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Some Properties of Fuzzy Distance two Labeling Graph

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Abstract—Graph theoretical concepts are hugely used by the applications of computer science. Especially in research areas of computer science such as networking, image capturing, data mining, image segmentation etc. Fuzzy labeling graphs produce more precision, flexibility, and compatibility to the system compared to the classical and fuzzy graphs. They have large number of applications in Physics, Chemistry, Computer Science, and other branches of mathematics. In this paper a new concept of fuzzy distance two labeling is introduced. Some properties related to product fuzzy graph and fuzzy distance two labeling graph have been discussed. This paper also considers the properties of fuzzy distance two labeling circular graph with appropriate explanation.

Keywords— Fuzzy distance two labeling graph, product fuzzy graph, fuzzy bridge and fuzzy cut node, fuzzy circular graph.

I. INTRODUCTION

There are several reasons for the acceleration of interest in graph theory. It has became fashionable to mention that there are several applications of graph theory for solving combinatorial problems in different areas such as operation research, electrical and civil engineering, number theory, topology, algebra and computer science.

Euler became the father of graph theory when in 1736 he settled a famous unsolved problem of his day called the Konigsberg Bridge Problem is considered to be the first theorem of graph theory. The idea of fuzzy sets and fuzzy relations on a set was first explained by Zadeh in 1965 [9] is a mathematical implement for handling doubt like vagueness, ambiguity and imprecision in linguistic variables. A fuzzy set is described mathematically by assigning to each possible individual in the universe of address a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or adaptable with the concept interpreted by the fuzzy set.

In 1973, Kaufmann introduced the first definition of a fuzzy graph, based on Zadeh's fuzzy relations in 1971. A more elaborate definition is because of Azriel Rosenfeld [1] who considered fuzzy relations on fuzzy sets and designed the theory of fuzzy graphs structure in 1975 and obtained several graph theoretical concepts like bridges and trees. The various connectedness concepts in fuzzy graph theory and its applications introduced by Yeh and Bang [13], during the same time. Further Ramaswamy and Poornima [15] introduced product fuzzy graphs and proved several results which are analogous to fuzzy graphs. Nagoorgani et. al [4] discussed the properties of fuzzy labeling Graphs.

In fuzzy graph theory, the assignment of nodes and arcs of a fuzzy graph has great importance in various interesting problems like traffic light problem, job allocation problem etc. In the area of graph theory, mathematically a graph labeling represents the assignment of numbers to the arcs or nodes or both of a graph. The present paper is also deals with the fuzzy labeling graph.

In this research article on the basis of assignment of nodes and arcs of a fuzzy graph, we have developed a new idea for fuzzy distance two labeling graph with proper explanation. A graph is said to be a fuzzy distance two labeling graph if it has fuzzy distance two labeling. In addition to it, relation between product fuzzy graph and fuzzy distance two labeling graphs has been derived. Some essential properties of fuzzy distance two labeling circular graphs and related theorems are given in this paper.

II. PRELIMINARIES

In this section, some basic definitions and notations related to fuzzy graph have been introduced which are helpful for the present work of the paper.

A fuzzy set F defined on a non-empty set X is characterized by a mapping $m: X \to [0,1]$, which is called the membership function such that $m(x) = 1$ if $x \in F$, $m(x) = 0$ if $x \notin F$ and any intermediate value represents the degree in which x could belong to F . Fuzzy set is usually denoted by $F = (X, m)$.

A crisp graph G is a finite non-empty set of objects called nodes or vertices together with a set of unordered pair of distinct nodes of G called arcs or edges. The node set and the arc set of G are denoted by $N(G)$ *or* N and $A(G)$ *or* A respectively.

A fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is a pair of functions $\sigma_N : N \to [0,1]$ and $\mu_A : N \times N \to [0,1]$ such that for all $x, y \in N$ we have $\mu_A(x, y) \le \sigma_N(x) \wedge \sigma_N(y)$, where $\wedge$ stands for minimum. Also, we denote the underlying crisp graph of $F(G)$ by $G^* = (\sigma_N{}^*, \mu_A{}^*) = G^*(N, A)$ where $\sigma_N{}^* = \{x \in N : \sigma_N(x) > 0\}$ and $\mu_A{}^* = \{(x, y) \in N \times N : \mu_A(x, y) > 0\}$.

A sequence of distinct nodes $x_1, x_2, \ldots\ldots, x_n$ such that $\mu_A(x_r, x_{r+1}) > 0$; $1 \le r \le n-1$ is called a path P in a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ and here $n > 1$ is defined as the length of the path P . The consecutive pairs $\mu_A(x_r, x_{r+1})$ are called the arcs of the path P . If $x_1 = x_n$ and $n \ge 3$ then the path P is known as a cycle. Two nodes in a fuzzy graph $F(G)$ that are joined by a path are said to be connected.

The strength of a path P is defined to be the degree of membership of a weakest edge of the path P i.e the strength of a path P can be defined as $\bigwedge_{r=1}^{n-1} \mu_A(x_r, x_{r+1})$. The maximum of the strength of all paths between the nodes x and y in a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is defined as the strength of connectedness between x and y , and it is denoted by $\mu_A{}^{\infty}(x, y)$.

An arc of a fuzzy graph $F(G)$ is called strong if its weight is at least as great as the strength of the connectedness of its end nodes when it is deleted. A path consisting of only strong arcs is said to be a strong path in $F(G)$.

An edge is called a fuzzy bridge of $F(G)$ if its removal reduces the strength of connectedness between some pair of nodes in $F(G)$.

A node is a fuzzy cut node of $F(G) = (\sigma_N, \mu_A)$ if removal of it reduces the strength of the connectedness between some other pair of nodes. A node in a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is called fuzzy end node if it has exactly one strong neighbor in graph $F(G) = (\sigma_N, \mu_A)$.

The order of a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is defined as $OrderF(G) = \sum_{x \in N} \sigma_N(x).$ The size of a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is defined as $SizeF(G) = \sum_{x, y \in N} \mu_A(x, y).$
The degree of a node x in a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is defined by $d_{F(G)}(x) = \sum_{\substack{x \ne y \\ y \in N}} \mu_A(x, y)$.
The degree of an arc (x, y) in a fuzzy graph $F(G) = (\sigma_N, \mu_A)$ is denoted by $d_{F(G)}(x, y)$ and defined by $d_{F(G)}(x, y) = d_{F(G)}(x) + d_{F(G)}(y) - 2\mu_A(x, y)$.

III. FUZZY DISTANCE TWO LABELING GRAPH

In this section, new concept related to fuzzy distance two labeling graph has been described with the appropriate figure as follows:

A. Fuzzy distance two labeling graph

Consider a fuzzy graph $F(G)^{\psi} = (\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ with crisp fundamental graph $G^* = (N, A)$ for which the membership functions are $\sigma_N{}^{\psi} : N \to [0, 1]$ and $\mu_A{}^{\psi} : N \times N \to [0, 1]$. Then $F(G)^{\psi} = (\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ is called a fuzzy distance two labeling graph if arcs and nodes of this graph have different assignment such that

1. $\mu_A{}^{\psi}(r, s) < \sigma_N{}^{\psi}(r) \wedge \sigma_N{}^{\psi}(s)$ for all $r, s \in N$.
2. $|\sigma_N{}^{\psi}(r) - \sigma_N{}^{\psi}(s)| \ge \mu_A{}^{\psi}(r, s)$ if $d(r, s) = 1$ and $|\mu_A{}^{\psi}(r, p) - \mu_A{}^{\psi}(p, s)| \le \sigma_N{}^{\psi}(p)$ if $d(r, s) = 2$, where p is a node on the path connected by the nodes r and s.

