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Path Related Balanced Divided Square Difference Cordial Graphs

A.Alfred Leo[1]*, R. Vikramaprasad[2]

[1] Research and Development Centre, Bharathiar University, Coimbatore, India
[2]Department of Mathematics, Government Arts College, Salem, India

*Corresponding author:lee.ancy1@gmail.com,Tel: 09865834054

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***Abstract*—**In this article, we have investigated the balanced divided square difference cordial behavior of some path related graphs such as fan graph, half gear graph, triangular snake, double triangular snake, alternate triangular snake, $V_D(P_n)$.

***Keywords*—**fan graph, half gear graph, triangular snake, double triangular snake, alternate triangular snake.

I. INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiple edges. For standard terms we refer to Harary [7]. In 1967, Rosa [9] introduced a labeling of G called β-valuation. A dynamic survey on different graph labeling along with an extensive bibliography was found in Gallian [6]. The motivation behind this work is due to Dhavaseelan et.al [5] who introduced the concept of even sum cordial labeling graphs. In this article, we have investigated the balanced divided square difference cordial behaviour of some path related graphs such as fan graph, half gear graph, triangular snake, double triangular snake, alternate triangular snake,$V_D(P_n)$.

The article is organized as follows, section I gives the introduction of the article, section II contain the related works in cordial labeling, section III gives the preliminaries required for the main results, section IV contains the main results and section V concludes the research work with future directions.

II. RELATED WORK

The concept of cordial labeling was introduced by Cahit [3]. A.Alfred Leo et.al [1] introduced the concept of divided square difference cordial labeling graphs. V.J.Kaneria et.al [8] introduced the concept of balanced cordial labeling. A.Alfred Leo et.al [2] introduced the concept of balanced divided square difference cordial graphs. R.Varatharajan, et.al [11] introduced the notion of divisor cordial labeling. S.S.Sandhya et.al [10] has discussed the root square mean graphs of triangular snake and double triangular snake. S.N.Daoud et.al discussed edge odd graceful labeling of fan graph and half gear graph in [4].

III. PRELIMINARIES

Definition 3.1 [6]

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition. If the domain of the mapping is the set of vertices then the labeling is called *vertex labeling*.

Definition 3.2 [6]

A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(V)$ is called the label of the vertex v of G under f.

Definition 3.3 [3]

A binary vertex labeling f of a graph G is called a C*ordial labeling* if $\left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 3.4 [4]

A *fan graph* $F_{m,n}$ is defined as the join of two graphs $\overline{K_m} + P_n$ where $\overline{K_m}$ is the empty set on m vertices and P_n is the path graph on n vertices. When m = 1, $F_{m,n}$ is usual fan graph and when m = 2, $F_{m,n}$ is double fan graph. The new edges are called the spokes of the fan.

Definition 3.5 [4]

The half gear graph HG_n is a graph obtained from the fan graph F_n by inserting a vertex between any two adjacent vertices in its path P_n.

Definition 3.6 [10]

The *triangular snake* T_n is obtained from the path $v_1 v_2 v_3 \dots v_n$ by joining v_i and v_{i+1} to a new vertex u_i for $1 \leq i \leq n-1$. Therefore we will get $n-1$ triangles C_3.

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Definition 3.7 [10]

The *double triangular snake* $D(T_n)$ consists of two triangular snakes that have a common path. ie) obtained from the path $v_1v_2v_3 \dots v_n$ by joining v_i and v_{i+1} to a new vertices u_i and w_i for $1 \le i \le n-1$. Therefore we will get $2n-2$ triangles C_3.

Definition 3.8 [10]

The *alternate triangular snake* $A(T_n)$ is obtained from the path $v_1v_2v_3 \dots v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex u_i. That is every alternate edge is replaced by triangle C_3.

Definition 3.9 [7]

The graph $V_D(P_n)$ is obtained from the path $v_1v_2v_3 \dots v_n$ by joining v_1 and v_3 to a new vertex u.

