ \{(a_i, a_j) : a_i < a_j\}\ |$ then the $arc(\omega_i) = asc(\omega_i)$. Hence

$$arc(\omega_i) = p - i.$$

□

Remark 4.12

The $arc(\omega_i)$ is equidistributed with the $asc(\omega_i)$ for any arbitrary permutation group and its in decreasing order for $G_P^{\Gamma_1}$

Theorem 4.13

Let $\omega_i \in G_P^{\Gamma_1}$, the $Arc(\omega_i) = \bigcup_{j \square 1}^{p-i} \{(j, j+i)\}$.

Proof.

The difference between any pair $(j,k) \in Arc(\omega_i)$ is i .Therefore any Arc can be written as $(j, j+i)$. Since 1 is the least of all numbers in any $\omega_i \in G_P^{\Gamma_1}$ and the cardinality of the $Arc(\omega_i) = p - i$, this implies that

$$Arc(\omega_i) = \bigcup_{j \square 1}^{p-i} \{(j, j+i)\}$$

□

Corollary 4.14

Suppose that $G_P^{\Gamma_1}$ is Γ_1-non deranged permutations. Then the, $Arc(\omega_{p-i}) = \{(1, p)\}$.

Proof.

We have from theorem 4.13 that

$Arc(\omega_i) = \bigcup_{j \square 1}^{p-i} \{(j, j+i)\}$,then for $i = p - 1$, we have $p - i = p - (p-1) = 1$, therefore $j = 1$. Since

$$Arc(\omega_i) = \{(j, j+i)\}$$

$$Arc(\omega_{p-1}) = \{(1, 1+(p-1))\}$$

$$= \{(1, 1 + p - 1)\}$$
$$= \{(1, p)\}.$$

□

Proposition 4.15

Let $\omega_i \in G_P^{\Gamma_1}$, then the $\bigcup_{i=1}^{p-1} Arc(\omega_i) = pair(\omega_1)$.

Proof.

Suppose $\omega_i = 123\ldots p$, the pair of ω_i is all pair (i, j) such that $i < j$ and $a_i < a_j$. Since the $Arc(\omega_i) = \bigcup_{i=1}^{p-1} \{(j, j+i)\}$. So this union contains every pair (i, j) of ω_1, where $i < j$

□

Proposition 4.16

Let $\omega_i, \omega_k \in G_p^{\Gamma_1}$, where $i \neq k$, then the $Arc(\omega_i) \cap Arc(\omega_k) = \phi$

Proof.

From theorem 3.13 $Arc(\omega_i) = \bigcup_{j=1}^{p-i} \{(j, j+i)\}$. since $i \neq k$, then $j + i \neq j + k$. Therefore $Arc(\omega_i) \cap Arc(\omega_k) = \phi$

□

Proposition 4.17

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, then the $Isv(\omega_{p-1}) = p - 2$.

Proof.

Given that $\omega_i \in G_P^{\Gamma_1}$, the $Isv(\omega_{p-1}) = p - 2$, we have $\omega_{P-1} = 1p(p-1)\ldots 2$. So the only arc that we have is $(1, p)$ because $1 < p > (p-1) > (p-2) > \ldots > 2$ therefore all other vertices will not be the endpoints of any arc, which means that the only vertices that are not isolated are 1 and p. Hence $Isv(\omega_{p-1}) = p - 2$.

□

Proposition 4.18

Let $\omega_i \in G_P^{\Gamma_1}$, then $Isv(\omega_j) = 2j - p$. Where $i = \frac{p+1}{2}$.

Proof.

It is clear that the number of isolated vertex is odd. Then for $i = \frac{p+1}{2}$ we have that $Isv(\omega_i) = 2\left(i - \frac{p+1}{2}\right) + 1$ for $k = 1$, $Isv(\omega_{i+1}) = 2\left(i + 1 - \frac{p+1}{2}\right) + 1$. Then for $k \geq 0$, $Isv(\omega_{i+k}) = 2\left(i + k - \frac{p+1}{2}\right) + 1$. Now let $i + k = j$, then

$$\begin{aligned} Isv(\omega_j) &= 2\left(j - \frac{p+1}{2}\right) + 1 \\ &= 2j - 2\left(\frac{p+1}{2}\right) + 1 \\ &= 2j - p. \end{aligned}$$

Proposition 4.19

Let $\omega_i \in G_P^{\Gamma_1}$, then $Cr_2(\omega_1) = Cr_2(\omega_{p-1}) = 0$.

Proof.

For $\omega_1 = 12\ldots p$, each number a_i is less than a_{i+1}, therefore there is no crossing. For $(\omega_{p-1}) = 1p(p-1)(p-2)\ldots 2$, then there is only one arc which is $(1, p)$ and this implies that we have one arc This implies that

$$Cr_2(\omega_1) = Cr_2(\omega_{p-1}) = 0$$

V. CONCLUSION

This paper has provided very useful theoretical properties of this scheme called Γ_1-non deranged permutations in relation to ascent block we also study standard representation of Γ_1 non-deranged permutations by using using prime numbers $p \geq 5$ and partitioning the permutation set, using ascent block. A recursion formula for generating maximum block and minimum block were generated by using ascent block we also established that the number of ascent block in ω_i is i .

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