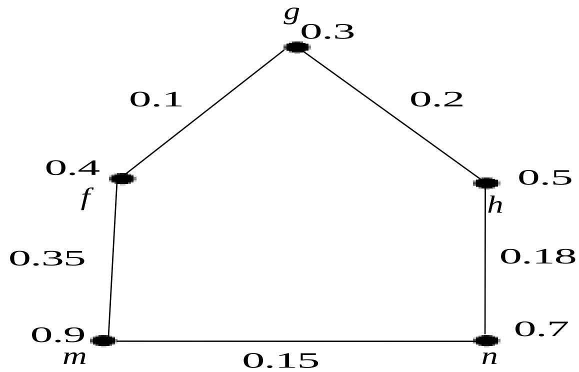

Figure 1: Fuzzy distance two labeling graph

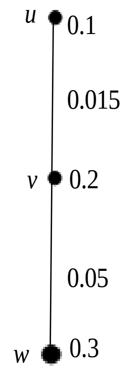

Figure 2: Product fuzzy graph

B. Example of fuzzy distance two lableing graph

Fuzzy distance two labeling graph $F(G)^{\psi} = (\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ is described by Figure 1 in which $\sigma_N{}^{\psi} = \{ f\,|\,0.4,\ g\,|\,0.3,\ h\,|\,0.5,\ m\,|\,0.9,\ n\,|\,0.7 \}$ and $\mu_A{}^{\psi} = \{ (f, g)\,|\,0.1, (g, h)\,|\,0.2, (h, n)\,|\,0.18, (m, n)\,|\,0.15, (f, m)\,|\,0.35 \}$.

C. Product fuzzy graph

Let $G^* = (N, A)$ be a underlying crisp graph, σ_N be a fuzzy subset of N and μ_A be a fuzzy subset of $N \times N$. We call graph $F(G)^P = (\sigma_N, \mu_A)$ be a product fuzzy graph if $\mu_A(x, y) \le \sigma_N(x) \times \sigma_N(y)$ for all $x, y \in N$.

D. Example of product fuzzy graph

Figure 2 illustrates a product fuzzy graph $F(G)^P = (\sigma_N, \mu_A)$ where $\sigma = \{u\,|\,0.1, v\,|\,0.2,\ w\,|\,0.3\}$ and $\mu_A = \{(u,\ v)\,|\,0.015, (v,\ w)\,|\,0.05\}$.

E. Remark

In general each fuzzy distance two labeling graph represents a fuzzy graph but converse need not be true.

IV. RELATION BETWEEN PRODUCT FUZZY GRAPH AND FUZZY DISTANCE TWO LABELING GRAPH

In general, product fuzzy graph need not be a fuzzy distance two labeling graph but under some certain conditions it can be made a fuzzy distance two labeling graph. These conditions have been established in the following theorem:

A. Theorem

Every product fuzzy graph is a fuzzy distance two labeling graph if the membership value of nodes and arcs are distinct but converse need not be true.

Proof: A fuzzy graph $F(G)^P = (\sigma_N, \mu_A)$ with crisp fundamental graph $G^* = (N, A)$ is defined a product fuzzy graph if $\mu_A(s, t) \le \sigma_N(s) \times \sigma_N(t)$ for all $s, t \in N$. Since $\sigma_N(s)$ and $\sigma_N(t)$ lies within $[0, 1]$, then obviously $\sigma_N(s) \times \sigma_N(t) \le \sigma_N(s) \wedge \sigma_N(t)$ for all $s, t \in N$. Hence, if we use $\sigma_N(s) \times \sigma_N(t)$ in place of $\sigma_N(s) \wedge \sigma_N(t)$ and assign distinct membership values to the nodes and arcs then clearly all the condition of fuzzy distance two labeling graph will be satisfied by such a product fuzzy graph. Hence every product fuzzy graph is a fuzzy distance two labeling graph if the membership value of nodes and arcs are distinct.

But converse need not be true. To prove this consider the following observation: Since the assignment of nodes and arcs of a fuzzy graph has great importance in fuzzy graph theory, so fuzzy distance two labeling graph may or may not be a product fuzzy graph depends only on the assignment of nodes and arcs of that graph.