Definition 3.10 [8]

A cordial graph G with a cordial labeling f is called a *balanced cordial graph* if

$$\left|e_f(0)-e_f(1)\right| = \left|v_f(0)-v_f(1)\right| = 0.$$

It is said to be *edge balanced cordial graph* if

$$\left|e_f(0)-e_f(1)\right| = 0 \text{ and } \left|v_f(0)-v_f(1)\right| = 1.$$

Similarly it is said to be *vertex balanced cordial graph* if

$$\left|e_f(0)-e_f(1)\right| = 1 \text{ and } \left|v_f(0)-v_f(1)\right| = 0.$$

A cordial graph G is said to be *unbalanced cordial graph* if

$$\left|e_f(0)-e_f(1)\right| = \left|v_f(0)-v_f(1)\right| = 1.$$

Definition 3.11 [1]

Let $G=(V,E)$ be a simple graph and $f: V \to \{1,2,3,\dots |V|\}$ be bijection. For each edge uv, assign the label 1 if $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ is odd and the label 0 otherwise. f is called divided square difference cordial labeling if $\left|e_f(0)-e_f(1)\right| \le 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1respectively.

A graph G is called divided square difference cordial if it admits divided square difference cordial labeling.

Definition 3.12 [2]

A divided square difference cordial graph G is called a *balanced divided square difference cordial* if $\left|e_f(0)-e_f(1)\right| = 0$.

A divided square difference cordial graph G is called a *unbalanced divided square difference cordial* if $\left|e_f(0)-e_f(1)\right| = 1$.

Proposition 3.13 [1]

Any path P_n is a divided square difference cordial graph.

IV. RESULTS AND DISCUSSION

Proposition 4.1

The fan graph $F_{1,n}$ is a unbalanced divided square difference cordial.

Proof

Let G be a fan graph $F_{1,n}$ with $|V(G)| = n+1$, $|E(G)| = 2n-1$. Let $u_1, u_2, \dots, u_n$ are the vertices of path P_n and w is the vertex of $\overline{K_1}$. We define the labeling $f: V(G) \to \{1,2,\dots,n+1\}$ as follows.

First we draw the path P_n by Proposition 3.13. Then label the vertex w as $f(w) = n+1$ and join w to P_n.

Thus, we get $\left|e_f(0)-e_f(1)\right| \le 1$.

In particular, we get $\left|e_f(0)-e_f(1)\right| = 1$. Hence G is a unbalanced divided square difference cordial graph.

Example 4.2

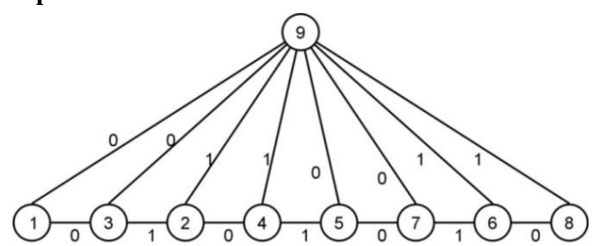

Fig 1. *Fan graph* $F_{1,8}$

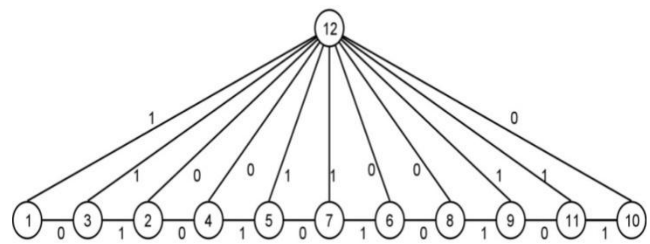

Fig 2. *Fan graph* $F_{1,11}$

Proposition 4.3

The half gear graph HG_n is a balanced divided square difference cordial when n is even.

Proof

Let G be a half gear graph HG_n with $|V(G)| = 2n$ and $|E(G)| = 3n-2$. Now we define the labeling $f: V(G) \to \{1,2,\dots,2n\}$ as follows.