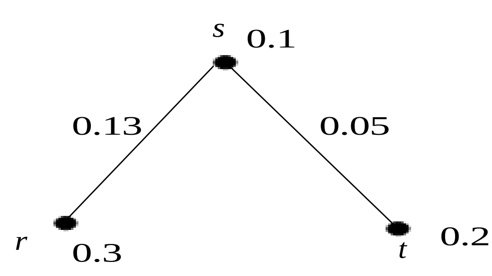

Figure 3: Fuzzy distance two labeling graph but not a product fuzzy graph

Fuzzy graph $F(G)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ in figure 3 is a fuzzy distance two labeling graph with $\sigma_N{}^{\psi}=\{r\,|\,0.3, s\,|\,0.1, t\,|\,0.2\}$ and $\mu_A{}^{\psi}=\{(r, \text{s})\,|\,0.13, (s, \text{t})\,|\,0.05\}$ which is not a product fuzzy graph because the required condition $\mu_A{}^{\psi}(s, \text{t}) \leq \sigma_N{}^{\psi}(s) \times \sigma_N{}^{\psi}(t)\ \forall\ s, t \in N$ does not hold for this graph as $\mu_A{}^{\psi}(r, \text{s}) = 0.13 > 0.03 = \sigma_N{}^{\psi}(r) \times \sigma_N{}^{\psi}(s)$.

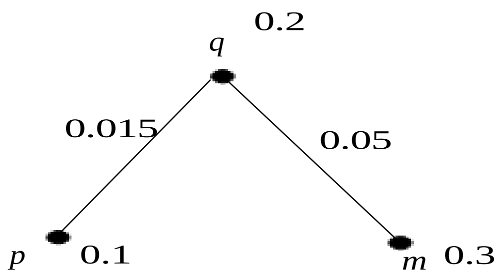

Figure 4: Fuzzy distance two labeling graph which is also a product fuzzy graph

Fuzzy graph $F(G)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ in figure 4 formed a fuzzy distance two labeling graph with membership functions $\sigma_N{}^{\psi}=\{p\,|\,0.1, q\,|\,0.2, m\,|\,0.3\}$ and $\mu_A{}^{\psi}=\{(p, \text{q})\,|\,0.015, (q, \text{m})\,|\,0.05\}$ which is also represent a product fuzzy graph because for this graph the condition $\mu_A{}^{\psi}(s, \text{t}) \leq \sigma_N{}^{\psi}(s)\times\sigma_N{}^{\psi}(t)\ \forall\ s, t \in N$ good hold. Hence fuzzy distance two labeling graph may or may not be a product fuzzy graph.

B. Theorem

Every complete product fuzzy graph is a distance two labeling fuzzy graph if each node of the graph assigns a distinct membership value but not converse.

Proof: Suppose $G^*=(N, A)$ denote a crisp fundamental graph for the fuzzy graph $F(G)^P=(\sigma_N, \mu_A)$. Then the graph $F(G)^P=(\sigma_N, \mu_A)$ is a complete product fuzzy graph if $\mu_A(s, t)=\sigma_N(s)\times\sigma_N(t)$ for all $s, t \in N$. By above theorem, it is obvious that every product fuzzy graph $F(G)^P=(\sigma_N, \mu_A)$ is a fuzzy distance two labeling graph if each nodes and arcs of the graph have distinct assignment number. Hence by assigning distinct membership value to each node $s, t \in N$ of complete product fuzzy graph and using membership value $\mu_A(s, t)=\sigma_N(s)\times\sigma_N(t)$ for every arc, the resulting fuzzy graph will follow all the condition of fuzzy distance two labeling graph and the result is obvious. Converse part is also obvious by above discussion.

V. PROPERTIES OF FUZZY DISTANCE TWO LABELING CIRCULAR GRAPH

Some basic properties of fuzzy distance two labeling circular graph have been discussed in this section.

A. Circular graph or cycle

A crisp graph in which finite number of nodes connected in a closed chain is called a circular graph or a cycle (i.e) a cycle or a circular graph is a crisp graph that consists of a single cycle. A circular crisp graph is denoted by $C^*=(N, A)$.

B. Fuzzy distance two labeling cycle

A circular graph $C^*=(N, A)$ is said to be a fuzzy distance two labeling cycle graph if all nodes and arcs of $C^*=(N, A)$ assign membership values according to the definition of fuzzy distance two labeling graph and it is denoted by $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$.

C. Theorem

If $C^*=(N, A)$ be an underlying crisp circular graph with $|N|=|A|=\alpha$ then exactly $\alpha-1$ bridges appeared in the fuzzy distance two labeling circular graph $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$.

Proof: Let $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ be a fuzzy distance two labeling cycle with $C^*=(N, A)$ as underlying crisp circular graph such that $|N|=|A|=\alpha$. Consider (p, q) be an arc of $C^*=(N, A)$ with assignment number $\mu_A{}^{\psi}(p, q)=\wedge\{\mu_A{}^{\psi}(p_i, q_i):1\leq i\leq\alpha\}$ where symbol $\wedge$ is used for minimum. Since graph $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ has fuzzy distance two labeling, then there will be only single arc in C^* which is assigned by the number $\mu_A{}^{\psi}(p, q)$. If we eject $\mu_A{}^{\psi}(p, q)$ from $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$ then it will not decrease the strength of connectedness $\mu_A{}^{\infty}(p, q)$ between the nodes p and q this implies that $\mu_A{}^{\psi}(p, q)$ is a weakest arc. This proves that only one weakest arc occurs in each fuzzy distance two labeling cycle $F(C)^{\psi}=(\sigma_N{}^{\psi}, \mu_A{}^{\psi})$. Now clearly the weakest arc obtained in the fuzzy distance two labeling cycle graph cannot form a fuzzy bridge of graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Which implies that the ejection of any arc from the fuzzy distance two labeling cycle $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ except the weakest arc will give the decrement in the value of strength of connectedness $\mu_A^{\infty}(p, q)$ between the nodes p and q. Therefore all the arcs in the fuzzy distance two labeling cycle (except only one weakest arc) will be fuzzy bridge of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Since $|N| = |A| = \alpha$, then the total number of arcs of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ after omitting this weakest arc is $\alpha - 1$. Hence exactly $\alpha - 1$ bridges appeared in the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$.

D. Theorem

If $C^* = (N, A)$ denote an underlying crisp circular graph with $|N| = |A| = \alpha$ then exactly $\alpha - 2$ cut nodes occurred in the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$.

Proof: Let $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is a fuzzy distance two labeling cycle with $C^* = (N, A)$ as an underlying crisp circular graph where $|N| = |A| = \alpha$. We know that any node in a fuzzy graph defined a fuzzy cut node of that fuzzy graph if its ejection gives the decrement in the value of strength of the connectedness for some another combined nodes. This implies that a node in a fuzzy graph is a fuzzy cut node for that graph iff it serve as a common node of any two fuzzy bridges. By theorem 6.4.2, exactly $\alpha - 1$ bridges occurred in each fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ (i.e) fuzzy distance two labeling cycle graph will have single weakest arc only. Let $\mu_A^{\psi}(p, q)$ denote the allocation of this single weakest arc of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ and let $\sigma_N^{\psi}(p)$ and $\sigma_N^{\psi}(q)$ respectively be the assignment of the nodes p and q. Therefore excluding $\sigma_N^{\psi}(p)$ and $\sigma_N^{\psi}(q)$ nodes from the graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$, remaining all $\alpha - 2$ nodes of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ serve as a common node of two fuzzy bridges of the fuzzy distance two labeling cycle $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Therefore all these $\alpha - 2$ nodes of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ will be cut nodes of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Hence exactly $\alpha - 2$ cut nodes appeared in every fuzzy distance two labeling cycle $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$.