First we can construct the fan graph $F_{1,n}$ by Proposition 4.1. Then insert a vertex between any two adjacent vertices of the path P_n and the new vertices are $v_1, v_2, \dots, v_{n-1}$. Label the new vertices by taking $f(v_i) = n+i+1, 1 \le i \le n-1$.

Thus, we get $\left|e_f(0)-e_f(1)\right| \le 1$. In particular, when n is even we get $\left|e_f(0)-e_f(1)\right| = 0$ and when n is odd we get $\left|e_f(0)-e_f(1)\right| = 1$.

Hence G is a unbalanced divided square difference cordial graph when n is odd and balanced divided square difference cordial graph when n is even.

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Example 4.4

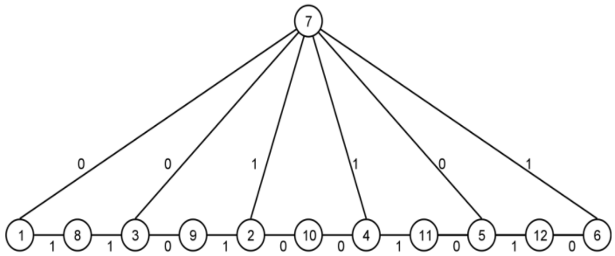

Fig 3. Half gear graph HG_6

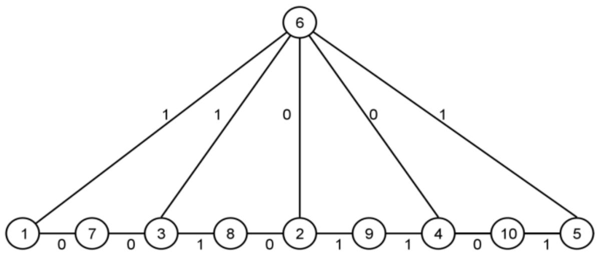

Fig 4. Half gear graph HG_5

Proposition 4.5

The double fan graph $F_{2,n}$ is a balanced divided square difference cordial when n is odd.

Proof

Let G be a double fan graph $F_{2,n}$ with $|V(G)| = n + 2$, $|E(G)| = 3n - 1$. Let $u_1, u_2, \dots, u_n$ are the vertices of path P_n and x, y are the vertices of $\overline{K_2}$. Join the vertices of the path P_n to the vertices x and y to get the double fan graph.

Now we define a map $f: V(G) \to \{1,2, \dots, n + 2\}$ as follows.

First we can label the path P_n by Proposition 3.13. Then label the vertices x and y by taking $f(x) = n + 1$ and $f(y) = n + 2$. Then, we get $\left|e_f(0) - e_f(1)\right| \le 1$. In particular, we get $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd and $\left|e_f(0) - e_f(1)\right| = 1$when n is even.Hence G is a unbalanced divided square difference cordial when n is even and balanced divided square difference cordial when n is odd.

Example 4.6

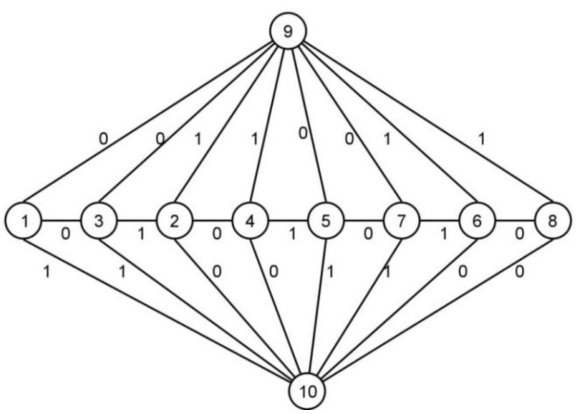

Fig 5. Double fan graph $F_{2,8}$

Proposition 4.7

The triangular snake graph T_n $(n \not\equiv 3 \bmod 4)$ is a balanced divided square difference cordial when n is odd.

Proof

Let G be a triangular snake graph T_n with$|V(G)| = 2n - 1$ and $|E(G)| = 3(n - 1)$.