E. Remark

Every fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ has exactly only one weakest arc, say $\mu_A^{\psi}(p, q)$. Which implies that $\sigma_N^{\psi}(p)$ and $\sigma_N^{\psi}(q)$ are two end nodes. Hence exactly two end nodes obtained in every fuzzy distance two labeling cycle graph.

F. Remark

The node in a fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is either a cut node or end node.

G. Theorem

If $C^* = (N, A)$ denote the fundamental crisp circular graph with $|N| = |A| = \alpha$, then every bridge in the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is strong.

Proof: Let $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is a fuzzy distance two labeling cycle with $C^* = (N, A)$ as a crisp fundamental circular graph where $|N| = |A| = \alpha$. Suppose $N = \{t_j : 1 \le j \le \alpha\}$ denote the class of all nodes of crisp circular graph $C^* = (N, A)$. Then by theorem 6.4.2 the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ have single weakest arc and $\alpha - 1$ fuzzy bridges exactly. Now it is remain to show that all these $\alpha - 1$ fuzzy bridges of fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ are strong. Let us consider an arc (t_j, t_{j+1}) from these $\alpha - 1$ fuzzy bridges of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ where $j = 1, 2, 3, ...\alpha - 1$ and (t_α, t_1) be the weakest arc of this fuzzy distance two labeling cycle graph. Since $C^* = (N, A)$ is a circular crisp graph then exactly two distinct path exist there which combined the nodes t_j and t_{j+1} (i.e) one path with assignment number $\mu_A^{\psi}(t_j, t_{j+1}) > 0$ and another path with assignment number $\mu_A^{\psi}(t_{j+1}, t_1, t_2, ..., t_j) > 0$ where $j = 1, 2, 3, ...\alpha - 1.$ Obviously the path $(t_{j+1}, t_1, t_2, ..., t_j)$ will consist the weakest arc (t_α, t_1) of the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. Therefore the strength of connectedness for the nodes t_j and t_{j+1} in the fuzzy distance two labeling cycle graph will be equal to the membership value of the arc (t_j, t_{j+1}), this implies that $\mu_A^{\infty}(t_j, t_{j+1}) = \mu_A^{\psi}(t_j, t_{j+1})$. Hence (t_j, t_{j+1}) arc in the fuzzy graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ represent a strong arc of the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$. By repeating this process for the remaining arcs we get every arc in $\alpha - 1$ fuzzy bridges of $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ are strong arcs. Hence every bridge in the fuzzy distance two labeling cycle graph $F(C)^{\psi} = (\sigma_N^{\psi}, \mu_A^{\psi})$ is strong.

VI. CONCLUSION

Fuzzy graphs play an important role in many areas, including decision-making, computer networking and management sciences. A fuzzy distance two labeling graph is a generalization of the fuzzy graph. The main purpose of this research article is to explore the role of fuzzy distance two labeling graph in Computer Science field. Fuzzy distance labeling graphs is a powerful tool that makes things ease in various field of computer science as said above. The fuzzy distance two labeling graphs give more precision, flexibility and compatibility to the system when more than one agreements are to be dealt with. The fuzzy distance two labeling graph is useful for handling various interesting problems like traffic light problem, networking problems and job allocation problem etc.

It is known that graphs are actually the natural models of relations. A graph is an standard process to describe data involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness or uncertainty in illustration of objects or in its relationships or both, we require to construct a fuzzy graph model. The real world problems often involve distance restrictions, unmatched assignment and multi-objects. Then fuzzy distance two labeling graphs give more accuracy and precision as compared to fuzzy graphs.

In this research article, a new idea of fuzzy distance two labeling graphs has been introduced with sufficient illustrations. Some important properties on fuzzy distance two labeling circular graph and the relation between product fuzzy graphs and fuzzy distance two labeling graphs have been discussed.

We are extending our research work to (1) operations on fuzzy distance two labeling graphs (2) complement of fuzzy distance two labeling graphs.

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