Let $v_1, v_2, \dots, v_n$ are the vertices of the path P_n and let $u_1, u_2, \dots, u_{n-1}$ are the vertices joined to $v_1v_2, v_2v_3 \dots, v_{n-1}v_n$ respectively. Now, we define the label $f: V(G) \to \{1,2, \dots ,2n - 1\}$ as follows.

First, we can construct the path P_n by Proposition 3.13, then assign label values for the vertices $u_1, u_2, \dots, u_{n-1}$ by taking $f(u_i) = n + i, 1 \le i \le n - 1$.

Thus, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

In particular, $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd and $\left|e_f(0) - e_f(1)\right| = 1$when n is even.

Hence G is a unbalanced divided square difference cordial when n is even and balanced divided square difference cordial when n is odd.

Example 4.8

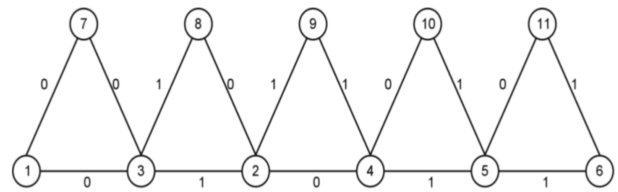

Fig 6. Triangular snake graph T_6

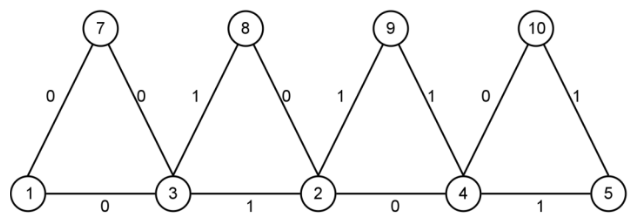

Fig 7. Triangular snake graph T_5

Proposition 4.9

The double triangular snake graph $D(T_n)(n \not\equiv 3 \bmod 4)$ is a balanced divided square difference cordial when n is odd.

Proof

Let G be a double triangular snake graph $D(T_n)$ with $|V(G)| = 3n - 2$ and $|E(G)| = 5(n - 1)$.

Let $v_1, v_2, \dots, v_n$ are the vertices of the path P_n and let $u_1, u_2, \dots, u_{n-1}, w_1, w_2, \dots, w_{n-1}$ are the vertices joined to $v_1v_2, v_2v_3 \dots, v_{n-1}v_n$ respectively. Now, we define the label $f: V(G) \to \{1,2, \dots ,3n - 2\}$ as follows.

First, we can draw the path P_n by Proposition 3.13, then assign label values for the other vertices by taking $f(u_i) = n + i$, and $f(w_i) = 2n - 1 + i, 1 \le i \le n - 1$.

Thus, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

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In particular, $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd and $\left|e_f(0) - e_f(1)\right| = 1$when n is even.
Hence G is a unbalanced divided square difference cordial when n is even and balanced divided square difference cordial when n is odd.

Example 4.10

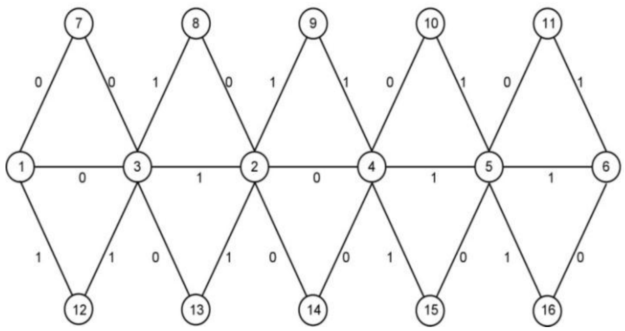

Fig 8. Double triangular snake graph $D(T_6)$

Proposition 4.11

The alternate triangular snake graph $A(T_n)$,$n \equiv 1(mod\ 4)$ is a balanced divided square difference cordial.

Proof

Let G be a alternate triangular snake graph $A(T_n)$, $n \equiv 1(mod\ 4)$ with $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n - 2$.
Let $v_1, v_2, \dots, v_n$ are the vertices of the path P_n and let $u_1, u_2, \dots, u_{\frac{n-1}{2}}$ are the vertices joined to $v_1v_2, v_3v_4 \dots, v_{n-2}v_{n-1}$ respectively. Now, we define the label $f: V(G) \to \left\{1,2, \dots, \frac{3n-1}{2}\right\}$ as follows.
First, we can construct the path P_n by Proposition 3.13, then assign label values for the vertices $u_1, u_2, \dots, u_{\frac{n-1}{2}}$ by taking $f(u_i) = n + i, 1 \le i \le \frac{n-1}{2}$.
Thus, we get $\left|e_f(0) - e_f(1)\right| \le 1$.
In particular, $\left|e_f(0) - e_f(1)\right| = 0$
Hence G is a balanced divided square difference cordial graph.

Example 4.12

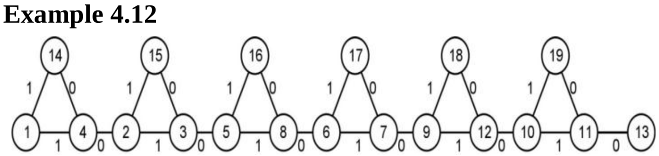

Fig 9. Alternate triangular snake graph $A(T_{13})$

Proposition 4.13

The graph $V_D(P_n), n \ge 4$ is a balanced divided square difference cordial when n is odd.

Proof

Let G be a $V_D(P_n)$ graph with $|V(G)| = n + 1$ and $|E(G)| = n + 1$. Let $u, v_1, v_2, \dots, v_n$ are the vertices of $V_D(P_n)$. Now, we define a map $f: V(G) \to \{1,2, \dots, n + 1\}$ as follows.
We can construct the path P_n by Proposition 3.13, then assign label value for the vertex u by taking $f(u) = n + 1$.
Thus, we get $\left|e_f(0) - e_f(1)\right| \le 1$.
In particular, $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd and $\left|e_f(0) - e_f(1)\right| = 1$ when n is even.
Hence G is a unbalanced divided square difference cordial when n is even and balanced divided square difference cordial when n is odd.

Example 4.14

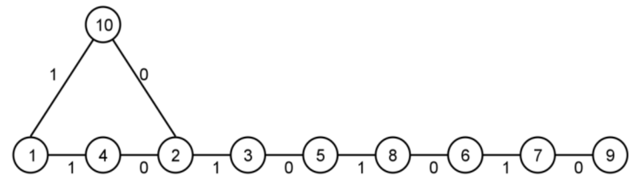

Fig 10. Graph $V_D(P_9)$

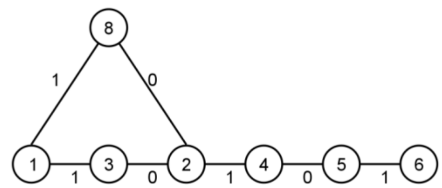

Fig 11. Graph $V_D(P_6)$

V. CONCLUSION

In this article, it is proved that some of the path related graphs such as fan graph, half gear graph, triangular snake, double triangular snake, alternate triangular snake,$V_D(P_n)$are balanced or unbalanced divided square difference cordial. The readers can construct different algorithm for each graph to prove them balanced or unbalanced divided square difference cordial. Also readers can investigate similar results for other graph families.

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Authors Profile

A.Alfred Leo pursed Bachelor of Science and Master of Philosophy in Mathematics from St.Joseph's College, Trichy, Tamil Nadu. He Pursed his Master of Science in Mathematics from Sacred Heart college, Tirupattur, TamilNadu. Now currently he is a research scholar in Bharathiar University, Coimbatore.

Dr.R.Vikramaprasad pursed Bachelor of Science in Mathematics from Govt.Arts College,Salem,Tamil Nadu. He Pursed his Master of Science in Mathematics from National College,Trichy, TamilNadu. He is currently working as a Assistant Professor of Mathematics in Government Arts College, Salem – 7, Tamil Nadu. His main area of research is Graph Theory.

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